5. INTEGRATED THICKNESS, SHAPE AND TOPOLOGY OPTIMIZATION

5.1 GENERAL PERSPECTIVE

Thickness, Shape, and topology optimization are the objectives of structural optimization. Till today, thickness and shape optimization are combined and topology optimization is performed separately. The reason for topology optimization carried separately is it is used as a tool for finding initial optimal shapes. Thickness and shape optimization are tools for detailed final designs. Depending on the practical considerations in the industry, it is recommended to consider the combination of some vital thickness and shape parameters along with topology optimization.
Thickness/Shape optimization and Topology optimization dealt separately produce improved new designs. However, integration of Thickness, Shape and Topology optimization offer more efficient tool to be used. This was illustrated with inverse model and cylindrical shell examples.

5.2 INTEGRATED THICKNESS, SHAPE AND TOPOLOGY OPTIMIZATION-THEORY

Past research considered these optimization techniques separately, seeking an initial optimal material layout and then refinement of the shape. The method used in this work combines both optimization techniques, where the shape/thickness of the shell structure and material distribution are optimized simultaneously. The aim of combining the optimization modules is to arrive at the optimum design that maximizes the stiffness of the shell statically and dynamically.

In the past, the automated integration of topology and shape optimization is rarely been considered and very few authors attempted topology and shape optimization in the sequence.

In this section an automated approach for simultaneous Thickness/shape and topology optimization of shell structures is presented. For attaining maximum stiffness of the shell, both optimization techniques are combined and are optimized simultaneously. This formulation involves a variable ground structure for topology optimization, since the shape of the shell is modified later. The method
has been implemented into a mathematical model and the feasibility is demonstrated using several examples.

5.3 ALGORITHM FOR INTEGRATED THICKNESS/SHAPE AND TOPOLOGY OPTIMIZATION

In the present study, an integrated approach for structural thickness/shape optimization and topology design is proposed to produce an optimal structural design. The architecture of the proposed structural optimization system is given in Fig. 5.1.

The procedure used consists of three stages as follows:

**Stage 1 Shape Optimization**

In this stage the problem is defined in terms of the finite element model of the material properties, the loads and the objective function. The constraints are selected from a set of predefined constraints. Thickness/Shape optimization is performed to arrive at the optimized shape.

The following steps are carried out in this stage.

- Define the initial Model, discretize the model with finite elements, define material properties, Define the loads and boundary conditions
- Perform the analysis
- Define the design variables
- Define the objective and constraint functions
- Specify the optimization method
Obtain the optimum model with respect to Thickness/Shape

**Stage 2 Investigation of Shape optimization results to integrate**

**Thickness/shape and topology optimization**

In this stage the optimum model obtained in the first phase is checked for various conditions i.e deflections, stresses are within allowable limits.

**Stage 3 Topology optimization**

In the last stage topology optimization is performed on the thickness/shape optimized models to obtain the final layout of the shell models.

The following are the steps in this phase.

- Select proper element type
- Specify optimized and non-optimized regions
- Define the loads and boundary conditions
- Define the objective and constraint functions
- Control the optimization process

The architecture of the integrated thickness/shape and topology optimization is shown in figure 5.1.

**5.4 ILLUSTRATIVE EXAMPLES**

**5.4.1 Example-1 -Inverse models**

**5.4.1.1 Problem definition**

The same example inverse model generated from a flat plate with a central concentrated load is considered as in the previous chapters. Initially and the deflection profile is inverted by 180° to get the deflection
free inverse model for a specified loading. Free vibration analysis is carried out on the inverse model Block Lanczo’s method in ANSYS software. The support conditions considered are

- fixed corners
- simply supported corners
- fixed Edges
- simply supported edges

**5.4.1.2 Objective functions**

In this chapter shape and topology optimization are integrated to achieve the optimum models. The two cases of objective functions for which optimization is carried out are

**Case 1:** Maximization of static stiffness can be achieved by minimization of structural volume, First thickness optimization is performed and the shell model is topologically optimized. The solution approach used for minimum volume problem is optimality criteria approach, which is by default in ANSYS topology optimization module.

**Case 2:** Maximization of Dynamic stiffness can be achieved by minimizing the weighted frequency (for first five frequencies) with a constraint that total volume of the shell should be reduced to 50% of the initial volume. Prior to that shape optimization is performed with thickness as the design variable. The solution approach used for minimum weighted frequency problem is sequential convex programming approach (SCP).
5.4.1.3 Initial Geometry

Taking the symmetry of the structure as an advantage, a quarter portion of the shell has been modeled by applying symmetry boundary conditions in ANSYS. Initially quarter of the flat plate (5m x 5m) is modeled using 4 key points. It is discretized into number of finite elements using 4-noded shell93 elements. Material properties are considered as that of isotropic steel. A 10 kN concentrated load is applied at the geometric centre of the plate.

