Chapter 3 Effect of Beam Pre-modulation and Guide Magnetic Fields on Slow Wave Free Electron Laser: Non Local Theory

3.1 Introduction

The effect of a pre-bunched electron beam on gain and efficiency enhancement in Compton and Raman regime FEL along with the finite boundary effects has been discussed in detail in Chapter 2. A free electron laser with a pre-bunched electron beam combines the best characteristics of amplifiers and oscillators. However, the generation of much shorter wavelengths employing moderate energy pre-bunched electron beams has yet to be discussed.

The FEL is a frequency tunable device. The wavelength of the emitted FEL radiation is given by \( \lambda = \frac{\lambda_w}{2\gamma_0^2} \), where \( \gamma_0 \) is the relativistic gamma factor and \( \lambda_w \) is the wiggler wavelength. Hence either by changing the beam energy or by changing the wiggler wavelength, the wavelength hence the frequency of FEL radiation can be tuned. The beam energies usually employed in a FEL vary from sub–MeV (at millimeter/sub-millimeter wavelengths) to several MeV (infrared to ultraviolet). As the shorter wavelength generation is becoming more demanding, the requirements on the electron beam energy is becoming important. Recently, considerable efforts have been made to generate higher frequencies with
moderate energy beams, as it has certain distinct advantages like use of alternate accelerator technology, compact system size etc.

For this, two schemes have been proposed

1. Lowering the wiggler period

2. Employing a slow wave structure.

Most of the work performed for the generation of sub-millimeter waves focused on employing a slow wave structure in a free electron laser, as an array of permanent magnets (most commonly used) can be used as a wiggler without requiring the need for alternate wiggler. A slow wave free electron laser employs a slow wave medium to slow down the phase velocity of Transverse electric (TE) or Transverse magnetic (TM) fields to less than ‘c’, the velocity of light, so that they can be excited by a moderately relativistic electron beam. This could be accomplished by using a dielectric lining in a waveguide. The effects of loaded dielectrics in free electron lasers have been investigated in the Compton and Raman regime [23]. The main effect of loading dielectrics in free electron lasers is shortening of operating wavelengths by using same beam energy as in vacuum free electron lasers. In the Raman regime a free electron laser using a dielectric lined metallic waveguide has been considered to get shorter wavelengths. The detailed analytical study of a dielectric loaded Raman type free electron laser has been performed by Shibuya and Shiozawa [23]. They have demonstrated the up-shifting of operating frequencies in the dielectric loaded free electron laser in the Raman regime. The Raman type FEL consisted of a relativistic electron beam contained in a dielectric loaded parallel plate waveguide and an array of permanent magnets. With the aid of numerical
illustrations, they have shown that when a dielectric sheet is loaded on the waveguide the maximum growth rate and the oscillation frequency obtained can be greater than those for the vacuum Raman type FEL. It was also shown that by choosing proper values for the relative permittivity of the dielectric sheet, one can greatly lower the beam energy without degrading the oscillation characteristics. Tripathi and Liu [31] have considered a dielectric loaded waveguide as a slow wave structure. They have proposed the operation of a free electron laser in a dielectric loaded waveguide and have shown that one can approach large $\omega_1$ as

$$v_z \rightarrow v_{ph} \left( \omega_1 = \frac{k_w v_z}{1 - v_z / v_{ph}} \right),$$

where $k_w$ is the wiggler wave vector, $v_z$ is the axial beam velocity and $v_{ph}$ is the phase velocity of the radiation wave.

A plasma lining having a dielectric constant $\varepsilon$ given by

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

can also act as a slowing down medium for $\omega_p \gg \omega$ so that $|\varepsilon| \gg 1$.

Jaiman et al. [54] have studied the millimeter wave generation in a plasma-filled slow wave device.

In slow wave FELs, the requirement of low beam velocity makes the FEL interaction possible at lower beam energies for the generation of far infrared frequencies. In this chapter we have explored the effects of finite boundaries on slow wave FEL in addition to the effect of
beam pre-modulation. The effect of a strong axial guide magnetic field on slow wave FEL is also studied.

