CHAPTER 4
MATHEMATICAL MODEL FOR TWO STAGE SUPPLY CHAIN INVESTIGATIONS

4.1 A Supply Chain Model

The standard periodic review based stock OUT replenishment policy is used. External demand for a single item occurs at the retailer, where the fundamental demand process faced by the retailer is an AR (1) process. The retailer’s demand from the customer is a demand pattern

![Diagram of two stage supply chain]

Fig 4.1: A model of two stage supply chain

4.2 Demand Process

In this work, considered a two-stage supply chain that consists of one supplier and two Retailers, and aimed to measure the bullwhip effect in the simple supply chain. The two Retailers order and replenish the stock from a supplier in each period $t$. The two Retailers’ market share are considered as $\alpha$ and $1 - \alpha$, respectively. Also assumed that the order up to inventory policy for Retailer_1 and Retailer_2 both employ an autoregressive AR(1) model:

$$D_t = \delta + \phi D_{t-1} + \epsilon_t.$$  \hspace{1cm} \text{(4.1)}

Assuming $\alpha$ as market share of retailer_1 and when employed autoregressive model AR (1) to forecast the demand, the forecasting equation can be written as

$$D_{r1,t} = \alpha \delta_1 + \phi_1 D_{r1,t-1} + \alpha \epsilon_{r1,t}.$$  \hspace{1cm} \text{(4.2)}
In equation (4.2), $D_{r1,t}$ is the $t^{th}$ period demand, $\delta_1$ is the autoregressive constant. $\phi_1$ is the first-order autocorrelation coefficient, where $-1 < \phi_1 < 1$. $\epsilon_{r1,t}$ is the forecast error for period $t$ and $\epsilon_{r1,t}$ is independent and identically distributed from a symmetric distribution with mean 0 and variance $\sigma^2_{r1}$. According to a first-order autocorrelation property of time series model, for any period $t$, we must have

$$E(D_{r1,t}) = E(D_{r1,t-1}) = \mu_{1,d} = \frac{a \delta_1}{1 - \phi_1}, \quad \text{------------------------ (4.3)}$$

$$\text{Var} (D_{r1,t}) = \text{Var} (D_{r1,t-1}) = \sigma^2_{1,d} = \frac{\sigma^2_{1} \sigma^2_{\epsilon_1}}{1 - \phi_1^2}.$$ 

Similarly, as the market share of retailer_2 will be 1-$\alpha$, and when AR(1) autoregressive is applied to predict the demand at retailer_2, forecasting equation can now be written as

$$D_{r2,t} = (1 - \alpha) \delta_2 + \phi_2 D_{r2,t-1} + (1 - \alpha) \epsilon_{r2,t}. \quad \text{------------------------ (4.4)}$$

In equation (4.4), $D_{r2,t}$ is the $t^{th}$ period demand. $\delta_2$ is the autoregressive constant, $\phi_2$ is the first-order autocorrelation coefficient, where $-1 < \phi_2 < 1$. $\epsilon_{r2,t}$ is the forecast error for period $t$ and $\epsilon_{r2,t}$ is independent and identically distributed from a symmetric distribution with mean 0 and variance $\sigma^2_{r2}$. Similarly we also have

$$E(D_{r2,t}) = E(D_{r2,t-1}) = \mu_{2,d} = \frac{(1 - \alpha) \delta_2}{1 - \phi_2}, \quad \text{------------------------ (4.5)}$$

$$\text{Var} (D_{r2,t}) = \text{Var} (D_{r2,t-1}) = \sigma^2_{2,d} = \frac{(1 - \alpha) \sigma^2_{\epsilon_2}}{1 - \phi_2^2}.$$ 

### 4.3 Inventory Policy

In order to meet the dynamic needs of the supply chain model, the supply chain model shown in Fig 4, employs the order-up-to inventory policy and assumed order lead times while ordering by both retailers. The goal of the inventory policy is to maintain inventory levels at the target inventory levels $q_t$. At the start of period $t$, the order quantity $q_{r1,t}$ by Retailer_1 can be written as

$$q_{r1,t} = S_{r1,t} - S_{r1,t-1} + D_{r1,t-1}. \quad \text{------------------------ (4.6)}$$

In equation (4.6), $S_{r1,t}$ is the order-up to-level and it can be determined through a lead-time demand by
In equation (4.7), $\hat{D}_{rl,t}^j$ is the value of lead-time demand and forecast based on historical sales data. $z$ is the normal z-score that can be determined based on the desired service level of the inventory policy. $\hat{\sigma}_{rl,t}^j$ is the standard deviation of forecast error lead-time demand? Analogously the order of quantity of Retailer_2 as follows:

$$q_{r2,t} = S_{r2,t} - S_{r2,t-1} + D_{r2,t-1}.$$  

(4.8)

The order-up-to level $S_{r2,t}$ can be as follows:

$$S_{r2,t} = \hat{D}_{r2,t}^j + z\hat{\sigma}_{r2,t}^j$$  

(4.9)

In equation (4.9), $\hat{D}_{r2,t}^j$ is the value of lead-time demand and forecast based on historical sales data? $z$ is the normal z-score that can be determined based on the preferred service level of the inventory policy. $\hat{\sigma}_{r1,t}^j$ is the standard deviation of lead-time demand forecast error.

### 4.4 Forecasting Method

Looking at the operating doctrine of inventory policy the accuracy of the demand forecasting for the future lead time $l$ period is the most significant factor that affects the inventory level of the Retailer’s in supply chain model. When each forecasting error is present, the impacts of various forecasting methods on the bullwhip effect are not the same. Hence, three forecasting methods are introduced to minimize the errors. They are 1) Minimum mean square errors (MMSE) 2) Moving average (MA) and 3) Exponential smoothing (ES).

#### 4.4.1 Minimum mean square error (MMSE) Forecasting Method

Under the MMSE forecast method, the lead-time demand is as follows:

$$D_t^l = D_t + D_{t+1} + \cdots + D_{t+l-1} = \sum_{i=0}^{l-1} D_{t+i}.$$  

(4.10)

When used the MMSE method to predict the future lead-time demand, the total demand expectations within the lead-time $l$ are

$$\hat{D}_t^l = \hat{D}_t + \hat{D}_{t+1} + \cdots + \hat{D}_l = \sum_{i=0}^{l-1} \hat{D}_{t+i}.$$  

(4.11)

Where $\hat{D}_{t+i}$ can be characterized as
\[
\hat{D}_{r,i} = E[D_{r,i} | D_{t-1}, D_{t-2}, \ldots] .
\]

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(4.12)

### 4.4.2. Moving Average (MA) Forecasting Method

In the MA forecasting method, the lead-time demand can be determined as follows:

\[
\hat{D}'_{r,i} = \frac{1}{k} \sum_{i=1}^{k} D_{t-i} .
\]

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(4.13)

In equation (4.13), \( k \) is the span (number of periods) for the MA forecasting method. \( D_{t-i} \) is the actual demand in period \( t-i \).

### 4.4.3. Exponential Smoothing (ES) Forecasting Method

When ES forecasting method, the lead-time demand can be written as follows:

\[
\hat{D}'_{r,i} = \beta D_{r,i} + (1 - \beta) \hat{D}'_{r,i-1} .
\]

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(4.14)

In equation (14), \( \beta \) is the smoothing exponent?

### 4.5 THE MEASURE OF THE BULLWHIP EFFECT

In this section measure of the bullwhip effect under the MMSE, MA and ES are derived mathematically. The demand both retailers is written as

\[
D_t = D_{r,1,t} + D_{r,2,t} .
\]

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(4.15)

### 4.5.1. The Bullwhip Effect Measure under the MMSE Forecasting Method.

Under the MMSE forecasting method, the total demand expectations within the lead-time 1 of Retailer_1 can be determined as

\[
\hat{D}'_{r,1,t} = \frac{1}{1 - \phi_1} \alpha \delta_1 - \frac{\phi_1 (1 - \phi_1^{1/2})}{(1 - \phi_2)^2} \alpha \delta_1 + \frac{\phi_1 (1 - \phi_1^{1/2})}{1 - \phi_1} D_{r,1,t-1} .
\]

----------------------

(4.16)

It is known that \( \sigma^2_{r,1,t} \) does not depend on \( t \), so the order quantity of Retailer_1 can be described as follows:

\[
q_{r,1,t} = (1 + \frac{\phi_1 (1 - \phi_1^{1/2})}{1 - \phi_1}) D_{r,1,t-1} - \frac{\phi_1 (1 - \phi_1^{1/2})}{1 - \phi_1} D_{r,1,t-2} .
\]

----------------------

(4.17)

Where \( Y_1 = \phi_1 (1 - \phi_1^{1/2})/(1 - \phi_1) \).