5.4.1.4 Discussion of Results

5.4.1.4.1 Corners fixed

In the case of inverse model fixed on four corners

Case 1: With an objective function of minimizing the strain energy, shape optimization is performed with the thickness as design variable and the thickness values at four corners of the plate are obtained. The thickness optimized model is then subjected to topology optimization with an objective of minimizing the structural compliance or strain energy with a volume reduction of 50%, the structural compliance was reduced from an initial value of 28146.2 to a final value of 7173.8 in 8 iterations. The percentage reduction being 74.51%.

Case 2: With an objective function of maximizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of weighted frequency was 0.1206 and it was reduced to 0.1205 with a
percentage reduction of only 0.1% in 40 iterations. Thus the fundamental frequency which was 0.1205 without any volume change was reduced (as there is a volume reduction of 50%) to 0.1189. Much improvement is obtained from the integrated optimization.

4.1.4.2 Corners simply supported

In the case of inverse model simply supported on four corners

Case 1: Shape optimization is performed minimizing the strain energy and thickness as design variable and the optimum thickness values at four corners of the plate are obtained. The thickness optimized model is then subjected to topology optimization with an objective of minimizing the structural compliance or strain energy with a volume reduction of 50%, the structural compliance was reduced from an initial value of $0.2195 \times 10^7$ to a final value of $0.0738 \times 10^7$ in 26 iterations. The percentage reduction being 66.38%.

Case 2: With an objective function of minimizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of weighted frequency was 0.2035 and it was reduced to 0.1888 with a percentage reduction of only 7.21% in 18 iterations. Thus the fundamental frequency is increased from the initial value of $1.6222 \times 10^{-2}$ to $1.8841 \times 10^{-2}$

5.4.1.4.3 Edges fixed

In the case of inverse model fixed on all edges
Case 1: Shape optimization is performed minimizing the strain energy and thickness as design variable and the optimum thickness values at four corners of the plate are obtained. Topology optimization was performed with an objective of minimizing the structural compliance or strain energy with a volume reduction of 50%, the structural compliance was reduced from an initial value of 99674.6 to 33577.1 in 12 iterations. The percentage reduction was 66.31%.

Case 2: With an objective function of maximizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of weighted frequency was 0.9053 and it was reduced to 0.7557 (since there is a 50% percent volume reduction) in 25 iterations.

5.4.1.4.4 Edges simply supported

In the case of inverse model simply supported on all edges

Case 1: With an objective function of minimizing the strain energy, shape optimization is performed with the thickness as design variable and the thickness values at four corners of the plate are obtained. Topology optimization with an objective of minimizing the structural compliance or strain energy is performed on the shape optimized model with a volume reduction of 50%, in 32 iterations the structural compliance was reduced from an initial value of 174280 to a final value of 90667. The percentage reduction being 47.98%.
Case 2: With an objective function of maximizing the weighted frequency with a constraint on volume reduction by 50%, the optimized value of weighted frequency was 0.5740. Thus the fundamental frequency is increased from the initial value of 0.10252 to 0.1271 which shows an increase of 23.96% even for a volume reduction of 50%.

The density plots and iteration histories are presented for the two cases of objective function in figures 5.2-5.5.

For case 2 the initial and optimized values of fundamental frequencies are presented in table 5.1.

**5.4.2 Example-2 -Cylindrical shells**

**5.4.2.1 Problem definition**

The same example of cylindrical shell with various boundary conditions is considered as in the previous chapters. The shell can be supported on the straight edges, on the curved ones or on both of them at the same time. Thickness and Topology optimization of the shell are carried out, under two different objective functions.

**Case 1:** Maximization of static stiffness can be achieved by minimization of structural volume. Shape optimization is performed and in topology optimization the constraint is on the structural compliance of the structure, which should be increased to 50% of the initial value. The solution approach used for minimum compliance problem is optimality criteria approach, which is by default in ANSYS topology optimization module.
**Case 2:** Maximization of Dynamic stiffness can be achieved by maximizing the weighted frequency (for first five frequencies) with a constraint that volume of the structure should be reduced to half of the initial volume. The solution approach used for minimum weighted frequency problem is sequential convex programming approach (SCP).

