CHAPTER 7

A NEW INTUITIONISTIC FUZZY ENTROPY MEASURE AND ITS APPLICATION IN MULTI-CRITERIA DECISION MAKING

7.1. INTRODUCTION

The process of decision making involves three types of characteristics, namely uncertainty, complexity and credibility. Entropy is an effective tool to quantify the degree of uncertainty in an intuitionistic fuzzy environment. Burillo and Bustince [74] extended the concept of fuzzy to intuitionistic fuzzy settings and firstly proposed the intuitionistic fuzzy entropy, which measures the uncertainty attached with the IFS. Thereafter, many authors have shown their interest in defining a wide range of intuitionistic fuzzy entropy measures from different aspects of applications.

In this chapter, the concept of entropy is extended from fuzzy sets to intuitionistic fuzzy sets and proposes a new entropy measure for IFS and studies some of its important properties. The findings of the proposed measure are applied to the decision making problem in mobile phone selection through a numerical example and experimental results demonstrate the feasibility, efficiency and validity of the proposed measures.

7.2. A NEW INTUITIONISTIC FUZZY ENTROPY MEASURE

Let $A$ be an intuitionistic fuzzy set in universe of discourse $X = \{x_i; i = 1, 2, \ldots, n\}$ and represent the family of all Atanassov intuitionistic fuzzy sets in $X$ by $AIFS(X)$. We propose a new measure of fuzzy entropy for the Atanassov intuitionistic fuzzy set $A$ based on the exponential function is given by

$$E(A) = \frac{1}{n(e-1)} \sum_{i=1}^{n} \left[ e(1 - \mu_A(x_i) - \nu_A(x_i)) - 1 + \mu_A(x_i)e^{1-\mu_A(x_i)+\nu_A(x_i)} + \nu_A(x_i)e^{1-\nu_A(x_i)+\mu_A(x_i)} \right]$$

(7.1)
The intuitionistic fuzzy entropy measure $E(A)$ satisfies the following axioms as given by Szmidt and Kacprzyk [115]:

A1. $E(A) = 0$ (minimum), if and only if $A$ is a crisp set, i.e., $\mu_A(x_i) = 0, \nu_A(x_i) = 1$ or $\nu_A(x_i) = 0, \mu_A(x_i) = 1$ for all $x_i \in X$.

A2. $E(A) = 1$ (maximum), if and only if $\mu_A(x_i) = \nu_A(x_i)$ for all $x_i \in X$.

A3. $E(A) \leq E(B)$ if and only if $A \subseteq B$, i.e., $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for $\mu_B(x_i) \leq \nu_B(x_i)$ or $\mu_A(x_i) \geq \mu_B(x_i)$ and $\nu_A(x_i) \leq \nu_B(x_i)$ for $\mu_B(x_i) \geq \nu_B(x_i)$ for all $x_i \in X$.

A4. $E(A) = E(A^C)$.

Proof.

A1. Let $A$ be a crisp set with membership value being either 0 or 1, i.e., $\mu_A(x_i) = 0, \nu_A(x_i) = 1$ or $\nu_A(x_i) = 0, \mu_A(x_i) = 1$ for all $x_i \in X$.

Proof. Let us assume $A$ be a crisp set, then $E(A) = 0$.

Now, let

$$E(A) = 0$$

$$\Rightarrow \frac{1}{n(e-1)} \sum_{i=1}^{n} \left[ e(1 - \mu_A(x_i) - \nu_A(x_i)) - 1 + \mu_A(x_i) e^{1 - \mu_A(x_i) + \nu_A(x_i)} + \nu_A(x_i) e^{1 - \nu_A(x_i) + \mu_A(x_i)} \right] = 0$$

$$\Rightarrow e(1 - \mu_A(x_i) - \nu_A(x_i)) - 1 + \mu_A(x_i) e^{1 - \mu_A(x_i) + \nu_A(x_i)} + \nu_A(x_i) e^{1 - \nu_A(x_i) + \mu_A(x_i)} = 0$$

$$\Rightarrow \mu_A(x_i) \left( \frac{e^{\nu_A(x_i)} - e^{\mu_A(x_i)}}{e^{\mu_A(x_i)}} \right) + \nu_A(x_i) \left( \frac{e^{\mu_A(x_i)} - e^{\nu_A(x_i)}}{e^{\nu_A(x_i)}} \right) = \frac{1-e}{e}$$

