CHAPTER 3

$G_{fr}$-evaluation of by
FE Nodal Release technique.
3.0 Introduction
This chapter presents the conceptual evaluation of $G_f$ by nodal release technique using Finite Element model of a standard fracture testing specimen. Nodal Release is already an established method in which actual crack propagation is simulated by successive release of FE nodes in the direction of the crack propagation. In this method, successive “loading” and “unloading” cycles are required to reach to a finite crack growth level. An appropriate FE model, incorporated with the mathematical formulation for elasto-plastic fracture behaviour of ductile material, will generate nearly accurate values of stress, strain in the vicinity of the crack tip. By definition, $G_f$ is measured directly from the energy consumption rate for plastic deformation ahead of the crack tip, the same can be evaluated from the stress, strain values within a localized area near the crack tip. The technique is more elaborated in the next few sections.

3.1 FE modeling for nodal release.

Fig. 3.1 The Nodal release technique to simulate the actual crack growth phenomenon
The basic structure of the elasto-plastic FE code is developed by following the FE algorithms as given by Bathe & Wilson (1977) and Hinton & Owen (1977). It is incorporated with elastic and plastic stress-strain relationship, hardening rule, plastic work rule and plastic flow rule and their corresponding constitutive equations for FE modelling. A generalised approach is followed here so that crack tip modelling can be changed easily to suit the requirement of a particular EPFM problem. For example, Cohesive Model needs a layer of collapsed elements whereas Nodal Release technique requires 8-noded elements with fine mesh.

For the FE simulation by Nodal Release method, following experimental data are required:

1. Load versus displacement (Crack Mouth Opening Displacement, CMOD or Load line Displacement, LLD)
2. Displacement versus crack growth (Δa).
A step-by-step description of the nodal release technique can be presented as below:

- **LOADING**: With the FE model, the specimen is “loaded” (incremental load is applied) till it attains the displacement level corresponding to the crack growth equals to 0.2 mm from the initial crack length, \(a_0\). This displacement data is obtainable from the experimental data. Now it is assumed that the specimen is loaded to a value of J-integral equals to \(J_i\), i.e., the J-value at crack initiation. Here, \(J_i\) is considered as the value of J-integral when crack growth is equal to 0.2 mm. Thus \(J_i\) can also be denoted as \(J_{0.2}\). This state is corresponding to point \(P_1\) in the load-displacement curve (Fig.-3.2). Here extra input energy is pumped into crack tip by restraining the crack growth.

- **UNLOADING**: Now “unloading” is carried out in a fixed number of iterative steps say, 50. Prior to the start of unloading, nodal reaction forces, PLY1 and PLY2 (Fig.-3.1) which are normal to the crack plane and acting at the crack tip node, NC1 and its adjacent node NC2 respectively, are evaluated. In each iteration, nodal reaction forces are reduced successively. Displacement (CMOD), \(u\) is kept at the same value as it had reached during the previous “loading” phase. Obviously for maintaining equilibrium, external load should fall now. After completion of all the iterations PLY1 and PLY2 will become infinitesimally small. Theoretically, the load-displacement point from the FE calculation should come down to point \(P_2\) on the experimental curve. At this stage the above two nodes will be “released” and the crack is assumed to grow by \(\Delta a\), which is equal to the length of one crack tip element.
**RELOADING**: Next, the specimen is “reloaded” from the previous point \( \text{P2} \) to reach to the displacement level corresponding to the crack length \( (a_i + \Delta a) \) where \( a_i = a_0 + \Delta a \). The displacement data is obtained from the experimental data. Like the previous “loading” phase the simulated curve will again prevail above the experimental one and will finish at \( \text{P3} \).

- Again the specimen will be “unloaded” like the previous unloading phase and again it will meet the experimental curve at \( \text{P4} \).

- Now these “reloading” and “unloading” cycles can be continued to reach to a finite amount of crack growth say, \( \lambda = 2.5 \text{ mm} \).

Thus the program in totality will try to trace the actual curve for the growing crack (i.e., the experimental load-displacement curve).