Similarly we get the order quantity of Retailer_2:
\[ q_{r2t} = (1 + \frac{\phi_2 (1 - \phi_2^{12})}{1 - \phi_2})D_{r2,t-1} - \frac{\phi_2 (1 - \phi_2^{12})}{1 - \phi_2}D_{r2,t-2} \]
\[ = (1 + Y_2)D_{r2,t-1} - Y_2D_{r2,t-2}, \]  \hspace{1cm} \text{(4.18)}

where \( Y_2 = \frac{\phi_2 (1 - \phi_2^{12})}{1 - \phi_2} \).

Since total demand at two retailers is
\[ D_t = D_{r1,t} + D_{r2,t}, \] \hspace{1cm} \text{(4.19)}

The variance of the total demand,
\[ \text{Var}(D_t) = \text{Var}(D_{r1,t} + D_{r2,t}) \] \hspace{1cm} \text{(4.20)}
\[ = \text{Var}(D_{r1,t}) + \text{Var}(D_{r2,t}) + 2 \text{Cov}(D_{r1,t}, D_{r2,t}) \]

The covariance between \( D_{r1,t} \) and \( D_{r2,t} \) can be described as
\[ \text{Cov}(D_{r1,t}, D_{r2,t}) = \frac{\alpha}{1-\alpha} \text{Var}(D_{r2,t}). \] \hspace{1cm} \text{(4.21)}

Total order quantity by two Retailers’s written as
\[ q_t = (1 + Y_1)D_{r1,t-1} - Y_1D_{r1,t-2} + (1 + Y_2)D_{r2,t-1} - Y_2D_{r2,t-2} \] \hspace{1cm} \text{(4.22)}

Then the variance of the total order quantity will be
\[ \text{Var}(q_t) = (1 + 2Y_1 + 2Y_2^2) \text{Var}(D_{r1,t-1}) + (1 + 2Y_2 + 2Y_2^2) \times \text{Var}(D_{r2,t-1}) - 2Y_1(1 + Y_1) \text{Cov}(D_{r1,t-1}, D_{r1,t-2}) + 2(1 + Y_1)(1 + Y_2) \text{Cov}(D_{r1,t-1}, D_{r2,t-1}) - 2Y_2(1 + Y_1) \text{Cov}(D_{r1,t-2}, D_{r1,t-1}) + 2Y_1Y_2 \text{Cov}(D_{r1,t-2}, D_{r2,t-2}) - 2Y_2(1 + Y_2) \text{Cov}(D_{r2,t-1}, D_{r2,t-2}) \] \hspace{1cm} \text{(4.23)}

It can be prove that
\[ \text{Cov}(D_{r1,t-1}, D_{r1,t-2}) = \phi_1 \text{Var}(D_{r1,t}), \]
\[ \text{Cov}(D_{r1,t-1}, D_{r2,t-1}) = \frac{\alpha}{1-\alpha} \text{Var}(D_{r2,t}), \]
\[ \text{Cov}(D_{r1,t-1}, D_{r2,t-2}) = \phi_2 \frac{\alpha}{1-\alpha} \text{Var}(D_{r2,t}), \]
\[ \text{Cov}(D_{r1,t-2}, D_{r2,t-1}) = \phi_2 \frac{\alpha}{1-\alpha} \text{Var}(D_{r2,t}), \]
\[ \text{Cov}(D_{r1,t-2}, D_{r2,t-2}) = \phi_2 \text{Var}(D_{r2,t}). \] \hspace{1cm} \text{(4.24)}

Therefore, (4.23) can be described as
Var \( (q_t) \)
\[
= (1 + 2Y_1 + 2Y_2^2 - 2Y_1(1 + Y_1)\phi_1) \text{Var}(D_{r1t}) + (1 + 2Y_2 + 2Y_2^2 - 2Y_2(1 + Y_2)\phi_2) \text{Var}(D_{r2t}) + (2 + 2(Y_1 + Y_2) + 3Y_1Y_2 - 2\phi_1Y_2(1 + Y_1) - 2\phi_2Y_1(1 + Y_2)) \frac{\alpha}{1 - \alpha} \text{Var}(D_{r2t})
\]
\-------------------------- (4.25)

For simplicity (4.25) can be written as
\[
\text{Var} \ (q_t) = P_1 \text{Var} \ (D_{r1t}) + P_2 \text{Var} \ (D_{r2t}) + P_3 \frac{\alpha}{1 - \alpha} \text{Var} \ (D_{r2t}) \hspace{1cm} (4.26)
\]

In equation (26) \( P_1 \) is the coefficient of \( \text{Var} \ (D_{r1t}) \), \( P_2 \) is the coefficient of \( \text{Var} \ (D_{r2t}) \), and \( P_3 \) is the coefficient of \( \alpha/(1 - \alpha) \text{Var} \ (D_{r2t}) \). Then, the BWE in MMSE forecasting method is
\[
BWE_{MMSE} = \frac{\text{Var} \ (q_t)}{\text{Var} \ (d_t)} = \frac{\alpha^2 P_1 + (1 - \alpha)^2 P_2 + \alpha(1 - \alpha)P_3}{\alpha^2 + (1 - \alpha)^2 + 2\alpha(1 - \alpha)}
\]
\------------------------ (4.27)

### 4.5.1. MATLAB Code to Measure the Bullwhip Effect in MMSE Forecast Method

Market share \( (\alpha) \) is kept constant, autoregressive coefficient \( (\phi_1) \) retailers _1 is varied, to find Bullwhip effect for sets of \( (\phi_1, \alpha) \), and remaining values of lead times of retailers _1 and retailers _2 are same (=3), autoregressive coefficient \( (\phi_2) \) of retailers _2 (=0.6) and BWE is measured by incrementing \( \alpha \) again. MATLAB software code written here can perform arithmetic and produce graphs efficiently. The code written to perform the computations and to produce graphs is depicted under

```matlab
myfile = xlsread(‘C:\Users\DHANASREE\Documents\MATLAB\Dist_Data.xlsx’);
wks = myfile(:,1);
D1t = int32(myfile(:,2));  \%----- Retailer-1 demand as in Inputfile
D2t = int32(myfile(:,3));  \%----- Retailer-2 demand as in Inputfile
Dem1t = D1t(9:end);
Dem2t = D2t(9:end);
totDem_t = Dem1t + Dem2t;
\% ----Measurement of BWE w.r.t MMSE---
\%----- Minimum Mean Square Error Method--------------
Leadt1=3;
```
Leadt2 = 4;
alpa = 0.4;
py1 = 0.6;
py2 = 0.6;
delt1 = 1; delt2 = 1;
D1t_1 = Dem1t(1:end-1);
D1t_2 = Dem1t(1:end-2);

% ---- For Graph1 ---- BWE Vs py1 -----
Leadt1 = 3; Leadt2 = 3;
ap = [0.4 0.6 0.8];
p = 0.1:0.1:0.9;

for j = 1:length(p)
    py2 = 0.6;
    py1 = p(j);
    for i = 1:length(ap)
        alpa = ap(i);
        BWE_MM = bwe_mmse(py1, py2, Leadt1, Leadt2, alpa);
        asd(:, i) = [BWE_MM];
    end
    gdata1(j,:) = [p(j) asd];
end

disp('Measurement of BWE w.r.t MMSE')
disp('influence of (phy & alpha) on Bullwhip Effect')
disp(gdata1)
figure
subplot(2,2,1)
xaxis = gdata1(:,1);
yaxis = gdata1(:,2:4);
plot(xaxis, yaxis);
title('influence of (phy & alpha) on Bullwhip Effect')
xlabel('Autoregressive coefficient \phi')
ylabel('Bullwhip Effect')
Table 4.1: Influence of \( \Phi \) & \( \alpha \) on Bullwhip Effect