$$\Rightarrow \mu_A(x_i) \left( \frac{1 - e^{\mu_A(x_i) - \nu_A(x_i)}}{e^{\mu_A(x_i) - \nu_A(x_i)}} \right) + \nu_A(x_i) \left( \frac{1 - e^{\nu_A(x_i) - \mu_A(x_i)}}{e^{\nu_A(x_i) - \mu_A(x_i)}} \right) = \frac{1-e}{e}$$

$$\Rightarrow \phi(\mu_A(x_i), \nu_A(x_i)) + \phi(\nu_A(x_i), \mu_A(x_i)) = \phi(1,0)$$

$$\Rightarrow \phi(\nu_A(x_i), \mu_A(x_i)) + \phi(\mu_A(x_i), \nu_A(x_i)) = \phi(0,1)$$

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Therefore, \( E(A) = 0 \) if and only if \( A \) is a crisp set.

**A2.** Let \( \mu_A(x_i) = \nu_A(x_i) \) for all \( x_i \in X \). Then from (7.1), we have \( E(A) = 1 \)

Now, suppose that

\[
E(A) = 1,
\]

\[
\Rightarrow \frac{1}{n(e-1)} \sum_{i=1}^{n} \left[ e(1 - \mu_A(x_i) - \nu_A(x_i)) - 1 + \mu_A(x_i)e^{1-\mu_A(x_i)+\nu_A(x_i)} + \nu_A(x_i)e^{1-\nu_A(x_i)+\mu_A(x_i)} \right] = 1
\]

\[
\Rightarrow \mu_A(x_i)e^{1-\mu_A(x_i)+\nu_A(x_i)} + \nu_A(x_i)e^{1-\nu_A(x_i)+\mu_A(x_i)} = e(\mu_A(x_i) + \nu_A(x_i))
\]

\[
\Rightarrow \mu_A(x_i)e^{-\mu_A(x_i)+\nu_A(x_i)} + \nu_A(x_i)e^{-\nu_A(x_i)+\mu_A(x_i)} = (\mu_A(x_i) + \nu_A(x_i))
\]

(7.2)

Let us consider

\[
\mu_A(x_i) - \nu_A(x_i) = w_1
\]

(7.3)

and

\[
\mu_A(x_i) + \nu_A(x_i) = w_2
\]

(7.4)

From equation (7.2), we have

\[
\left( \frac{w_1 + w_2}{2} \right)e^{-w_1} + \left( \frac{w_2 - w_1}{2} \right)e^{w_1} = w_2
\]

\[
\Rightarrow w_1(e^{-w_1} - e^{w_1}) + w_2(e^{-w_1} + e^{w_1} - 2) = 0
\]

(7.5)

Now, substitute \( e^{-w_1} = z \) in equation (7.5), we have

\[
w_1 \left( z - \frac{1}{z} \right) + w_2 \left( z + \frac{1}{z} - 2 \right) = 0
\]

\[
\Rightarrow \frac{z-1}{z} \left( w_1 (z + 1) + w_2(z - 1) \right) = 0
\]

\[
\Rightarrow \frac{z-1}{z} \left( z(w_1 + w_2) + (w_1 - w_2) \right) = 0
\]
\[ \Rightarrow \frac{e^{-w_1} - 1}{e^{-w_1}} \left( e^{-w_1} (w_1 + w_2) + (w_1 - w_2) \right) = 0 \]

\[ \Rightarrow \frac{e^{v_A(x_i) - \mu_A(x_i)}}{e^{v_A(x_i) - \mu_A(x_i)}} \left( 2\mu_A(x_i)e^{v_A(x_i) - \mu_A(x_i)} - 2\nu_A(x_i) \right) = 0 \]

\[ \Rightarrow e^{v_A(x_i) - \mu_A(x_i)} - 1 = 0 \text{ and } 2\mu_A(x_i)e^{v_A(x_i) - \mu_A(x_i)} - 2\nu_A(x_i) = 0 \]

\[ \Rightarrow e^{v_A(x_i) - \mu_A(x_i)} - 1 = 0 \text{ and } \mu_A(x_i)e^{-\mu_A(x_i)} = \nu_A(x_i)e^{-v_A(x_i)} \]

\[ \Rightarrow \mu_A(x_i) = \nu_A(x_i) \text{ and } \mu_A(x_i) = \nu_A(x_i) \]

(Using the fact that \( f(x) = xe^{-x} \) is a bijection)

\[ \Rightarrow \mu_A(x_i) = \nu_A(x_i). \]

Therefore, \( E(A) = 1 \) attains the maximum value when \( \mu_A(x_i) = \nu_A(x_i) \) for all \( x_i \in X \).