### 3.2 Mathematical representation

The total change in plastic energy during one loading and one unloading cycle can be written as:

\[
\partial U^\text{pl} = \frac{\partial U^\text{pl}}{\partial u} \left|_u \right| du + \frac{\partial U^\text{pl}}{\partial a} \left|_a \right| da
\]  

(3.1)

First term of the RHS of the above equation is the extra energy that is pumped at the crack tip by not allowing the crack to grow. The other term, which is a negative one, represents the energy consumed for plastic deformation when the crack is assumed to grow by a finite amount ‘da’. In general these two terms are not equal. The balance amount of plastic energy contributes to trailing plasticity which is termed as global plastic dissipation. (Kanninen et. al., 1979) The change in plastic energy is calculated in a control volume around the crack tip.
Within the control volume, which is shifted with the crack tip during crack growth, a measure of plastic dissipation energy required for a predetermined crack growth can be obtained. This procedure is discussed in detail in the next section.

### 3.3 Measuring the plastic dissipation energy rate

As the plastic dissipation energy is measured within a control volume, the first job is to identify the area. It is observed from FE simulation that for the material SA333 Gr.6 Steel and for a crack growth size of 0.2 mm an area of (2mm×2mm) around the crack tip is getting affected during unloading. That is to say that during unloading the plastic energy changes over this area. Now the plastic dissipation energy rate is calculated by Gauss-integration of stress multiplied by change in plastic strain values at the plastically yielded Gauss-points within the square area (2mm×2mm) during a fixed amount of crack growth as given below:

\[
\Delta E_{pl} = \frac{\partial U_{pl}}{\partial a} \left|_{unloading} \right. .dA = \sum \left( \int (\sigma \cdot d\varepsilon_{pl}) \right)_{unloading}
\]

(3.2)

![Fig.-3.3 The control volume, denoted by Area-A (per unit thickness), which contains the plastically deformed zone near the crack tip.](image)
For the next “unloading” step, for a crack extension of \( \Delta a = 0.2 \text{mm} \) with respect to the previous one, the area-A is to be shifted by 0.2mm in the direction of the crack propagation (towards left as per the above figure). If the change in plastic deformation energy during unloading (for crack growth) is considered as the work done by an average internal dissipative force \( F(u) \), depends on plastic deformation level, to move a barrier of length \( l_c \) then,

\[
\Delta E_{\text{pl}} = \frac{\partial U_{\text{pl}}}{\partial a} |_{u} da = F(u)l_c \quad (3.3)
\]

Where \( l_c \) is a material specific length parameter. For this FE computation, \( l_c \) is considered as **twice the inter-particle distance for the material**. For SA333 Gr.6 steel, the inter-particle distance is reported to be equal to 0.1 mm so, \( l_c \) is equal to 0.2mm. It is observed that after certain amount of crack propagation (for the SA333 Gr.6 steel it is about 2.0 mm) the steady state condition is reached. At this stage the above energy, \( \Delta E_{\text{pl}} \) and also the force, \( F(u) \) attain nearly a constant value. This is reflected on the FE results, which can be observed in Fig.-3.7

At this stage, from the definition of \( G_f \) and \( G_c \) as given by Marie & Chapuliot (1998,1999), one can write that for the entire crack growth process from crack length ‘a’ to crack length ‘a+da’ (change in displacement from ‘u’ to ‘u+du’) the work done by the average dissipative internal force ‘F’ is equal to the plastic dissipation at the crack tip. Thus,

\[
F(u)du = G_c B \, da
\]

Thus by using Eq.-\((3.3)\),

\[
G_c = \frac{\Delta E_{\text{pl}}}{B l_c} \times \frac{du}{da} \quad (3.4)
\]

where, \( B \) is the specimen thickness and \( \frac{du}{da} \) is the slope of the displacement versus crack growth curve, which is to be calculated from
the experimental data. It is shown by Marie and Chapuliot that the slope \( \frac{du}{da} \) becomes constant for steady crack growth.