<table>
<thead>
<tr>
<th>( \phi_1 )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha=0.4 )</td>
<td>2.35</td>
<td>2.41</td>
<td>2.49</td>
<td>2.57</td>
<td>2.65</td>
<td>2.72</td>
<td>2.747</td>
<td>2.7</td>
<td>2.53</td>
<td>2.37</td>
</tr>
<tr>
<td>( \alpha=0.6 )</td>
<td>1.99</td>
<td>2.1</td>
<td>2.24</td>
<td>2.4</td>
<td>2.57</td>
<td>2.72</td>
<td>2.803</td>
<td>2.76</td>
<td>2.46</td>
<td>2.16</td>
</tr>
<tr>
<td>( \alpha=0.8 )</td>
<td>1.61</td>
<td>1.8</td>
<td>2.02</td>
<td>2.28</td>
<td>2.56</td>
<td>2.83</td>
<td>2.997</td>
<td>2.96</td>
<td>2.51</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Fig 4.2 Influence of (Autoregressive Coefficient \( \Phi \) & \( \alpha \)) on Bullwhip Effect in MMSE
4.5.1.1. The Parameters are Analyses by the MMSE Forecasting Method.

Figs 4. 2 – 4. 4 simulates the expression of the bullwhip effect under the MMSE forecasting method and shown the impact of parameters on the bullwhip effect. Indicated that the bullwhip effect occurs only when a positive autoregressive relationship exists in the demand process under MMSE forecasting method. So, we just consider the circumstance that $\phi_1$ varies from 0 to 1. Fig. 4.5.1 when lead times are same for retailer_1 & retailer_2, $\phi_2$ value is 0.6 constant indicates that the bullwhip effect increases slowly with the increase of $\phi_1$, and the bullwhip effect begins to decrease rapidly when it reaches the maximum value. The bullwhip effect is low only for lower value of $\phi_1$ and for higher values $\alpha$. The bullwhip effect becomes larger with $\alpha$ becoming larger, and when $\phi_1$ is larger than another certain value, the bullwhip effect becomes smaller with $\alpha$ becoming larger.

4.5.1.2 MATLAB code for varied (LTr_1 & $\alpha$) on bullwhip effect

Lead time of retailers_1 is kept constant, market share ($\alpha$) value are varied, to find Bullwhip effect for sets of (LTr_1, $\alpha$), lead times of retailers_1 and retailers_2 are same (=3), ($\phi_1$)= ($\phi_2$) (=0.6) and BWE is measured by changing lead time of retailers_1 again.

```matlab
Ldt1= [2 3 4];
Leadt2 =3;
ap = 0:0.1:1.0;
py1 =0.6; py2 =0.6;
for j =1: length(ap)
    alpa = ap(j);
    for i = 1:length(Ldt1)
        Leadt1 = Ldt1(i);
        BWE_MM = bwe_mmse(py1,py2,Leadt1,Leadt2,alpa);
        asd(:,i) = [ BWE_MM];
    end
    gdata2(j,:) = [ap(j) asd];
end
```
disp('Measurement of BWE w.r.t MMSE')
disp('Effect of (LTr1 & \alpha) on bullwhip effect')
disp(gdata2)
subplot(2,2,2)
xaxis = gdata2(:,1);
yaxis = gdata2(:,2:4);
plot (xaxis,yaxis);
title('Effect of (LTr1 & \alpha) on bullwhip effect')
xlabel(' Market share \alpha ')
ylabel('Bullwhip Effect')
legend('LTr_1=2','LTr_1=3','LTr_1=4');

<table>
<thead>
<tr>
<th>α</th>
<th>Bullwhip effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.047 2.752 2.640 2.551 2.485 2.442 2.423 2.427 2.455 2.505</td>
</tr>
<tr>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Table: 4.2 Impact LTr_1 & α on bullwhip effect

Fig 4.3 Effect of (LTr_1& \alpha) on Bullwhip effect in MMSE
Fig 4.3. Reveals that the Bullwhip Effect decreases gradually to the minimum value and then it begins to increase with the increase of $\alpha$. For the different LTr_1, the larger it is, the larger the bullwhip effect is. The graph suggests that when retailer_1 occupies half of the market share and for short lead time, the bullwhip effect value will be the lowest.

**4.5.1.3. MATLAB programme code for varying (lead time retailer_1 & $\phi_1$) and to measure bullwhip effect.**

Lead time of retailers_1 is kept constant, $\phi_1$ value is varied, to find Bullwhip effect for sets of (LTr_1, $\alpha$), lead times of retailers _2 (=3),$\phi_2 $ =0.6 $\alpha=0.4$ and BWE is measured by changing lead time of retailers_1 again.

```matlab
p1= 0.1:0.1:1.0;
p2=0.6;
L1= [2 3 4];
L2= 3;
alpha = 0.4;
for j = 1:length(p1)
    py1 = p1(j);
    for i =1: length(L1)
        alpha = L1(i);
        BWE_MM = bwe_mmse(py1,p2,L1,L2,alpa);
        asd(:,i) = [ BWE_MM];
    end
    gdata3(j,:) = [p1(j) asd];
end
disp('Measurement of BWE w.r.t MMSE')
disp('Impact of ($\alpha$ & LTr_1)on bullwhip effect ')
disp(gdata3)
subplot(2,2,3)
xaxis = gdata3(:,1);
yaxis = gdata3(:,2:4);```
```matlab
plot(xaxis,yaxis);
title('Impact of (\alpha & LTr_1) on bullwhip effect');
xlabel('Market share \alpha');
ylabel('Bullwhip Effect');
legend('Ltr_1=2','LTr_1=3','LTr_1=4');
```

Table 4.3: Impact of LTr_1 and $\phi_1$ on bullwhip effect

<table>
<thead>
<tr>
<th>$\phi_1$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTr_2</td>
<td>2.351</td>
<td>2.406</td>
<td>2.461</td>
<td>2.509</td>
<td>2.543</td>
<td>2.551</td>
<td>2.519</td>
<td>2.433</td>
<td>2.270</td>
<td>2.153</td>
</tr>
<tr>
<td>LTr_3</td>
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<td>2.413</td>
<td>2.486</td>
<td>2.566</td>
<td>2.647</td>
<td>2.715</td>
<td>2.747</td>
<td>2.704</td>
<td>2.530</td>
<td>2.370</td>
</tr>
<tr>
<td>LTr_4</td>
<td>2.352</td>
<td>2.415</td>
<td>2.493</td>
<td>2.589</td>
<td>2.702</td>
<td>2.820</td>
<td>2.919</td>
<td>2.945</td>
<td>2.793</td>
<td>2.598</td>
</tr>
</tbody>
</table>

Fig 4.4 Impact of ($\phi_1$ & LTr_1) on Bullwhip effect in MMSE
Fig 4.3 reveals the impact of $\phi_1$ on the bullwhip effect, and observed that the bullwhip effect increases gradually to the maximum value with the increase of $\phi_1$, and the bullwhip effect begins to decrease rapidly when it reaches the maximum value. The bullwhip effect does not occur only when $\phi_1$ increases to a certain value, and those values are different for different $LT_{r_1}$. For different $Lr_1$, the larger it is, the larger the bullwhip effect.

4.5.2. The Measure of the Bullwhip Effect under the MA Forecasting Method.

In the MA forecasting method, the lead-time demand of Retailer_1 can be determined as follows:

$$D_{r1,t}^i = \frac{ln}{k} \sum_{i=1}^{k} D_{r1,t-i}$$  \hspace{1cm} (4.28)

Therefore, the total order quantity of the two Retailers’ is

$$q_t = (1 + \frac{l_1}{k})D_{r1,t-1} - \frac{l_1}{k}D_{r1,t-k-1} + (1 + \frac{l_2}{k})D_{r2,t-1} - \frac{l_2}{k}D_{r2,t-k-1}.$$

Var ($q_t$)

$$= (1 + 2\frac{l_1}{k} + 2(\frac{l_1}{k})^2 - 2\phi_1^k(1 + \frac{l_1}{k})^2) \text{Var} (D_{r1,t}) + (1 + 2\frac{l_2}{k} + 2(\frac{l_2}{k})^2 - 2\phi_2^k(1 + \frac{l_2}{k})^2) \text{Var} (D_{r2,t}) + 2((1 + \frac{l_1}{k})(1 + \frac{l_2}{k}) - (1 + \frac{l_1}{k})\frac{l_2}{k} \alpha_1^k - \frac{l_1}{k}(1 + \frac{l_2}{k})\alpha_2^k + \frac{l_1l_2}{k}\alpha_{1-\alpha}) \text{Var} (D_{r2,t})$$

\hspace{1cm} (4.30)

For simplicity above can be written as

$$\text{Var} (q_t) = G_1 \text{Var} (D_{r1,t}) + G_2 \text{Var} (D_{r2,t}) + G_3 \frac{\alpha}{1-\alpha} \text{Var} (D_{r2,t}).$$ \hspace{1cm} (4.31)

In above $G_1$ is the coefficient of $\text{Var} (D_{r1,t}), G_2$ is the coefficient of $\text{Var} (D_{r2,t}),$ and $G_3$ is the coefficient of $\alpha/(1-\alpha) \text{Var} (D_{r2,t}).$

Then, the BWE in MA forecasting method is

$$BWE_{MA} = \frac{\text{Var} (q_t)}{\text{Var} (D_{r2,t})} = \frac{\alpha^2 \phi_1 + (1-\alpha)^2 \phi_2 + \alpha(1-\alpha)\phi_3}{\alpha^2 + (1-\alpha)^2 + 2\alpha(1-\alpha)}.$$

\hspace{1cm} (4.32)

4.5.2.1. MATLAB code to Measurement of BWE Impact of ($\phi_1$ & $\alpha$) on Bullwhip effect

Market share ($\alpha$) is kept constant, autoregressive coefficient ($\phi_1$) retailers _1 value is varied, to find Bullwhip effect for sets of ($\phi_1$, $\alpha$), and reaming values of
lead times of retailers _1 and retailers _2 are same (=3), autoregressive coefficient ($\phi_2$) retailers _2 (=0.6) and BWE is measured by changing $\alpha$ again, since MATLAB software can perform efficiently the arithmetic and produce graphs. code is developed for the model in MATLAB.