**A3.** In order to show that (7.1) accomplish the requirement of A3, it is necessary to prove that the functions

\[ L(x, y) = \frac{1}{(e-1)} \left[ e(1-x-y)-1+xe^{1-x+y}+ye^{1-y+x} \right] \quad (7.6) \]

where \( x, y \in [0,1] \), is increasing with respect to first argument \( x \) and decreasing for \( y \). Take the partial derivatives of \( L \) with respect to \( x \) and \( y \), respectively, gives

\[ \frac{\partial L(x, y)}{\partial x} = (1-x)e^{1-x+y} + ye^{1-y+x} - e \quad (7.7) \]

\[ \frac{\partial L(x, y)}{\partial y} = xe^{1-x+y} + (1-y)e^{1-y+x} - e \quad (7.8) \]

To obtain the critical points of \( L \), we set

\[ \frac{\partial L(x, y)}{\partial x} = 0, \quad \frac{\partial L(x, y)}{\partial y} = 0 \quad (7.9) \]

From equation (7.7) and (7.8), we have
(1 - x)e^{1-x+y} + ye^{1-y+x} = e \quad (7.10)

xe^{1-x+y} + (1 - y)e^{1-y+x} = e \quad (7.11)

On adding (7.10) and (7.11), we get

\[ e^{-x+y} + e^{-y+x} = 2 \quad (7.12) \]

Substitute \( y - x = \theta \) in (7.12), we have

\[ e^\theta + e^{-\theta} = 2 \quad (7.13) \]

Now, let \( e^\theta = z \), then from the equation (7.13), we have

\[
\frac{1}{z} + z = 2
\]

\[ \Rightarrow (z - 1)^2 = 0 \]

\[ \Rightarrow z = 1 \]

\[ \Rightarrow e^\theta = 1 \]

\[ \Rightarrow e^{y-x} = e^0 \]

\[ \Rightarrow x = y \]

Therefore, we have

\[
\frac{\partial L(x, y)}{\partial x} \geq 0, \quad \text{for } x \leq y
\]

and

\[
\frac{\partial L(x, y)}{\partial x} \leq 0, \quad \text{for } x \geq y
\]

For any \( x, y \in [0,1] \), \( L(x, y) \) is increasing with respect to \( x \) for \( x \leq y \) and decreasing when \( x \geq y \).

Similarly, we obtain that

\[
\frac{\partial L(x, y)}{\partial y} \leq 0, \quad \text{for } x \leq y
\]
and

\[ \frac{\partial L(x, y)}{\partial y} \geq 0, \quad \text{for } x \geq y \]

Now, consider two sets \( A, B \in AIFS(X) \) with \( A \subseteq B \). Let \( X \) be a finite universe of discourse is partitioned into two disjoint sets \( X_1 \) and \( X_2 \) with \( X_1 \cup X_2 = X \).

Further, we assume all \( x_i \in X_1 \) being dominated by

\[ \mu_A(x_i) \leq \mu_B(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i), \]

while holds for all \( x_i \in X_2 \),

\[ \mu_A(x_i) \geq \mu_B(x_i) \geq \nu_B(x_i) \geq \nu_A(x_i). \]

Then \( E(A) \leq E(B) \) immediately holds from the monotonicity of \( L(x, y) \) and from (7.1), gives \( E(A) \leq E(B) \) when \( A \subseteq B \).

A4. Considering \( A^C = \{ (x, \nu_A(x_i), \mu_A(x_i)) \mid x \in X \} \) for all \( x_i \in X \), i.e., \( \mu_{A^C}(x_i) = \nu_A(x_i) \) and \( \nu_{A^C}(x_i) = \mu_A(x_i) \) in (7.1), we get

\[ E(A) = E(A^C). \]

The above axioms A1- A4 have shown that the proposed measure satisfies all the properties given by Szmidt and Kacprzyk [115]. Therefore, the proposed measure \( E(A) \) is a valid measure of intuitionistic fuzzy entropy.

### 7.2.1. NUMERICAL EXAMPLES

In this section, we will demonstrate the effectiveness and feasibility of the proposed intuitionistic fuzzy entropy measure through a numerical example and compare it with the some existing entropy measures as discussed in chapter 1. As recalling from first chapter, some of the IFE measures are listed below.