In section 3.2, a brief discussion of slow wave free electron laser is given. We obtain the cyclotron resonance condition in the presence of the strong axial guide magnetic field in section 3.3. In section 3.4, we study the growth rate of pre-bunched slow wave FEL. The comparison of growth rate obtained for slow wave FEL (non-local theory) with the growth rate for FEL (non-local theory) has been given in section 3.5. A brief discussion of the results is also given in section 3.6.

3.2 Slow wave FEL

A slow wave free electron laser employs a slow wave medium to slow down the phase velocity of TE or TM fields to less than 'c', the velocity of light, so that they can be excited by a moderately relativistic electron beam.

Figure 3.1 The schematic diagram of slow wave FEL.
A sketch of a slow wave FEL is shown in Figure 3.1. A pre-modulated electron beam having beam radius 0.2cm, beam energy 0.07MeV and beam thickness 1mm propagates through a dielectric loaded cylindrical waveguide. Electron beam tends to spread due to mutual repulsion between the electrons. In order to reduce the space charge spreading and hence to reduce the energy spread of the beam, some beam focusing mechanism is required. For this, the interaction region is comprised of a strong guide magnetic field to guide the beam, hence to reduce the beam spread. Moreover, this strong guide magnetic field along with the wiggler magnetic field produces cyclotron resonance. At high beam current levels space charge effects of the beam become dominant. The radial spread in the beam is kept low so that dc and ac space charge effects can be neglected. We have also neglected the dc self generated magnetic field as we have chosen the beam parameters (e.g. beam current, beam energy and beam cross-section), in such a way that dc self-generated magnetic field may not play an important role.

3.3 Cyclotron Resonance Condition

Consider a axial pre-modulated electron beam having radius \( r_0 \), density \( n_{b0} \), equilibrium beam velocity \( v_{b0 z} \), relativistic gamma factor \( \gamma = \gamma_0 (1 + \Delta \sin \omega_0 \tau) \) propagating through a dielectric loaded (with a dielectric of permittivity \( \varepsilon_1 \)) cylindrical wave-guide of radius \( a_1 \), as
shown in Figure 3.1, where \( \Delta = \frac{V_1}{V_b} \) is the modulation index and its value ranges from 0 to 1,

\( V_b \) is the beam voltage, \( \omega_0 \left( = \frac{k_{z0}}{z_0} v_{b0} \right) \) and \( k_{z0} \) are the modulation frequency and wave number of the pre-modulated electron beam, respectively.

As discussed earlier in the previous chapter, that for the electrons to lose energy to the electromagnetic wave, the bunching of the electrons takes place in the decelerating zone. Hence we have assumed that all the beam electrons are in the decelerating zone, i.e., phase of the pre-bunched beam \( \psi \) is \(-\pi/2\). The beam electrons acquire an oscillatory velocity \( v_0 \) due to wiggler field \( B_w = B_w (\hat{x} - i\hat{y}) e^{-ik_w z} \) and in the presence of guide magnetic field \( B_S \hat{z} \), the oscillatory velocity produces **cyclootron resonance** condition.

The oscillatory velocity, governed by the equation of motion is given as

\[
m_e \frac{d}{dt} (\gamma v_0) = -\frac{e}{c} v_0 \times \vec{B} \quad ,
\]

where \( m_e \) is the mass of electrons and \( \vec{B} = B_w (\hat{x} - i\hat{y}) e^{-ik_w z} + B_S \hat{z} \).

Expanding, equation (3.1) we get

\[
\frac{\partial}{\partial t} (\gamma v_0) + v_{b0} \nabla (\gamma v_0) = -\frac{e}{m_e c} \vec{v}_0 \times [B_w (\hat{x} - i\hat{y}) e^{-ik_w z} + B_S \hat{z}] \quad ,
\]

where \( \gamma = \gamma_0 (1 + \Delta \sin \omega_0 t) \) is the relativistic gamma factor.

Taking the derivative of equation (3.2), and using \( \nabla = \frac{\partial}{\partial z} = -ik_w \), we obtain
as, phase of the pre-bunched beam \( \omega_0 \tau \) is \( -\pi/2 \). Thus we get \( \cos \omega_0 \tau = 0 \), so equation (3.3) now becomes

\[
-ik_w v_{0x} \gamma v_{0y} = -v_0 \times \omega_{ce} \hat{z} - \frac{e}{m_c} v_0 \times B_w (\hat{x} - i\hat{y}) e^{-ik_w z}
\]

where \( \omega_{ce} = \frac{eB_s}{m_c} \).