```matlab
p1= -1.0:0.1:1.0;
py2=0.6;
Leadt1= 3; Leadt2= 3;
k=3;
ap = [0.4 0.6 0.8];

for j = 1: length(p1)
    py1 = p1(j);
    for i = 1: length(ap)
        alpa = ap(i);
        BWE_MA = bwe_ma(py1,py2,Leadt1,Leadt2,alpa,k);
        asd(:,i) = [ BWE_MA];
    end
    mdata1(j,:) = [p1(j) asd];
end
%disp(mdata1)
figure
xaxis = mdata1(:,1);
yaxis = mdata1(:,2:4);
subplot(2,3,1)
plot (xaxis,yaxis);
title('Effect of (LTr1 & alfa) on bullwhip effect ')
xlabel(' Autoregressive coefficent \( \phi \)')
ylabel('Bullwhip Effect')
legend('LTr_1=2','LTr_1=3','LTr_1=4');
```
Table 4.4: Impact ($\alpha$ and $\phi_1$) on bullwhip effect

<table>
<thead>
<tr>
<th>$\phi_1$</th>
<th>$\alpha=0.4$</th>
<th>$\alpha=0.6$</th>
<th>$\alpha=0.8$</th>
<th>$\phi_1$</th>
<th>$\alpha=0.4$</th>
<th>$\alpha=0.6$</th>
<th>$\alpha=0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6.0816</td>
<td>7.0544</td>
<td>6.5808</td>
<td>0.1</td>
<td>4.48</td>
<td>4.652</td>
<td>4.1784</td>
</tr>
<tr>
<td>-0.9</td>
<td>5.648</td>
<td>6.404</td>
<td>5.9304</td>
<td>0.2</td>
<td>4.4688</td>
<td>4.6352</td>
<td>4.1616</td>
</tr>
<tr>
<td>-0.8</td>
<td>5.3008</td>
<td>5.8832</td>
<td>5.4096</td>
<td>0.3</td>
<td>4.4384</td>
<td>4.5896</td>
<td>4.116</td>
</tr>
<tr>
<td>-0.7</td>
<td>5.0304</td>
<td>5.4776</td>
<td>5.004</td>
<td>0.4</td>
<td>4.3792</td>
<td>4.5008</td>
<td>4.0272</td>
</tr>
<tr>
<td>-0.6</td>
<td>4.8272</td>
<td>5.1728</td>
<td>4.6992</td>
<td>0.5</td>
<td>4.2816</td>
<td>4.3544</td>
<td>3.8808</td>
</tr>
<tr>
<td>-0.5</td>
<td>4.6816</td>
<td>4.9544</td>
<td>4.4808</td>
<td>0.6</td>
<td>4.136</td>
<td>4.136</td>
<td>3.6624</td>
</tr>
<tr>
<td>-0.4</td>
<td>4.584</td>
<td>4.808</td>
<td>4.3344</td>
<td>0.7</td>
<td>3.9328</td>
<td>3.8312</td>
<td>3.3576</td>
</tr>
<tr>
<td>-0.3</td>
<td>4.5248</td>
<td>4.7192</td>
<td>4.2456</td>
<td>0.8</td>
<td>3.6624</td>
<td>3.4256</td>
<td>2.952</td>
</tr>
<tr>
<td>-0.2</td>
<td>4.4944</td>
<td>4.6736</td>
<td>4.2</td>
<td>0.9</td>
<td>3.3152</td>
<td>2.9048</td>
<td>2.4312</td>
</tr>
<tr>
<td>-0.1</td>
<td>4.4832</td>
<td>4.6568</td>
<td>4.1832</td>
<td>1</td>
<td>2.8816</td>
<td>2.2544</td>
<td>1.7808</td>
</tr>
</tbody>
</table>

Fig 4.5: Impact of ($\phi_1$ & $\alpha$) on Bullwhip effect under the MA

4.5.2.1 The Analysis of Parameters under the MA Forecasting Method.

Fig 4.5 - 4.9 simulate the expression of the bullwhip effect under the MA forecasting method to illustrate the impact of parameters on the bullwhip effect.
Fig 4.4 shows that the bullwhip effect decreases with the Increase of $\phi_1$. With different $\alpha$ the bullwhip effect changes only a little. So $\alpha$ hardly affects the bullwhip effect in the circumstance of different $\phi_1$.

4.5.2.2. Measure the impact of ($\phi_1$ & $k$) on Bullwhip effect under MA

Moving average ($k$) is kept constant, autoregressive coefficient ($\phi_1$) retailers _1 value is varied, to find Bullwhip effect for sets of ($\phi_1$, $k$), and reaming values of lead times of retailers _1 and retailers _2 are same (=3), autoregressive coefficient ($\phi_2$) retailers _2 (=0.6) and BWE is measured by changing $\alpha$ again, MATLAB software can perform efficiently of arithmetic and produce graphs. code is developed for the model in MATLAB.

4.5.2.2 MATLAB CODE: Measure the Impact Of ($\phi_1$ & $K$) On Bullwhip Effect under MA

```matlab
py1=0.6; py2=0.6;
Leadt2= 3; Leadt1 = 3;
k1 = [3 5 7];
ap = 0:0.1:1.0;
for j = 1:length(ap)
    alpa = ap(j);
    for i = 1:length(k1)
        k = k1(i);
        BWE_MA = bwe_ma(py1,py2,Leadt1,Leadt2,alpa,k);
        asd(:,i) = [ BWE_MA ];
    end
    mdata4(j,:) = [ap(j) asd];
end
%disp(mdata4);
xaxis = mdata4(:,1);
yaxis = mdata4(:,2:4);
subplot(2,3,4)
plot (xaxis,yaxis);
title('Effect of (LTr1 & $\alpha$) on bullwhip effect ')
xlabel(' Autoregressive cofficent $\phi$');```
Table: 4.5 Impact of ($\phi_1$ & k) on Bullwhip effect