If the total crack extension is \( \lambda \), \( G_f \) can be written as,

\[
G_f = \frac{G_e}{\lambda} = \frac{\Delta E_{pl}}{l_c B \lambda} \times \frac{du}{da} \quad \text{(unit=N/mm}^2 \text{ or MPa)}
\]  

(3.5)

### 3.4 Surface energy release rate (\( \Delta E_{sur} \))

Here 'surface energy release rate' refers to the amount of energy release for creating the new crack surface for a pre-defined crack extension, say, \( \Delta a=0.2 \text{mm} \). This energy is assumed to be equal to the total work done by the two nodal reaction forces PLY1 and PLY2 (Fig.-3.1) during unloading phase. During “unloading”, the forces, PLY1 and PLY2 are reduced by 1/50 in each iteration when unloading is carried out in 50 iterations. The total displacements of the two nodes, NC1 and NC2 are also calculated and stored at each iteration. After completion of the unloading process, total area summation of the load versus displacement curve for the two nodes, NC1 and NC2 will be approximately equal to the amount of surface energy for 0.2 mm crack growth.

### 3.5 RESULTS

Following three specimens are studied here with the FE Nodal Release technique:

1. Full Compact Tension (CT) specimen CT25_5C
2. Half CT specimen : CT12_2C
3. Small CT specimen : CT08_2C

**Material**: All the specimens are made from **SA333 Gr-6 Carbon Steel** which is used for making the primary heat transport (PHT) pipes used in Indian pressurised heavy water nuclear reactors.
Material property:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus (E)</td>
<td>2.03 Gpa</td>
</tr>
<tr>
<td>Poisson’s Ratio (ν)</td>
<td>0.3</td>
</tr>
<tr>
<td>Yield Stress (σ_y)</td>
<td>312 MPa</td>
</tr>
<tr>
<td>Strain hardening exponent (n)</td>
<td>0.24</td>
</tr>
<tr>
<td>Hardening Constant (k), for σ = k.ε. n</td>
<td>790 Mpa</td>
</tr>
<tr>
<td>Ji (ie., J_{0.2})</td>
<td>230 kJ/m²</td>
</tr>
</tbody>
</table>

Table 3.1

Chemical composition:

<table>
<thead>
<tr>
<th>Element</th>
<th>C</th>
<th>Mn</th>
<th>Si</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Ni</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.14</td>
<td>0.9</td>
<td>0.25</td>
<td>0.016</td>
<td>0.018</td>
<td>805 ppm</td>
<td>525 ppm</td>
</tr>
<tr>
<td>Al</td>
<td></td>
<td>Cu</td>
<td>Pb</td>
<td>V</td>
<td>H</td>
<td>O</td>
<td>N</td>
</tr>
<tr>
<td>&lt; 0.1</td>
<td>540 ppm</td>
<td>80 ppm</td>
<td>90 ppm</td>
<td>&lt;5 ppm</td>
<td>0.03</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2

In percent or ppm (parts per million)

Experimental data:

Experiments on the above fracture specimens were carried out at NML, Jamshedpur and all the relevant data was supplied by BARC, Mumbai.
3.5.1 Full CT Specimen CT25_5C

Geometry

Fig.-3.4 The Full-CT Specimen: CT25_5C

B = Thickness, \( B_N = \) Net thickness after Side Grooving (20% here)
W = Width, \( a_0/W = \) Initial crack length to width ratio

\[ B = 25 \quad B_N = 20 \quad (with \ 20\% \ SG) \]
\[ W = 50 \]
\[ a_0/W = 0.5 \]

ALL DIMENSIONS ARE IN MM

Fig.-3.5 FEM generated curve superimposed on the experimental curve for the specimen: CT25_5C
FEM Computation of energy release rate for plastic deformation ($\Delta E_{pl}$) and crack extension ($\Delta E_{sur}$) is given in tabular form as below.