Burillo and Bustince [74] intuitionistic fuzzy entropy

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\[ E_{BB}(A) = \frac{1}{n} \sum_{i=1}^{n} \pi_A(x_i). \]

Zhang and Jiang [118] intuitionistic fuzzy entropy

\[ E_{ZJ1}(A) = -\frac{1}{n} \sum_{i=1}^{n} \left( \frac{\mu_A(x_i) + 1 - \nu_A(x_i)}{2} \log \left( \frac{\mu_A(x_i) + 1 - \nu_A(x_i)}{2} \right) + \frac{\nu_A(x_i) + 1 - \mu_A(x_i)}{2} \log \left( \frac{\nu_A(x_i) + 1 - \mu_A(x_i)}{2} \right) \right), \]

Vlachos and Sergiadis [44] intuitionistic fuzzy entropy

\[ E_{VS}(A) = -\frac{1}{n} \sum_{i=1}^{n} \left[ \mu_A(x_i) \log \mu_A(x_i) + \nu_A(x_i) \log \nu_A(x_i) - (1 - \pi_A(x_i)) \log (1 - \pi_A(x_i)) - \pi_A(x_i) \right]. \]

Wei et al. [120] intuitionistic fuzzy entropy

\[ E_{W}(A) = \frac{1}{n} \sum_{i=1}^{n} \left[ \sqrt{2} \cos \left( \frac{\mu_A(x_i) - \nu_A(x_i)}{4} \pi - 1 \right) \times \frac{1}{\sqrt{2} - 1} \right]. \]

Next, we evaluate the above entropy measures and proposed entropy for following two examples to check the validity of the entropy measures for IFS.

Example 7.1: Let us calculate the entropy for the following intuitionistic fuzzy sets:

\[ I_1 = \langle 0.2, 0.5 \rangle, I_2 = \langle 0.3, 0.5 \rangle \text{ and } I_3 = \langle 0.5, 0.5 \rangle \]

Calculate the values of \(I_1, I_2\) and \(I_3\) for the measures \(E_{VS}(A), E_{ZJ1}(A), E_{W}(A), E_{BB}(A)\) and the proposed measure \(E(A)\) presented in Table 7.1.

**Table 7.1:** Comparison of the different intuitionistic fuzzy entropy measures under \(I_1, I_2\) and \(I_3\)

<table>
<thead>
<tr>
<th>IFS</th>
<th>(E(A))</th>
<th>(E_{VS}(A))</th>
<th>(E_{ZJ1}(A))</th>
<th>(E_{W}(A))</th>
<th>(E_{BB}(A))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_1)</td>
<td>0.9056</td>
<td>0.9042</td>
<td>1.201993</td>
<td>0.905665</td>
<td>0.3</td>
</tr>
<tr>
<td>(I_2)</td>
<td>0.9616</td>
<td>0.9635</td>
<td>1.087943</td>
<td>0.957965</td>
<td>0.2</td>
</tr>
<tr>
<td>(I_3)</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Table 7.1 shows that closer the membership degree $\mu_A(x_i)$ to the non-membership degree $\nu_A(x_i)$, the greater is the IFE and when $\mu_A(x_i) = \nu_A(x_i)$, i.e., $I_3 = \langle 0.5, 0.5 \rangle$, then the IFE attains the maximum entropy. It can be seen from the Table 7.1, all the measures except $E_{BB}(A)$ attains the maximum.

**Example 7.2.** Let $A = \{ (x_i, \mu_A(x_i), \nu_A(x_i)) | x_i \in X \}$ be an IFS on finite universe of discourse $X = \{ x_1, x_2, \ldots, x_n \}$. For any positive real numbers, De et al. [52] defined the IFS, $A^n$ as follows:

$$A^n = \{ (x_i, [\mu_A(x_i)]^n, 1- [1-\nu_A(x_i)]^n) | x_i \in X \}. \quad (7.14)$$

Using the operation as defined in (7.14), De et al. [52] also introduced the concentration and dilation of $A$ given by

Concentration: \quad CON(A) = A^2, \\

Dilation: \quad DIL(A) = A^{1/2}.

Like FS, $CON(A)$ and $DIL(A)$ may be considered as “very($A$)” and “more or less($A$)” respectively.