Taking \( \vec{v}_0 = \hat{x} v_{0x} + \hat{y} v_{0y} = \vec{v}_{0\perp} \) and keeping the values for \( x \) and \( y \) components which are calculated using equation (3.4) we obtain

\[
v_{0x} = \frac{eB_w e^{-ik_w z}}{m_c c k_w \gamma [1 - \frac{\omega_{ce}}{k_w v_{b0} \gamma}]} \quad \text{and} \quad v_{0y} = -iv_{0x},
\]

which further gives

\[
\vec{v}_{0\perp} = (\hat{x} - i\hat{y}) \frac{eB_w e^{-ik_w z}}{m_c c k_w \gamma [1 - \frac{\omega_{ce}}{k_w v_{b0} \gamma}]},
\]

where \( \omega_{ce} \) is the cyclotron frequency and when electron cyclotron frequency \( \sim \omega_{ce}/\gamma \) is in resonance with the wiggler frequency \( \sim k_w v_{b0} \), this is known as \textbf{cyclotron resonance condition}. Under this condition, when \( (\omega_{ce}/\gamma) \sim (k_w v_{b0}) \), the transfer electron quiver velocity

\[
\vec{v}_0 = \hat{x} v_{0x} + \hat{y} v_{0y} = \vec{v}_{0\perp}
\]

approaches infinity. Thus we can say that, when the slow wave FEL
has a guide magnetic field $B_s$, the transverse electron quiver velocity is enhanced via cyclotron resonance condition.

### 3.4 Growth Rate of Slow Wave FEL

As the electromagnetic wave $\vec{E}_1 = a_1 e^{-i(\omega_1 t - k_{1z} z)}$ is also present in the interaction region, then using equation of motion $m_e \frac{d}{dt} (\gamma \vec{v}_1^*) = -e \vec{E}_1$, perturbed electron velocity $\vec{v}_1 = \frac{e \vec{E}_1}{m_e \omega_1 \gamma}$ is obtained and exerts a ponderomotive force, given as

$$F_p = -\frac{e}{2c} (\vec{v}_0 \times \vec{B}_1 + \vec{v}_1 \times \vec{B}_w) = e\nabla \phi_p$$

(3.6)

where $\vec{B}_1 = \frac{c}{\omega_1} (k_{1z} \hat{z} \times \vec{E}_1)$

Using the relativistic equation of motion

$$m_e \frac{d}{dt} (\gamma \vec{v}) = -e \vec{E}$$

(3.7)

where $\vec{E} = -\nabla \phi_p$

and expanding $v = v_{b0} + v_2$ and $\gamma' = \gamma + \gamma^3 \frac{v_{b0} v_2}{c^2}$, equation (3.7) can be linearized to obtain

$$-i(\omega - k_{1z} v_{b0}) (\gamma v_2 + \gamma^3 \frac{v_{b0} v_2}{c^2} v_{b0}) = eikz \phi_p / m_e$$

(3.8)
This produces velocity perturbation $v_2$ as

$$v_2 = \frac{-ek_z\phi_p}{m_e\gamma^3(\omega_1 - k_zv_{b0})}.$$  \hspace{1cm} (3.9)

Using the equation of continuity $\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0$ and expanding $\vec{v} = v_{b0} + v_2$ and $n = n_{b0} + n_2$ we obtain density perturbation $n_2$

$$n_2 = -\frac{n_{b0}k_z^2e\phi_p}{m_e\gamma^3(\omega_1 - k_zv_{b0})^2}.$$  \hspace{1cm} (3.10)

The non-linear current density is given as

$$\vec{J}_1 = -n_{b0}ev_{1\perp} - n_1ev_{b0}\hat{z} - \frac{1}{2}n_2ev_{0\perp}.$$  \hspace{1cm} (3.11)

where $n_1 = -\frac{k_zJ_1}{\omega_1e}$.