<table>
<thead>
<tr>
<th>$\phi_1$</th>
<th>K=3</th>
<th>K=5</th>
<th>K=7</th>
<th>$\phi_1$</th>
<th>K=3</th>
<th>K=5</th>
<th>K=7</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.000</td>
<td>6.082</td>
<td>3.439</td>
<td>2.556</td>
<td>0.000</td>
<td>4.482</td>
<td>2.671</td>
<td>2.066</td>
</tr>
<tr>
<td>-0.900</td>
<td>5.648</td>
<td>3.231</td>
<td>2.423</td>
<td>0.100</td>
<td>4.480</td>
<td>2.670</td>
<td>2.065</td>
</tr>
<tr>
<td>-0.800</td>
<td>5.301</td>
<td>3.064</td>
<td>2.317</td>
<td>0.200</td>
<td>4.469</td>
<td>2.665</td>
<td>2.062</td>
</tr>
<tr>
<td>-0.700</td>
<td>5.030</td>
<td>2.935</td>
<td>2.234</td>
<td>0.300</td>
<td>4.438</td>
<td>2.650</td>
<td>2.052</td>
</tr>
<tr>
<td>-0.600</td>
<td>4.827</td>
<td>2.837</td>
<td>2.172</td>
<td>0.400</td>
<td>4.379</td>
<td>2.622</td>
<td>2.034</td>
</tr>
<tr>
<td>-0.500</td>
<td>4.682</td>
<td>2.767</td>
<td>2.127</td>
<td>0.500</td>
<td>4.282</td>
<td>2.575</td>
<td>2.005</td>
</tr>
<tr>
<td>-0.400</td>
<td>4.584</td>
<td>2.720</td>
<td>2.097</td>
<td>0.600</td>
<td>4.136</td>
<td>2.505</td>
<td>1.960</td>
</tr>
<tr>
<td>-0.300</td>
<td>4.525</td>
<td>2.692</td>
<td>2.079</td>
<td>0.700</td>
<td>3.933</td>
<td>2.408</td>
<td>1.898</td>
</tr>
<tr>
<td>-0.200</td>
<td>4.494</td>
<td>2.677</td>
<td>2.070</td>
<td>0.800</td>
<td>3.662</td>
<td>2.278</td>
<td>1.815</td>
</tr>
<tr>
<td>-0.100</td>
<td>4.483</td>
<td>2.672</td>
<td>2.066</td>
<td>0.900</td>
<td>3.315</td>
<td>2.111</td>
<td>1.709</td>
</tr>
<tr>
<td>0.000</td>
<td>4.482</td>
<td>2.671</td>
<td>2.066</td>
<td>1.000</td>
<td>2.882</td>
<td>1.903</td>
<td>1.576</td>
</tr>
</tbody>
</table>

Fig 4.6: Impact of ($\phi_1$ & k) on Bullwhip effect under the MA
Fig 4.6 indicates that the bullwhip effect decreases slowly all the time with the increase of $\phi_1$. And, in this situation, to shift the span (number of periods) $k$ of retailer_1, and to find that the bullwhip effect becomes smaller with the increase of $k$. The conclusion that $k$ is a key factor to affect the bullwhip effect.

4.5.2.3. MATLAB Code to study the impact of ($\Phi$ & Ltr_1) On Bullwhip Effect

Lead times of retailers _1 is kept constant, autoregressive coefficient ($\phi_1$) retailers _1 value is varied, to find Bullwhip effect for sets of ($\phi_1$,LTr_1), and rearming values of lead times of retailers _2 are same (=3), autoregressive coefficient ($\phi_2$) retailers _2 (=0.6) and BWE is measured by changing $\alpha$ again, MATLAB software can perform efficiently arithmetic and produce graphs. Code is developed for the model in MATLAB.

```matlab
py1=0.6; py2=0.6;
Leadt2= 3; Ld1 = [2 3 4];
k =3;
ap = 0:0.1:1.0;
for j = 1:length(ap)
    alpa = ap(j);
    for i = 1:length(Ld1)
        Leadt1 = Ld1(i);
        BWE_MA =   bwe_ma(py1,py2,Leadt1,Leadt2,alpa,k);
        asd(:,i) = [ BWE_MA];
    end
    mdata3(j,:) = [ap(j) asd];
end
%disp(mdata3);
xaxis = mdata3(:,1);
yaxis = mdata3(:,2:4);
subplot(2,3,3)
plot (xaxis,yaxis);
title('Effect of (LTr1 & $\phi$) on bullwhip effect')
```
xlabel('Autoregressive coefficient \(\phi\))
ylabel('Bullwhip Effect')
legend('LTr_1=2','LTr_1=3','LTr_1=4');

Table 4.6 Impact of (\(\phi\) & LTr_1) on bullwhip effect

<table>
<thead>
<tr>
<th>(\phi)</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTr_3</td>
<td>4.136</td>
<td>4.520</td>
<td>4.820</td>
<td>5.033</td>
<td>5.161</td>
<td>5.204</td>
<td>5.161</td>
<td>50.330</td>
<td>4.820</td>
<td>4.520</td>
<td>4.120</td>
</tr>
</tbody>
</table>

Fig: 4.7 Impact of (\(\phi\) & LTr_1) on bullwhip effect under the MA
By observing fig 4.7, when $\phi$ takes the value of zero; the bullwhip effect value is fixed no matter what value $LTr_1$ is. Then the bullwhip effect value increases to the maximum value firstly, and after that it begins to decrease with the increase of $\alpha$. In this situation, $LTr_1$ influences the BWE obviously, the smaller the $LTr_1$ is, and the smaller the BWE.

4.5.2.4. Impact of ($\alpha$ and k) on Bullwhip Effect

Market share of retailers _1 is kept constant, number of periods (k) retailers _1 value is varied, to find Bullwhip effect for sets of ($k \cdot LTr_1$), and remaining values of lead times of retailers _2 are same (=3), autoregressive coefficient ($\phi_2$) retailers _2 (=0.6) and BWE is measured by changing $\alpha$ again, MATLAB software is written for this purpose and presented.

```matlab
py1= -1.0:0.1:1.0; py2=0.6;
Leadt2 = 3; Ldt1 = 3
k = [2 3 4];
alpha = 0.4;
for j = 1: length(p1)
    py1 = p1(j);
    for i = 1:length(Ld1)
        Leadt1 = Ld1(i);
        BWE_MA = bwe_ma(py1,py2,Leadt1,Leadt2,alpha,k);
        asd(:,i) = [ BWE_MA];
    end
    mdata5(j,:) = [p1(j) asd];
end
%disp(mdata5);

xaxis = mdata5(:,1);
yaxis = mdata5(:,2:4);
subplot(2,3,5)
plot (xaxis,yaxis);
title('Effect of (PHY & $\alpha$) on bullwhip effect')
```
Table 4.7: Impact of (α and k) on Bullwhip Effect

<table>
<thead>
<tr>
<th>$\phi_i$</th>
<th>Bullwhip Effect</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k=3</td>
<td>k=5</td>
<td>k=7</td>
<td></td>
<td>k=3</td>
<td>k=5</td>
</tr>
<tr>
<td>-1.000</td>
<td>4.942</td>
<td>6.082</td>
<td>7.364</td>
<td>0.100</td>
<td>3.787</td>
<td>4.483</td>
</tr>
<tr>
<td>-0.900</td>
<td>4.629</td>
<td>5.648</td>
<td>6.790</td>
<td>0.000</td>
<td>3.786</td>
<td>4.482</td>
</tr>
<tr>
<td>-0.800</td>
<td>4.378</td>
<td>5.301</td>
<td>6.331</td>
<td>0.100</td>
<td>3.785</td>
<td>4.480</td>
</tr>
<tr>
<td>-0.700</td>
<td>4.183</td>
<td>5.030</td>
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<td>3.777</td>
<td>4.469</td>
</tr>
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<td>0.300</td>
<td>3.755</td>
<td>4.438</td>
</tr>
<tr>
<td>-0.500</td>
<td>3.931</td>
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<td>5.512</td>
<td>0.400</td>
<td>3.712</td>
<td>4.379</td>
</tr>
<tr>
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</tr>
<tr>
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<td>3.817</td>
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<td>0.600</td>
<td>3.537</td>
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</tr>
<tr>
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</tr>
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<td>0.800</td>
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<td>3.662</td>
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<td>5.248</td>
<td>0.900</td>
<td>2.944</td>
<td>3.315</td>
</tr>
<tr>
<td>0.100</td>
<td>3.785</td>
<td>4.480</td>
<td>5.246</td>
<td>1.000</td>
<td>2.631</td>
<td>2.882</td>
</tr>
</tbody>
</table>

Fig 4.8 Impact of (α and k) on Bullwhip Effect under the MA
The above Fig shows that the trend of the bullwhip effect is increased first to the maximum and declined gradually with the increase of $\alpha$. The larger the $k$ is, the smaller the BWE.

