Chapter 3
Optimal Pulse Shape Factors for Gaussian Pulse Derivatives

In this chapter, a method for finding optimal pulse shape factors for higher order Gaussian derivative pulse is presented, to obtain a new improved pulse shape that satisfies the UWB indoor emission mask requirements.

3.1 Approach and method for generating optimal pulse shape factors

As discussed earlier, that whenever a Gaussian pulse is transmitted the transmitter’s antenna output will be the first order derivative of Gaussian pulse because of the antenna’s derivative characteristics. So, it is useful to consider the derivatives of the Gaussian pulse for the pulse generation because most of the wideband antennas can differentiate the generated Gaussian pulse with respect to time (B. Hu et al., 2005). In time domain analysis, Gaussian pulse derivatives of higher order are similar to sinusoid waves modulated by the Gaussian signal. Also, with the increase in the derivative order, the zero crossing number also increases in time. Due to the increase in the number of crossings a high frequency signal is resulted. It shows that the centre frequency of the pulse increases by taking the derivative of the Gaussian pulse. All these observations guide us to consider the higher-order Gaussian derivatives for the transmission of Ultra Wide Band signals. By taking an appropriate pulse width and the derivative order, we can find a pulse shape that meets the emission mask requirements (Li Li et al., 2011) and (P. Li et al., 2011).

The \( n \)th order derivative of the Gaussian pulse given in equation (2.2) can be obtained recursively from the following expression (H. Nikookar et. al, 2009)

\[
P^{(n)}(t) = -\frac{n-1}{\sigma^2} P^{(n-2)}(t) \left(1 + \frac{t}{\sigma^2}\right)^{\frac{n-1}{2}}
\]

Also, the Fourier transform (FT) of the \( n \)th Gaussian pulse derivative can be given as:-

\[
\]
\[ |P_n(f)| = A(j2\pi f)^n e^{(-\frac{(2\pi f \sigma)^2}{2})} \] (3.2)

The amplitude spectrum of the \(n\)th Gaussian derivative pulse is given as:

\[ |P_n(f)| = A(2\pi f)^n e^{(-\frac{(2\pi f \sigma)^2}{2})} \] (3.3)

The frequency \((f)\) at which the highest value of the equation (3.3) occurs is denoted by the peak emission frequency \((f_M)\) and can be obtained by differentiating the above equation with respect to frequency and setting it equal to zero.

Differentiating the equation (4.3)

\[ \frac{d|P_n(f)|}{df} = A(n(2\pi f)^{n-1} 2\pi e^{(-\frac{(2\pi f \sigma)^2}{2})} - (2\pi f)^n 2\pi e^{(-\frac{(2\pi f \sigma)^2}{2})}(2\pi f)(\sigma)^2) \] (3.4)

\[ \frac{d|P_n(f)|}{df} = A(2\pi f)^{n-1} 2\pi e^{(-\frac{(2\pi f \sigma)^2}{2})}(n - (2\pi f \sigma)^2) \] (3.5)

The peak emission frequency \((f_M)\) should ensure that

\[ 2\pi f_M \sigma = \sqrt{n} \] (3.6)

Therefore, the maximum amplitude spectrum value using equation (3.3) is given as:

\[ |P_n(f_M)| = A(\sqrt{n})^n e^{(-\frac{n}{2})} \] (3.7)

The normalized Power Spectral Density (PSD) of the pulse shape is given as:

\[ |PSD_n(f)| = \frac{|P_n(f)|^2}{|P_n(f_M)|} = \frac{(2\pi f \sigma)^{2n} e^{-(2\pi f \sigma)^2}}{n^n} \] (3.8)

The transmitted signal’s Power Spectral Density \(PSD_t\) is represented by:

\[ |PSD_t(f)| = A_{max} |PSD_n(f)| = A_{max} \frac{(2\pi f \sigma)^{2n} e^{-(2\pi f \sigma)^2}}{e^{-n}} \] (3.9)

Where, \(A_{max}\) is the peak allowed power spectral density of the emission mask. Also, the values of \(n\) and \(\sigma\) can be taken to meet the emission mask requirements. Taking into Consideration, the ultra wideband spectral emission mask
it can be found out that it has some corner frequency points i.e. 0.96 GHz, 1.61 GHz, 1.99 GHz, 3.1 GHz, and 10.6 GHz and the spectrum of any pulse should be passing below or through this emission mask at these points.