Consider the IFS on $X = \{ x_1, x_2, \ldots, x_n \}$ as taken by De et al. [52]:

$$A = \{ \langle 6, 0.1, 0.8 \rangle, \langle 7, 0.3, 0.5 \rangle, \langle 8, 0.5, 0.4 \rangle, \langle 9, 0.9, 0.0 \rangle, \langle 10, 1.0, 0.0 \rangle \}. \quad (7.15)$$

Utilizing the operation proposed in (7.14), we can generate the following IFS:

$$A^{1/2}, A^2, A^3, A^4.$$  

The above notations are illustrated as follows, we have

$A^{1/2}$ may be considered as “More or less LARGE”,

$A^2$ may be considered as “Very LARGE”,

$A^3$ may be considered as “Quite very LARGE”,

$A^4$ may be considered as “Very very LARGE”.

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We use these IFS to compare the entropy measures $E_{VS}(A), E_{ZJ1}(A), E_{W}(A), E_{BB}(A)$ and $E(A)$ respectively. From the mathematical operations viewpoint, the entropy measures of these IFS need to follow the following requirement:

$$E(A^{1/2}) > E(A) > E(A^2) > E(A^3) > E(A^4).$$  \hspace{1cm} (7.16)

Compute the values for the IFS given in (7.15) for the measures $E_{VS}(A), E_{ZJ1}(A), E_{W}(A), E_{BB}(A)$ and the proposed measure $E(A)$. Comparisons are presented in Table 7.2.

**Table 7.2:** Comparison of the fuzziness with different entropy measures under $A$

<table>
<thead>
<tr>
<th>IFS</th>
<th>$E(A)$</th>
<th>$E_{VS}(A)$</th>
<th>$E_{ZJ1}(A)$</th>
<th>$E_{W}(A)$</th>
<th>$E_{BB}(A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^{1/2}$</td>
<td>0.5525</td>
<td>0.507</td>
<td>0.582</td>
<td>0.455</td>
<td>0.092</td>
</tr>
<tr>
<td>$A$</td>
<td>0.5263</td>
<td>0.493</td>
<td>0.572</td>
<td>0.438</td>
<td>0.12</td>
</tr>
<tr>
<td>$A^2$</td>
<td>0.3608</td>
<td>0.375</td>
<td>0.433</td>
<td>0.303</td>
<td>0.132</td>
</tr>
<tr>
<td>$A^3$</td>
<td>0.2541</td>
<td>0.297</td>
<td>0.332</td>
<td>0.216</td>
<td>0.135</td>
</tr>
<tr>
<td>$A^4$</td>
<td>0.1971</td>
<td>0.248</td>
<td>0.270</td>
<td>0.171</td>
<td>0.136</td>
</tr>
</tbody>
</table>

The results show that all the measures except $E_{BB}(A)$ satisfy the requirement of (7.16), i.e., the performance of the measures $E_{VS}(A), E_{ZJ1}(A), E_{W}(A)$ and $E(A)$ are satisfactory whereas $E_{BB}(A)$ is not. It can be seen from both the Tables 7.1 and 7.2, the proposed entropy measure $E(A)$ satisfy the requirement. Therefore, proposed measure $E(A)$ is a valid and effective measure from the structured linguistic variables perspective.

### 7.3. APPLICATION IN SELECTION OF MOBILE PHONE

#### 7.3.1. NUMERICAL ILLUSTRATION

In order to authenticate and analyze the proposed entropy measure, the following example is taken as considered by Xu [67].
Example - The steep development along with the most significant growth is being observed in information technology and communication since last two decades with the introduction of mobile phones. After reviewing, the key features to decide the best mobile phones are characterized into following six attributes -

$C_1$: Basic requirements such as the cost price, the quality and standard of the product.

$C_2$: Physical Characteristics such as the features, design, weight, dimension, attractiveness, sturdy nature, etc.

$C_3$: Technicality such as backup time, space available, RAM available, standby time, safety standards, etc.

$C_4$: Functionality includes accessibility to use.

$C_5$: Brand Choice which a customer relates to.

$C_6$: The additional features such as gaming, security services, languages, ring tone.

7.3.2. ALGORITHM

Algorithm for the selection of best mobile phones contains the following steps.