**5.4.2.2 Initial Geometry**

In the present analysis, the shell is modeled in ANSYS using nine key points, two straight lines for the right edges and the rest eight by segmented cubic splines as mentioned in section 3.9. Areas are generated and discretized using shell-93 elements. The rise of the shell structure is 3m. Various boundary conditions considered are

- Right edges supported
- Curved edges supported
- Right and Curved Edges supported

The shell structure is analysed and initial volume is found to be 5.3456m$^3$ for all the cases and the initially fundamental frequencies found from the modal analysis are 0.4269Hz, 0.9816 Hz, 3.2972Hz for straight edges supported, curved edges supported and all the four edges supported respectively.

**5.4.2.3 Results and Discussions**

**5.4.2.3.1 Shell supported on right edges:**

In the case of shell supported on right edges for
Case 1: After integrated shape and topology optimization with an objective function of minimizing the total volume of the structure, the optimized values of shape variables are $S_1=0.0768\, \text{m}$ and $S_2=0.1130\, \text{m}$ and the volume of the structure is $2.4759\, \text{m}^3$, achieved in 8 iterations. The total volume reduction was 53.68%.

Case 2: With an objective function of maximizing the weighted frequency with a constraint on volume reduction by 50%, the optimized values of shape variables are $S_1=0.0543\, \text{m}$ and $S_2=3.8181\, \text{m}$. Initially the value of weighted frequency was 133.339 and it was reduced to 78.099 in 30 iterations. Before optimization the fundamental frequency was 0.4267 and after integrated optimization the value is increased to 0.6940 with a percentage increase of 62.64%.

5.4.2.3.2 Shell supported on curved edges:

In the case of shell supported on curved edges for

Case 1: After integrated shape and topology optimization with an objective function of minimizing the total volume of the structure, the optimized values of shape variables are $S_1=0.034\, \text{m}$ and $S_2=1.6691\, \text{m}$ and the volume of the structure is $2.72855\, \text{m}^3$, achieved in 22 iterations. The total volume reduction was 78.22%.

Case 2: With an objective function of maximizing the weighted frequency with a constraint on volume reduction by 50%, the optimized values of shape variables are $S_1=0.7841\, \text{m}$ and $S_2=3.7201\, \text{m}$. Initially the value of
weighted frequency was 60.4038 and it was reduced to 35.3827. Before optimization the fundamental frequency was 0.97937 and after integrated optimization the value is increased to 1.0468 with a percentage increase of only 6.88 %.

5.4.2.3.3 Shell supported on four edges:

In the case of shell supported on four edges for

Case 1: With an objective function of minimizing the structural volume, initial volume of 5.3456 m$^3$ was reduced to 2.0556 m$^3$ in 22 iterations. The percentage reduction being 61.55 %. The shape variables attained the values as $S1=0.3973$ m and $S2=0.9733$ m.

Case 2: With an objective function of minimizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of weighted frequency was 283.76 and it was reduced to 184.83 with a percentage reduction of 34.86 % in 21 iterations, thus increasing the fundamental frequency from 3.2972 to 3.9364 with a percentage increase of 19.39%. For case 2 the initial and optimized values of fundamental frequencies are presented in table 5.2.

The density plots of topology optimization for case 1 and case 2 for all the boundary conditions are presented in figure 5.6 and figure 5.8 respectively. The iteration histories of case 1 and case 2 for objective function and constraint are presented in figure 5.7 and figure 5.9 respectively.
Table 5.1 Inverse Shell Model Problem: Initial and optimized frequencies of inverse models (with weighted frequency as objective)