### 4.5.2.5 Impact of ($\phi_1$ and $lr_1$) on bullwhip effect

Lead times of retailers _1 is kept constant, autoregressive coefficient ($\phi_1$) retailers _1 value is varied, to find Bullwhip effect for sets of ($\phi_1$, LTr_1), and remaining values of lead times of retailers _2 are same (=3), autoregressive coefficient ($\phi_2$) retailers _2 (=0.6), periods moving (k=3) and BWE is measured by changing $\alpha$ again, MATLAB software can perform efficiently arithmetic and produce graphs. Code is developed for the model in MATLAB.

<table>
<thead>
<tr>
<th>$\phi_1$</th>
<th>LTr_1=2</th>
<th>LTr_1=3</th>
<th>LTr_1=4</th>
<th>$\phi_1$</th>
<th>LTr_1=2</th>
<th>LTr_1=3</th>
<th>LTr_1=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.79</td>
<td>4.48</td>
<td>5.25</td>
<td>0</td>
<td>3.79</td>
<td>4.48</td>
<td>5.25</td>
</tr>
<tr>
<td>-0.1</td>
<td>3.79</td>
<td>4.48</td>
<td>5.25</td>
<td>0.9</td>
<td>2.94</td>
<td>3.32</td>
<td>3.71</td>
</tr>
<tr>
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<td>4.48</td>
<td>5.25</td>
<td>0.8</td>
<td>3.19</td>
<td>3.66</td>
<td>4.16</td>
</tr>
<tr>
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<td>3.79</td>
<td>4.48</td>
<td>5.25</td>
<td>0.7</td>
<td>3.39</td>
<td>3.93</td>
<td>4.52</td>
</tr>
<tr>
<td>-0.4</td>
<td>3.79</td>
<td>4.48</td>
<td>5.25</td>
<td>0.6</td>
<td>3.54</td>
<td>4.14</td>
<td>4.79</td>
</tr>
<tr>
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<td>4.48</td>
<td>5.25</td>
<td>0.5</td>
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</tr>
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<td>4.48</td>
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<td>3.71</td>
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<tr>
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<td>4.48</td>
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<td>3.76</td>
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<tr>
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<td>5.25</td>
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<td>5.25</td>
<td>0.1</td>
<td>3.79</td>
<td>4.48</td>
<td>5.25</td>
</tr>
<tr>
<td>-1.0</td>
<td>3.79</td>
<td>4.48</td>
<td>5.25</td>
<td>0</td>
<td>3.79</td>
<td>4.48</td>
<td>5.25</td>
</tr>
</tbody>
</table>
Fig 4.9 Impact of \((\phi_1 \text{ and } \text{LTr}_1)\) on bullwhip effect under the MA

Fig 4.9 which is similar to fig 4.6 simulates the impact of \(\phi_1\) on the bullwhip effect based on different \(\text{LTr}_1\). The bullwhip effect decreases slowly all the time with the increase of \(\phi_1\). And to find that the \(\text{LTr}_1\) influences the BWE obviously; the bullwhip effect becomes smaller with the decrease of \(\text{LTr}_1\).

4.5.3. The Measure of the Bullwhip Effect under the ES Forecasting Method.

Using the ES forecasting method, the lead-time demand of Retailer_1 can be determined as follows:

\[
\hat{D}_{r,t} = \beta D_{r,t-1} + (1-\beta)\hat{D}_{r,t-1}
\]

\[
\hat{D}_{r,t} = \hat{D}_{r,t} \geq 2, \text{ the total order quantity of two Retailers’ at period } t \text{ is}
\]

\[
q_t = (1 + \beta_1 l_1) D_{r1,t} - \beta_1 l_1 \hat{D}_{r1,t} + (1 + \beta_2 l_2) D_{r2,t} - \beta_2 l_2 \hat{D}_{r2,t}
\]

The variance of the total order quantity,
Var \( (q_t) = (1 + \beta_1 l_1)^2 \) \ Var \( (D_{r1,t}) + (\beta_1 l_1)^2 \) \ Var \( (\hat{D}_{r1,t}) + (1 + \beta_2 l_2)^2 \) \ Var \\
\( (D_{r2,t}) + (\beta_2 l_2)^2 \) \ Var \( (D_{r2,t}) -2\beta_1 l_1 (1 + \beta_1 l_1) \) \ Cov \( (D_{r1,t}, \hat{D}_{r1,t}) + 2(1 + \beta_1 l_1)(1 + \beta_2 l_2) \) \ Cov \( (D_{r1,t}, D_{r2,t}) -2\beta_2 l_2 (1 + \beta_1 l_1) \) \ Cov \( (D_{r1,t}, \hat{D}_{r2,t}) -2\beta_1 l_1 (1 + \beta_2 l_2) \) \ Cov \( (D_{r1,t}, D_{r2,t}) + 2\beta_1 l_1 \beta_2 l_2 \) \ Cov \( (\hat{D}_{r1,t}, \hat{D}_{r2,t}) -2\beta_2 l_2 (1 + \beta_1 l_1) \) \ Cov \( (D_{r2,t}, \hat{D}_{r2,t}) \).

\[ \text{------------------} \quad (4.35) \]

\[ \text{BWE}_{\text{ES}} = \frac{\text{Var}(q_t)}{\text{Var}(D_t)} \]

\[ = \frac{J_1 \text{Var}(D_{r1,t}) + J_2 + J_3 (\alpha/(1 - \alpha)) \text{Var}(D_{r2,t})}{\text{Var}(D_{r1,t}) + \text{Var}(D_{r2,t}) + 2(\alpha/(1 - \alpha)) \text{Var}(D_{r2,t})} \]

\[ = \frac{J_1 (\text{Var}(D_{r1,t})/\text{Var}(D_{r2,t})) + J_2 + J_3 (\alpha/(1 - \alpha))}{(\text{Var}(D_{r1,t})/\text{Var}(D_{r2,t})) + 1 + 2(\alpha/(1 - \alpha))} \]

\[ = \frac{J_1 (\alpha/(1 - \alpha))^2 + J_2 + J_3 (\alpha/(1 - \alpha))}{(\alpha/(1 - \alpha))^2 + 1 + 2(\alpha/(1 - \alpha))} \]

\[ = \frac{\alpha^2 J_1 (1-\alpha)+J_2 \alpha (1-\alpha) J_3}{\alpha^2 + (1-\alpha)^2 + 2\alpha (1-\alpha)} \quad \text{------------------} (4.36) \]

### 4.5.3.1. The Analysis of Parameters under the ES Forecasting Method

Fig 4.10 -4.15 simulates the expression of the bullwhip effect under the ES forecasting method to illustrate the impact of parameters on the bullwhip effect.

```matlab
p1 = -1:0.1:1.0;
p2 = 0.6;
Lead1 = 3;   Lead2 = 3;
b1 = 0.4;   b2 = 0.4;
ap = [0.4 0.7 0.8];
for j = 1: length(p1)
p1 = p1(j);
for i = 1: length(ap)
alpha = ap(i);
%\text{x}=[p1 p2 Lead1 Lead2 b1 b2 alpha]
BWE_ES = bwe_es(p1,p2,Lead1,Lead2,b1,b2,alpa);
```
asd(:,i) = [ BWE_ES];
end
Edat1(j,:) = [p1(j) asd];
end

disp(Edat1);
figure
xaxis = Edat1(:,1);
yaxis = Edat1(:,2:4);
subplot(2,3,1)
plot (xaxis,yaxis);
title('Effect of (LTr1 & \alpha) on bullwhip effect')
xlabel('Autoregressive coefficient \phi')
ylabel('Bullwhip Effect')
legend('k=3','k=5','k=7');