The optimal pulse is obtained by a method of finding the optimal pulse shape factors for the different derivative values by changing value of n and taking a fix value of frequency (f) as 10.6 GHz as one of the corner point.

For this the normalized Power Spectral Density equation can be written as:-

\[
20n \log_{10}(2\pi f \sigma) - \frac{10(2\pi f \sigma)^2}{\ln 10} - 10n \log_{10} n + \frac{10n}{\ln 10} - R_{dB} = 0
\]

Where \(R_{dB} = 10 \log_{10}|PSD_n(f)|\) denotes the emission mask back off value at a corner point of 10.6 GHz. \((R_{dB} = -10 \text{ dB for indoor UWB systems})\)

### 3.2 Simulation Results

To obtain the optimal pulse, simulation are carried out in MATLAB software for obtaining the optimal pulse shape factors values for first seven derivatives \((n= 1,2,\ldots,7)\) of Gaussian pulse by finding the roots of equation (3.10) taking a fix value of f as 10.6 GHz as one of the fixed corner point for meeting the emission mask requirements .

**Table 3.1** Optimal values of Pulse shape factors for first seven Gaussian derivatives

<table>
<thead>
<tr>
<th>Gaussian Derivative</th>
<th>Pulse shape factor value ((\alpha)) in seconds(s)</th>
<th>Peak Frequency (f_M) in GHz.</th>
<th>Power Spectral Density (PSD) in dBm/MHz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>0.1162 x10^{-9}</td>
<td>4.885</td>
<td>-44.42</td>
</tr>
<tr>
<td>Second</td>
<td>0.1363 x10^{-9}</td>
<td>5.886</td>
<td>-45.61</td>
</tr>
<tr>
<td>Third</td>
<td>0.1522 x10^{-9}</td>
<td>6.387</td>
<td>-43.54</td>
</tr>
<tr>
<td>Fourth</td>
<td>0.1657 x10^{-9}</td>
<td>6.887</td>
<td>-44.07</td>
</tr>
<tr>
<td>Fifth</td>
<td>0.1778 x10^{-9}</td>
<td>7.139</td>
<td>-42.65</td>
</tr>
<tr>
<td>Sixth</td>
<td>0.1887 x10^{-9}</td>
<td>7.389</td>
<td>-42.98</td>
</tr>
</tbody>
</table>
3.2.1 Pulse shapes of first seven Gaussian derivatives

Pulse shapes for the 1st-7th Gaussian derivative with optimal pulse shape factors values given in Table 3.1 are shown in figures 3.1-3.7.

**Figure 3.1** Pulse Shape of 1st derivative Gaussian Pulse ($\alpha=0.1162 \times 10^{-9} \text{s}$)
Figure 3.2 Pulse Shape of 2$^{nd}$ derivative Gaussian Pulse ($\alpha=0.1363 \times 10^{-9}$s)

Figure 3.3 Pulse Shape of 3$^{rd}$ derivative Gaussian Pulse ($\alpha=0.1522 \times 10^{-9}$s)
**Figure 3.4** Pulse Shape of 4th derivative Gaussian Pulse ($\alpha=0.1657 \times 10^{-9}$s)

**Figure 3.5** Pulse Shape of 5th derivative Gaussian Pulse ($\alpha=0.1778 \times 10^{-9}$s)
Figure 3.6 Pulse Shape of 6th derivative Gaussian Pulse ($\alpha=0.1887 \times 10^{-9}$s)

Figure 3.7 Pulse Shape of 7th derivative Gaussian Pulse ($\alpha=0.1988 \times 10^{-9}$s)

3.2.2 Power Spectral density simulation for the first seven Gaussian derivatives

Power Spectral density (PSD) simulations are carried out for the 1st-7th derivative of the Gaussian pulse for optimum $\alpha$ values given in Table 3.1 and for $\alpha$ values greater than and less than optimum value are shown in figures 3.8-3.14.
Figure 3.8 PSD of 1st derivative Gaussian Pulse for different $\alpha$ values ($\alpha = 0.0962 \times 10^{-9}, 0.1162 \times 10^{-9}, 0.1362 \times 10^{-9}$ and $0.1562 \times 10^{-9}$s).