Step 1. Construct the intuitionistic fuzzy decision matrix, in which each attribute $C_i (i=1,2,\ldots,6)$ obtain some ranking in terms of intuitionistic fuzzy sets by the decision maker/expert for the five mobile phones $M_l$, where $l=1,2,\ldots,5$. Taking a set of five mobile phones $M_1, M_2, M_3, M_4$ and $M_5$ and evaluate intuitionistic fuzzy information based on the above mentioned attributes, expressed by the following matrix

$$
\begin{pmatrix}
C_1 & (0.4, 0.2, 0.4) & (0.9, 0.1, 0.0) & (0.6, 0.2, 0.2) & (0.4, 0.3, 0.3) & (0.4, 0.6, 0.0) \\
C_2 & (0.5, 0.3, 0.2) & (0.2, 0.4, 0.4) & (0.7, 0.2, 0.1) & (0.1, 0.5, 0.4) & (0.3, 0.2, 0.5) \\
C_3 & (0.6, 0.1, 0.3) & (0.5, 0.4, 0.1) & (0.2, 0.5, 0.3) & (0.8, 0.1, 0.1) & (0.7, 0.2, 0.1) \\
C_4 & (0.8, 0.1, 0.1) & (0.3, 0.5, 0.2) & (0.4, 0.5, 0.1) & (0.5, 0.5, 0.0) & (0.6, 0.4, 0.0) \\
C_5 & (0.3, 0.4, 0.3) & (0.6, 0.3, 0.1) & (0.8, 0.2, 0.0) & (0.6, 0.3, 0.1) & (0.5, 0.3, 0.2) \\
C_6 & (0.4, 0.6, 0.0) & (0.4, 0.4, 0.2) & (0.3, 0.5, 0.2) & (0.2, 0.7, 0.1) & (0.5, 0.2, 0.3) \\
\end{pmatrix}
$$

Step 2. In the decision-making situation, decision maker/expert may believe that attribute of all the articles may not assumed to be of same significance. Therefore, to know the significance of
the attribute, first evaluate the weights with the condition $\sum_{k=1}^{6} w_k = 1$. Then evaluate the weight of each attribute by using the proposed entropy (7.1), given by the formula

$$w_k = \frac{1 - E(A)}{\sum_{k=1}^{6} (1 - E(A))}. \quad (7.17)$$

The weight of each attribute by using (7.17) is given by

$$w = (0.2386, 0.226, 0.2482, 0.2278, 0.2264, 0.2216)^T.$$

**Step 3.** Utilize the operator Intuitionistic fuzzy weighted averaging operator (IFWAO) [136] for the above obtained weight $w$ and aggregate decision to make a group valuation $d_l (l = 1, 2, \ldots, 5)$ of each mobile phone $M_l$ by the following formula

$$d_l = \left[1 - \prod_{k=1}^{6} (1 - \mu_{kl}) \prod_{k=1}^{6} \left(\nu_{kl}\right)^{w_k} \prod_{k=1}^{6} (1 - \mu_{kl}) - \prod_{k=1}^{6} \left(\nu_{kl}\right)^{w_k}\right]; \quad l = 1, 2, \ldots, 5. \quad (7.18)$$

where $l = 1, 2, \ldots, 5$, $k = 1, 2, \ldots, 6$.

The group valuations for the mobile phones by using (7.18) is given by

$$d_1 = (0.656, 0.126, 0.218), d_2 = (0.691, 0.198, 0.111), d_3 = (0.669, 0.203, 0.128),$$

$$d_4 = (0.617, 0.218, 0.165), d_5 = (0.640, 0.179, 0.182).$$

**Step 4.** Rank all the mobile phones with respect to the attributes. Larger value of $R(d_l)$, indicate the better mobile phone $d_l$

$$R(d_l) = \left(1 - \frac{1}{2} \pi_{d_l}\right)\left(\mu_{d_l} + \frac{1}{2} \pi_{d_l}\right); \quad l = 1, 2, \ldots, 5. \quad (7.19)$$

By using (7.19), we obtain the following values

$$R(d_1) = 0.681, R(d_2) = 0.705, R(d_3) = 0.686, R(d_4) = 0.642, R(d_5) = 0.664.$$
Therefore, we rank all the mobile phones based on the value of $R(d_i)$. From the above rankings, it is clear that the second mobile phone is the preferred one which agrees with the result with [136].

7.4. CONCLUSION

In this chapter we proposed a new intuitionistic fuzzy entropy measure. Two numerical examples are discussed to compare the proposed entropy with existing entropy measures and the results reveal that the proposed entropy measure is valid and reliable. Further we compared the proposed measure with the existing measures using the linguistic criterion and application point of view. The proposed entropy measure is applied to the decision making problem in the selection of mobile phones.