Using identity $\nabla \times (\nabla \times E) = \nabla (\nabla \cdot E) - \nabla^2 E$, we derive the wave equation for cylindrical co-ordinates for $\theta = 0$, (axial symmetric case) and using equation (3.11) in the wave equation for cylindrical co-ordinates, we obtain

$$\frac{\partial^2 E_{1z}}{\partial r^2} + \frac{1}{r}\frac{\partial E_{1z}}{\partial r} + p^2 E_{1z} = \frac{\omega_{p0}^2k_z^2}{4\omega_1^2\gamma^3(\omega_1 - k_zv_{b0})^2} E_{1z}p_1.$$  \hspace{1cm} (3.12)
where

\[ p^2 = \left( \frac{\omega_1^2}{c^2} \varepsilon_1 - k_{1z}^2 \right), \]

\[ p_1 = \frac{(1 - \frac{\omega_1 v_b b_0 \varepsilon_1}{k_{1z} c^2})}{(1 - \frac{\omega_1 v_b b_0 \varepsilon_1}{k_{1z} c^2})} k_{1z}^2. \]

Further by solving equation (3.12) using perturbation techniques as was done in the last chapter, we obtain the non-linear dispersion relation given as

\[ \left( \frac{\omega_1^2 - k_{1z}^2 c^2}{\varepsilon_1} \right) \left( \omega_1 - k_{1z} v_b b_0 \right)^2 = \frac{\omega_{pb}^2 e^2 k_{1z}^2 |v_{0\perp}|^2}{\omega_1^2 \gamma^3 \varepsilon_1^2} \left| \frac{p_1}{p} \right|^2 \]

(3.13)

Around simultaneous zeros of the factors on L.H.S, \( \omega_1 \sim \frac{kc}{\sqrt{\varepsilon_1}} \), \( \omega_1 \sim k_{1z} v_b b_0 \) giving radiation and beam modes respectively. Now we expand \( \omega_1 \) as \( \omega_1 = \frac{kc}{\sqrt{\varepsilon_1}} + \delta = k_{1z} v_b b_0 + \delta \), where \( \delta \) is the small modification in \( \omega_1 \) due to the finite right hand side of equation (3.13).

Hence the growth rate, i.e., the imaginary part of \( \delta \)

\[ \Gamma = \text{Im} \delta = \left( \frac{\omega_{pb}^2 k_{1z}^2 |v_{0\perp}|^2 c^2 p_1}{2 \omega_1^3 \gamma^3 \varepsilon_1^2} \right)^{1/3} \frac{\sqrt{3}}{2}. \]

(3.14)

The growth rate scales as one-third power of the beam density.
For typical parameters: electron beam energy $E_b = 0.071$ MeV, beam current $I_b = 1.1$ A, beam radius $r_0 = 0.2$ cm, waveguide radius $a = 0.2$ cm, mode number $n = 1$, i.e., the first zero of Bessel function, radiation frequency $\omega_1 = 2.87 \times 10^{11}$ rad./s, modulation frequency $\omega_0 = 2.9 \times 10^{11}$ rad./s, modulation wave vector $k_{z0} = 1.13 \text{ cm}^{-1}$, permittivity $\varepsilon_1 = 1.7$, wiggler field $B_w = 300$ G and modulation index = 0, i.e., without pre-modulated beam, growth rate is $\Gamma \sim 9 \times 10^8$ rad./sec.

The growth rate is found to increase with the increase in the guide magnetic field, and is maximum at 4.9 KG, when wiggler frequency ($k_w v_{b0}$) is comparable to cyclotron frequency ($\omega_{ce}/\gamma$). For the parameters given above and modulation index = 0.8, the growth rate as a function of magnetic field is plotted in Figure 3.2.

![Graph showing growth rate $\Gamma$ (rad./sec) as a function of magnetic field $B_z$ (in Gauss) for modulation index $\Delta = 0.80$.](image)

**Figure 3.2** Growth rate $\Gamma$ (rad./sec) as a function of magnetic field $B_z$ (in Gauss) for modulation index $\Delta = 0.80$. 

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70
We have plotted the variation of the growth rate of the slow wave FEL instability $\Gamma$ (rad./sec) as a function of the phase of pre-modulated electron beam in Figure 3.3 for the same parameters as used for Figure 3.2 with modulation index $\Delta = 0.8$. The growth rate of slow wave FEL instability is maximum when the phase of the pre-modulated beam is $-\pi / 2$, i.e., when the pre-modulated beam electrons are in the decelerating zone, the net energy transfer from the beam electrons to the radiation wave occurs. The growth rate becomes minimum when the phase of the pre-modulated beam is $+\pi / 2$, i.e., when the beam electrons are in the accelerating zone.