Figure 3.9 PSD of 2nd derivative Gaussian Pulse for different $\alpha$ values ($\alpha = 0.1163 \times 10^{-9}, 0.1363 \times 10^{-9}, 0.1563 \times 10^{-9}$ and $0.1763 \times 10^{-9}$s).

Figure 3.10 PSD of 3rd derivative Gaussian Pulse for different $\alpha$ values ($\alpha = 0.1322 \times 10^{-9}, 0.1522 \times 10^{-9}, 0.1722 \times 10^{-9}$ and $0.1922 \times 10^{-9}$s).
Figure 3.11 PSD of 4th derivative Gaussian Pulse for different $\alpha$ values ($\alpha = 0.1457 \times 10^{-9}$, $0.1657 \times 10^{-9}$, $0.1857 \times 10^{-9}$ and $0.2057 \times 10^{-9}$ s).

Figure 3.12 PSD of 5th derivative Gaussian Pulse for different $\alpha$ values ($\alpha = 0.157 \times 10^{-9}$, $0.177 \times 10^{-9}$, $0.197 \times 10^{-9}$ and $0.217 \times 10^{-9}$ s).
Figure 3.13 PSD of 6th derivative Gaussian Pulse for different α values (α = 0.1687 x10^-9, 0.1887 x10^-9, 0.2087 x10^-9 and 0.2287 x10^-9 s).

Figure 3.14 PSD of 7th derivative Gaussian Pulse for different α values (α = 0.1788 x10^-9, 0.1988 x10^-9, 0.2188 x10^-9 and 0.2388 x10^-9 s).
Above Figures 3.8-3.14, illustrate the obtained simulation results for the first seven derivatives of Gaussian pulse for various values of pulse shape factors (\(\alpha\)). The results show that the power spectral density of the pulses for the optimal values of \(\alpha\) satisfies the UWB emission mask closely as compared to other nearby taken values of \(\alpha\).

It was also found that for \(n=7\), the optimum value of \(\alpha\) does not closely matches the emission mask requirements at the corner frequency of \(f=10.6\text{GHz}\). For this reason, I have limited the waveforms to the 6\(^{th}\) order since higher order derivatives are not practical. In fact, even though the Gaussian pulse can be infinitely differentiated theoretically, it is generally limited to 5 or maximum 6 derivatives.

![Power Spectral Density](image)

**Figure 3.15** PSD of first 7 Gaussian pulse derivatives for optimal pulse shape factors

From the Figure 3.15, it is explicated that the Power Spectral Density results obtained for the 5\(^{th}\) order Gaussian derivative shown by green curve, matches the emission mask more closely as compared to the other derivatives. Therefore, I have considered the 5\(^{th}\) order derivative of Gaussian pulse as a Standard Gaussian Pulse in my thesis experimental work.
1.3 Summary

This chapter presents an approach for obtaining the optimal pulse shape factor ($\alpha$) values for the higher order derivative of Gaussian pulse. Using this method, the optimal pulse shape factor values for first seven Gaussian derivatives are obtained. From the simulation, the pulse shapes are generated for the first seven derivatives of Gaussian pulse using the optimal pulse shape factors values. The Power Spectral Density simulations results for the first seven Gaussian derivatives shows that the PSD of the pulses for the optimal $\alpha$ values matches the emission mask closely as compared to the other nearby taken values of $\alpha$. Also, the Power Spectral Density of fifth Gaussian pulse derivative matches more closely to the emission mask requirements as compared to other derivatives.