<table>
<thead>
<tr>
<th>(\phi_1)</th>
<th>Bullwhip effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha=0.4)</td>
<td>(\alpha=0.6)</td>
</tr>
<tr>
<td>-1</td>
<td>4.798</td>
</tr>
<tr>
<td>-0.9</td>
<td>4.782</td>
</tr>
<tr>
<td>-0.8</td>
<td>4.753</td>
</tr>
<tr>
<td>-0.7</td>
<td>4.722</td>
</tr>
<tr>
<td>-0.6</td>
<td>4.687</td>
</tr>
<tr>
<td>-0.5</td>
<td>4.649</td>
</tr>
<tr>
<td>-0.4</td>
<td>4.606</td>
</tr>
<tr>
<td>-0.3</td>
<td>4.558</td>
</tr>
<tr>
<td>-0.2</td>
<td>4.505</td>
</tr>
<tr>
<td>-0.1</td>
<td>4.444</td>
</tr>
<tr>
<td>0</td>
<td>4.376</td>
</tr>
</tbody>
</table>
From Fig 4.10 declares the impact of $\phi_1$ under various $\alpha$. The bullwhip effect decreases continuously with the increase of $\alpha$. When $\phi_1$ takes the value of 0.7 approximately, the bullwhip effect values of three different $\alpha$ are the same; when $\phi_1$ is less than 0.7, the smaller the $\alpha$ is, the smaller the bullwhip effect value is; when $\phi_1$ is more than 0.7, the result is opposite.

4.5.3.2 Impact of ($\phi_1, \beta_1$) on bullwhip effect.

As observed that from Fig 4.11, the bullwhip effect decreases all the time in pace with the increase of $\phi_1$. And, in this situation, to observe that the smaller the $\beta_1$ is, the smaller the bullwhip effect value is. This phenomenon shows that $\beta_1$ is an important factor to influence the bullwhip effect under the ES forecasting method; The MATLAB is developed for this model.

Programme code: Impact of ($\phi_1, \beta_1$) on bullwhip effect.

```
p1  = -1.0:0.1:1.0;
py2 = 0.6;
Leadt1 = 3;  Leadt2 = 3;
```
\( \alpha = 0.4; \ \beta_2 = 0.4; \)

\( b_1 = [0.4 \ 0.6 \ 0.8]; \)

\textbf{for} \( j = 1: \text{length}(p1) \)

\( py_1 = p1(j); \)

\textbf{for} \( i = 1: \text{length}(b1) \)

\( \beta_1 = b1(i); \)

\( \text{BWE\_ES} = \text{bwe\_es}(py1,py2,\text{Leadt1},\text{Leadt2},\beta_1,\text{beta2},\alpha); \)

\( \text{asd}(;i) = \{ \text{BWE\_ES} \}; \)

\textbf{end}

\( \text{Edata2}(j,:) = [p1(j) \text{ asd}]; \)

\textbf{end}

\%disp(Edata2);

\( xaxis = \text{Edata2}(;1); \)

\( yaxis = \text{Edata2}(;2:4); \)

subplot(2,3,2)

plot(xaxis,yaxis);

title('Impact of (PHY1 & \alpha) on bullwhip effect ')

xlabel('Autoregressive cofficent \phi')

ylabel('Bullwhip Effect')

legend('\alpha=0.4','\alpha=0.6','\alpha=0.8');

\begin{table}
\centering
\caption{Impact of \((\phi_1, \beta_1)\) on bullwhip effect.}
\begin{tabular}{|c|cccc|cccc|}
\hline
\( \Omega \) & \( \beta_1=0.4 \) & \( \beta_1=0.6 \) & \( \beta_1=0.8 \) & \( \Omega \) & \( \beta_1=0.4 \) & \( \beta_1=0.6 \) & \( \beta_1=0.8 \) \\
\hline
-1 & 4.798 & 6.271 & 8.239 & 0 & 4.376 & 5.490 & 6.719 \\
-0.9 & 4.782 & 6.262 & 8.161 & 0.1 & 4.298 & 5.356 & 6.502 \\
-0.8 & 4.753 & 6.202 & 8.033 & 0.2 & 4.210 & 5.208 & 6.269 \\
-0.7 & 4.722 & 6.137 & 7.898 & 0.3 & 4.107 & 5.041 & 6.016 \\
-0.6 & 4.687 & 6.067 & 7.757 & 0.4 & 3.988 & 4.852 & 5.739 \\
-0.5 & 4.649 & 5.991 & 7.608 & 0.5 & 3.848 & 4.637 & 5.434 \\
-0.4 & 4.606 & 5.908 & 7.451 & 0.6 & 3.682 & 4.389 & 5.094 \\
\hline
\end{tabular}
\end{table}
4.5.3.3 Impact of (α, β1) on bullwhip effect

p1 = -1.0:0.1:1.0;
py2 = 0.6;
alpa = 0.4;  Leadt2 = 3;
beta1 = 0.4; beta2 = 0.4;
Ld1 = [2 3 4];

for j = 1: length(p1)
    py1 = p1(j);
    for i = 1: length(Ld1)
        Leadt1 = Ld1(i);
        BWE_ES = bwe_es(py1,py2,Leadt1,Leadt2,beta1,beta2,alpa);
        asd(:,i) = [ BWE_ES];
    end
    Edata3(j,:) = [p1(j) asd];
end

Table: 4.11 Impact of (α, β1) on bullwhip effect

<table>
<thead>
<tr>
<th>α</th>
<th>Bullwhip effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.00</td>
</tr>
</tbody>
</table>
Fig 4.12: Impact of \((\alpha, \beta_1)\) on bullwhip effect under the ES

Fig 4.12 indicates the impact of \(\alpha\) on bullwhip effect for different \(\beta_1\) under the ES. When \(\beta_1\) takes the value of 0.4, the bullwhip effect decreases slowly first, after that it increases gradually. However, when the value of \(\beta_1\) is 0.6 and 0.8, the bullwhip effect keeps increasing rapidly.
4.5.3.4 Impact of \((\alpha, L_{T_1})\) on bullwhip effect

\[
p1 = -1.0:0.1:1.0;
\]
\[
py2 = 0.6;
\]
\[
alpa = 0.4; \quad \text{Leadt2} = 3;
\]
\[
\text{beta1} = 0.4; \quad \text{beta2} = 0.4;
\]
\[
Ld1 = [2 3 4];
\]
\[
\text{for } j = 1: \text{length}(p1)
\]
\[
\text{py1} = p1(j);
\]
\[
\text{for } i = 1: \text{length}(Ld1)
\]
\[
\text{Leadt1} = Ld1(i);
\]
\[
\text{BWE\_ES} = \text{bwe\_es(py1,py2,Leadt1,Leadt2,beta1,beta2,alpa)};
\]
\[
\text{asd(:,i)} = [\text{BWE\_ES}];
\]
\[
\text{end}
\]
\[
\text{Edata3(j,:)} = [p1(j) \text{ asd}];
\]
\[
\text{end}
\]
\[
\%\text{disp(Edata3)};
\]
\[
\text{xaxis} = \text{Edata3(:,1)};
\]
\[
\text{yaxis} = \text{Edata3(:,2:4)};
\]
\[
\text{subplot}(2,3,3)
\]
\[
\text{plot(xaxis,yaxis)}
\]
\[
\text{title('Impact of } (\phi_1 \& \beta_1) \text{ on bullwhip effect')}
\]
\[
\text{xlabel('\text{Autoregressive cofficient } \phi')}
\]
Fig 4.13 reveals that the impact of $\alpha$ on bullwhip effect for different LTr_1 under the ES. When LTr_1 takes the values of 0.4 and 0.6, the bullwhip effect decreases slowly first, after that it increases gradually. And the minimum value of the bullwhip effect occurs as the value of $\alpha$ is 0.5 approximately. This phenomenon indicates that the intense competition between two retailers can increase the
bullwhip effect. However, when the value of LTr_1 is 0.8, the bullwhip effect keeps increasing rapidly.