![Graph](image)

**Figure 3.3** Growth rate $\Gamma$ (rad./sec) as a function of phase angle $\Psi$.  

71
The growth rate of slow wave FEL increases with the increase in modulation index as shown in Figure 3.4 and has the maximum value when $\Delta \sim 0.8$, $\omega_0 \tau = -\frac{\pi}{2}$ (i.e., pre-modulated beam electrons are in the decelerating zones) and the wiggler frequency is approaching to the electron cyclotron frequency. By increasing the modulation index $\Delta$ further, the value of the growth rate instability goes down as the transverse electron velocity decreases, which decreases the growth rate of slow wave FEL.

![Graph showing growth rate $\Gamma$ (rad./sec) as a function of modulation index $\Delta$ for $E_b = 0.071$ MeV.](image)

**Figure 3.4** Growth rate $\Gamma$ (rad./sec) as a function of modulation index $\Delta$ for $E_b = 0.071$ MeV.
3.4.1 Gain

If $l$ is the length of the interaction region, then the gain (in dB) $G$ is given by

$$G = 10 \log \left( \frac{A}{A_0} \right) = 10 \frac{\Gamma l}{c}.$$  \hfill (3.15)

where, $A$ and $A_0$ are the amplitudes of the wave at the distances $z=0$ and $z=l$.

3.5 Comparison with Previous Chapter

For the un-modulated beam, i.e., with modulation index $\Delta = 0$, using beam energy $0.071\text{MeV}$ and the radiation frequency $2.87 \times 10^{11} \text{ rad./sec}$, the growth rate of pre-bunched slow wave FEL was found to be $9 \times 10^8 \text{ rad./sec}$. Whereas in case of pre-bunched FEL (non-local theory), with modulation index $\Delta = 0$, using the same beam energy $0.071\text{MeV}$ and at a lower radiation frequency $2.4 \times 10^{10} \text{ rad./sec}$, the growth rate was found to be $1.16 \times 10^7 \text{ rad./sec}$.

The comparative study shows that for slow wave FEL, it is possible to achieve higher growth rate, consequently the higher efficiency, at higher frequency with lower beam energy, i.e., $0.071\text{MeV}$ in this case. Thus we can say that the beam energy requirement has been reduced to achieve higher efficiency at higher frequency in slow wave FEL. The comparison is shown in Figures 3.5 & 3.6. Figure 3.5 shows the variation of growth rate with modulation index for pre-bunched slow wave FEL and Figure 3.6 gives the variation of growth rate with modulation index for pre-bunched FEL. It is seen that the growth rate obtained for slow wave FEL at
radiation frequency $\omega_1 = 2.87 \times 10^{11}$ rad./s is much higher than the growth rate obtained for FEL (non-local theory) at a lower radiation frequency $\omega_1 = 2.4 \times 10^{10}$ rad./s for the same beam energy, i.e., 0.071 MeV.

![Graph](image)

**Figure 3.5:** Growth rate $\Gamma$ vs $\Delta$ for Slow wave FEL.

**Figure 3.6:** Growth rate $\Gamma$ vs $\Delta$ for FEL (non-local th.)

### 3.6 Discussion

The beam pre-modulation on slow wave free electron laser offers considerable enhancement in growth rate, efficiency and gain when a guide magnetic field is used in the device in addition to when the frequency and wave number of the pre-modulated beam are comparable to that of the radiation wave. When the slow wave FEL has a guide magnetic field the transverse electron velocity is enhanced via cyclotron resonance. The non-local effects (finite geometry
effects) reduce the growth rate of the slow wave FEL instability. If we use plasma as a dielectric
medium of permittivity $\varepsilon_i[=1-\frac{\omega_p^2}{\omega^2-\omega_{ce}^2}]$ for magnetized plasma, then plasma can play an
important role in slowing down the phase velocity of radiation wave significantly reducing the
requirement on beam energy for generating frequency below the electron cyclotron
frequency. The scheme seems to work well at millimeter and sub-millimeter wavelengths.