<table>
<thead>
<tr>
<th>Support condition</th>
<th>S.no</th>
<th>Initial frequencies (Hz)</th>
<th>Optimized frequencies (Hz) after 50% volume reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corners fixed</td>
<td>1</td>
<td>0.1205</td>
<td>0.1389</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.1739</td>
<td>0.1805</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.2790</td>
<td>0.3324</td>
</tr>
<tr>
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<td>4</td>
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</tr>
<tr>
<td></td>
<td>5</td>
<td>0.3807</td>
<td>0.4252</td>
</tr>
<tr>
<td>Corners simply supported</td>
<td>1</td>
<td>0.0162</td>
<td>0.0188</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0283</td>
<td>0.0191</td>
</tr>
<tr>
<td></td>
<td>3</td>
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<td>0.1266</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.2067</td>
<td>0.1833</td>
</tr>
<tr>
<td>Edges fixed</td>
<td>S.no</td>
<td>Initial frequencies(Hz)</td>
<td>Optimized frequencies(Hz)(with 50% reduction in volume)</td>
</tr>
<tr>
<td>--------------</td>
<td>------</td>
<td>--------------------------</td>
<td>-------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.1733</td>
<td>0.1785</td>
</tr>
<tr>
<td></td>
<td>2</td>
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<tr>
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<td>0.5400</td>
<td>0.5454</td>
</tr>
<tr>
<td>Edges simply supported</td>
<td>1</td>
<td>0.10252</td>
<td>0.1271</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.1754</td>
<td>0.2830</td>
</tr>
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<td>0.3500</td>
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<td></td>
<td>5</td>
<td>0.4966</td>
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Table 5.2 Cylindrical shell Problem: Initial and optimized frequencies of cylindrical shells (with weighted frequency minimization as objective)
<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>2.4098</th>
<th>2.1197</th>
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<tr>
<td></td>
<td>4</td>
<td>2.4181</td>
<td>2.3638</td>
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<td></td>
<td>5</td>
<td>3.4521</td>
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<tr>
<td><strong>All edges simply supported</strong></td>
<td>1</td>
<td>3.2928</td>
<td>3.9364</td>
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<td></td>
<td>2</td>
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<td></td>
<td>4</td>
<td>6.2996</td>
<td>5.3720</td>
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<tr>
<td></td>
<td>5</td>
<td>6.4462</td>
<td>5.7184</td>
</tr>
</tbody>
</table>

**SHAPE OPTIMIZATION**

- Define the initial Design problem
- Perform Design Optimization
- Obtain Optimum Shape/thickness

**TOPOLOGY OPTIMIZATION**

- Define Shape/Thickness Optimization problem
- Perform topology optimization
- Check for various response
- Feature Based model

Optimum layout
Figure 5.1 Architecture of the integrated topology and shape optimization procedure

<table>
<thead>
<tr>
<th>Volume minimization as objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corners Fixed</td>
</tr>
<tr>
<td>Corners simply supported</td>
</tr>
</tbody>
</table>
Edges fixed

Edges simply supported

Figure 5.2 Inverse Shell Model Problem: Density Plots of optimum model for minimum volume as objective using integrated Thickness and topology Optimization

Volume minimization as objective
Corners Fixed | Corners simply supported

| Edges fixed | Edges simply supported |

Figure 5.3 Inverse Shell Model Problem: Iteration History for Set no Vs

Objective Function for minimum volume objective using integrated Thickness and topology Optimization

Weighted frequency minimization as objective
<table>
<thead>
<tr>
<th>Corners Fixed</th>
<th>Corners simply supported</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Corners Fixed" /></td>
<td><img src="image2" alt="Corners simply supported" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Edges fixed</th>
<th>Edges simply supported</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Edges fixed" /></td>
<td><img src="image4" alt="Edges simply supported" /></td>
</tr>
</tbody>
</table>

Figure 5.4 Inverse Shell Model Problem: Density Plots of optimum model for maximum weighted frequency as objective using integrated Thickness and topology Optimization

<p>| Weighted frequency as objective |</p>
<table>
<thead>
<tr>
<th>Corners Fixed</th>
<th>Corners simply supported</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edges fixed</td>
<td>Edges simply supported</td>
</tr>
</tbody>
</table>

Figure 5.5 Inverse Shell Model Problem: Iteration History for Set no Vs

Objective Function for maximum weighted frequency using integrated Thickness and topology Optimization

Volume minimization as objective
Figure 5.6 Cylindrical Shell Problem: Density Plots of optimum model for minimum volume as objective using integrated Shape and topology Optimization

<table>
<thead>
<tr>
<th>Straight edges supported</th>
<th>Curved edges supported</th>
<th>All edges supported</th>
</tr>
</thead>
</table>

Figure 5.7 Cylindrical shell Problem: Iteration History for Set no Vs

Objective Function for minimum volume using integrated Thickness and topology Optimization

<table>
<thead>
<tr>
<th>Straight edges supported</th>
<th>Curved edges supported</th>
<th>All edges supported</th>
</tr>
</thead>
</table>

Weighted frequency as objective
Figure 5.8 Cylindrical Shell Problem: Density Plots of optimum model for minimum weighted frequency as objective using integrated Shape and topology Optimization.

Weighted frequency as objective

Figure 5.9 Cylindrical Shell Problem: Iteration Histories for Set no Vs

Objective Function for maximum weighted frequency as objective using integrated Shape and topology Optimization.

6. CONCLUSIONS