### 4.5.3.5 Impact of $\phi_1$ & LTr_1 on Bullwhip Effect

```matlab
py1 = 0.6; py2 = 0.6;
beta1 = 2; Leadt2 = 3;
beta2 = 0.4; ap = 0.0:0.1:1.0;
Ld1 = [2 4 6];
for j = 1: length(ap)
    alpa = ap(j);
    for i = 1: length(Ld1)
        Leadt1 = Ld1(i);
        BWE_ES = bwe_es(py1,py2,Leadt1,Leadt2,beta1,beta2,alpa);
        asd(:,i) = [ BWE_ES];
    end
    Edata5(j,:) = [ap(j) asd];
end
%disp( Edata4);
xaxis = Edata5(:,1);
yaxis = Edata5(:,2:4);
subplot(2,3,5)
plot (xaxis,yaxis);
title('Comparison of forecasting methods ($\alpha$, (k = 3)) on bullwhip effect ')
xlabel('Autoregressive coefficient $\phi_1$')
ylabel('Bullwhip Effect')
legend(['LTr_1=2','LTr_1=3','LTr_1=4']);
```

Table: 4.13 Impact of ($\phi_1$ & Lr_1) on Bullwhip Effect

<table>
<thead>
<tr>
<th>$\phi_1$</th>
<th>LTr_1=2</th>
<th>LTr_1=3</th>
<th>LTr_1=4</th>
<th>$\phi_1$</th>
<th>LTr_1=2</th>
<th>LTr_1=3</th>
<th>LTr_1=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.9</td>
<td>3.990</td>
<td>4.782</td>
<td>5.637</td>
<td>0</td>
<td>3.698</td>
<td>4.376</td>
<td>5.106</td>
</tr>
<tr>
<td>-0.8</td>
<td>3.970</td>
<td>4.753</td>
<td>5.599</td>
<td>0.1</td>
<td>3.641</td>
<td>4.298</td>
<td>5.005</td>
</tr>
<tr>
<td>-0.7</td>
<td>3.948</td>
<td>4.722</td>
<td>5.558</td>
<td>0.2</td>
<td>3.575</td>
<td>4.210</td>
<td>4.890</td>
</tr>
</tbody>
</table>
Fig 4.14: Impact of \((\phi_1 \& \lambda_{-1})\) on bullwhip effect under the ES

Similar to fig 4.11-4.14 shows that the impact of \(\phi_1\) on the bullwhip effect based on different \(L_{Tr_1}\) under the ES. The Bullwhip Effect decreases all the time in pace with the increase of \(\phi_1\). For different \(L_{Tr_1}\), the smaller the \(L_{Tr_1}\)is, the smaller the bullwhip effect value is. This situation indicates \(L_{Tr_1}\)is also an important factor to influence the bullwhip effect under the ES forecasting.
4.5.4 The Comparison of the Forecasting Methods.

4.5.4.1 Forecasting methods comparison where $\text{MA}(k=3)$

Three forecasting methods are considered in the above analysis to select appropriate values of given parameters to minimum bullwhip effect. First set lead time of retailer_1 and retailer_2 is 3 and $\phi_1 = \phi_2 = 0.6$, study is conducted to choose appropriate $k$, $\beta_1$, and $\beta_2$. The MMSE method minimizes the variance of the forecasting error among all the linear forecasting methods. It obviously lead to the lowest average cost among the three forecasting approaches. When MA forecast data at $k=3$ and exponential smoothing forecast data $\beta = 0.5$ i.e. at $k=3$, BWE values are very close. From Fig 5.4.1-5.4.3, the trends of the three bullwhip effects are the same. However the values of them change obviously. According to fig 4.15, when $k = 3$, correspondingly $\beta_1 = \beta_2 = 0.5$, measured BWE in MMSE is found least. BWE in MMSE and BWE in ES decrease firstly to the minimum value and then increase with the increase of $\alpha$. However, BWE in MA has opposite trend. When $\beta_1$ is smaller than a certain value, BWE in MA is lower than BWE in ES; when $\beta_1$ is larger than the certain value and smaller than another certain value, BWE in MA is higher than BWE in ES; and when $\alpha$ is larger than another certain value, BWE in MA is lower than BWE in ES again. It means that the MMSE forecasting method is the best to forecast lead-time demand in this situation.

<table>
<thead>
<tr>
<th>Table 4.14: Forecasting methods comparison where $k=3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bullwhip Effect</td>
</tr>
<tr>
<td>alpha</td>
</tr>
<tr>
<td>MMSE</td>
</tr>
<tr>
<td>MA(k=3)</td>
</tr>
<tr>
<td>ES($\beta_1=\beta_2=0.5$)</td>
</tr>
</tbody>
</table>
Fig. 4.15: Comparison of three forecasting methods by varying $\alpha$ ($k=3$)

4.5.4.2 Comparison of three forecasting methods by varying $\alpha$ ($k=9$)

Fig. 4.16 reveals that the bullwhip effect under three forecasting methods of $k=9$ and $\beta_1 = \beta_2 = 0.2$ have the same trends with the circumstance of $k=3$ and $\beta_1 = \beta_2 = 0.5$. But BWE in MMSE is no longer the lowest of all, and it becomes the highest of the three. BWE in MA and BWE in ES are the same as fig 5.4.1. It means that when $\alpha$ is larger than a certain value and smaller than another certain value, the ES forecasting method is the best. In the other situation, the MA is the most attractive one. To conclude better adopt ES forecasting method when the intense competition between two retailers exists.

Table: 4.15 Comparison of three forecasting methods by varying $\alpha$ ($k=9$)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMSE</td>
<td>3.047</td>
<td>2.826</td>
<td>2.757</td>
<td>2.715</td>
<td>2.701</td>
<td>2.715</td>
<td>2.757</td>
<td>2.826</td>
<td>2.923</td>
<td>3.047</td>
</tr>
<tr>
<td>MA($k=9$)</td>
<td>1.880</td>
<td>2.145</td>
<td>2.28</td>
<td>2.358</td>
<td>2.367</td>
<td>2.355</td>
<td>2.310</td>
<td>2.235</td>
<td>2.071</td>
<td>1.897</td>
</tr>
<tr>
<td>ES($\beta_1=\beta_2=0.5$)</td>
<td>2.233</td>
<td>2.131</td>
<td>2.123</td>
<td>2.123</td>
<td>2.123</td>
<td>2.123</td>
<td>2.123</td>
<td>2.123</td>
<td>2.123</td>
<td>2.123</td>
</tr>
</tbody>
</table>
Fig 4.16: Comparison of three forecasting methods by varying $\alpha$ ($k=9$)

### 4.5.4.3 Comparison of three forecasting methods by varying $\alpha$ ($k=19$)

In Fig 4.17 set value of $k= 19$ and $\beta_1 = \beta_2 = 0.1$, and in this circumstance, BWE in MMSE is the highest all the time regardless of different $\alpha$. BWE in ES is a fixed value with the increase of $\alpha$. BWE in MA is the lowest of all. This phenomenon reveals that the MA method is the best forecasting method whatever $\alpha$ is as long as $k = 19$ or $k$ is larger.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMSE</td>
<td>3.047</td>
<td>2.826</td>
<td>2.757</td>
<td>2.715</td>
<td>2.701</td>
<td>2.715</td>
<td>2.757</td>
<td>2.826</td>
<td>2.923</td>
<td>3.047</td>
</tr>
<tr>
<td>MA($k=9$)</td>
<td>1.000</td>
<td>1.304</td>
<td>1.4</td>
<td>1.456</td>
<td>1.475</td>
<td>1.456</td>
<td>1.399</td>
<td>1.304</td>
<td>1.171</td>
<td>1.000</td>
</tr>
<tr>
<td>ES($\beta_1=\beta_2=0.5$)</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
</tr>
</tbody>
</table>
Fig 4.17: Comparison of three forecasting methods by varying $\alpha$ ($k=19$)