<table>
<thead>
<tr>
<th>After Unloading No</th>
<th>Crack growth (mm)</th>
<th>CMOD (experimental)</th>
<th>$\Delta E_{pl}$ (J)</th>
<th>$\Delta E_{sur}$ (J)</th>
<th>Ratio, $\frac{\Delta E_{pl}}{\Delta E_{sur}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>0.2</td>
<td>2.244</td>
<td>0.0358</td>
<td>1.9 x 10^{-3}</td>
<td>95:5</td>
</tr>
<tr>
<td>U2</td>
<td>0.4</td>
<td>3.123</td>
<td>0.0431</td>
<td>2.2 x 10^{-3}</td>
<td>95:5</td>
</tr>
<tr>
<td>U3</td>
<td>0.6</td>
<td>3.643</td>
<td>0.05075</td>
<td>3.4 x 10^{-3}</td>
<td>94:6</td>
</tr>
<tr>
<td>U4</td>
<td>0.8</td>
<td>4.515</td>
<td>0.0590</td>
<td>3.8 x 10^{-3}</td>
<td>94:6</td>
</tr>
<tr>
<td>U5</td>
<td>1.0</td>
<td>5.019</td>
<td>0.06288</td>
<td>4.5 x 10^{-3}</td>
<td>93:7</td>
</tr>
<tr>
<td>U6</td>
<td>1.2</td>
<td>5.366</td>
<td>0.06858</td>
<td>5.3 x 10^{-3}</td>
<td>93:7</td>
</tr>
<tr>
<td>U7</td>
<td>1.4</td>
<td>5.531</td>
<td>0.07031</td>
<td>6.1 x 10^{-3}</td>
<td>92:8</td>
</tr>
<tr>
<td>U8</td>
<td>1.6</td>
<td>5.895</td>
<td>0.06882</td>
<td>6.0 x 10^{-3}</td>
<td>92:8</td>
</tr>
<tr>
<td>U9</td>
<td>1.8</td>
<td>6.945</td>
<td>0.06779</td>
<td>5.9 x 10^{-3}</td>
<td>92:8</td>
</tr>
</tbody>
</table>

*Table-3.3 Results of the full CT Specimen CT25_5C by Nodal Release technique*

Now recalling the Eq.-(3.5),

$$G_f = \frac{\Delta E_{pl}}{l_c B \lambda} \times \frac{du}{da},$$

by plotting CMOD values versus the crack growth data as shown in Fig.-3.6 one can compute $\frac{du}{da}$ which is equal to the slope of the curve.

As reported by Marie & Chapuliot, this slope becomes constant for steady crack growth.
Fig.-3.6 Estimation of $\frac{du}{da}$ for the specimen: CT25_5C

Here, $\frac{du}{da} = 2.673$

Crack extension $\lambda$, in each step, is equal to 0.2 mm.

Net thickness ($B_N$) of the specimen = 20 mm

$l_c = 0.2$ mm as described earlier in Sec.-3.3 of this chapter.

So if anyone takes the stabilized value of $\Delta E_{pl}$ from the Table-3.3, i.e., equal to 0.06882 J,

Thus, \( G_f = 230 \) MPa
Fig. 3.7 The plastic deformation energy rate becomes nearly constant after some amount of crack growth.

3.5.2 Half CT Specimen CT12_2C

Geometry:

\[ B = 12.46 \quad B_N = 9.88 \]
\[ W = 24.98 \]
\[ a/W = 0.298 \]
\[ a = 7.44 \]

ALL DIMENSIONS ARE IN MM

Fig.-3.8 Half CT Specimen CT12_2C
After Unloading No | Crack growth (mm) | CMOD (mm) (experimental) | $\Delta E_{pl}$ (J) | $\Delta E_{sur}$ (J) | Ratio
--- | --- | --- | --- | --- | ---
U1 | 0.2 | 1.9626 | 0.01926 | $1.9 \times 10^{-3}$ | 91:9
U2 | 0.4 | 2.539 | 0.02543 | $2.4 \times 10^{-3}$ | 91:9
U3 | 0.6 | 3.120 | 0.03318 | $3.4 \times 10^{-3}$ | 91:9
U4 | 0.8 | 3.816 | 0.03632 | $3.8 \times 10^{-3}$ | 91:9
U5 | 1.0 | 4.263 | 0.03858 | $4.5 \times 10^{-3}$ | 90:10
U6 | 1.2 | 4.529 | 0.04491 | $5.3 \times 10^{-3}$ | 89:11
U7 | 1.4 | 5.028 | 0.04281 | $5.6 \times 10^{-3}$ | 88:12
U8 | 1.6 | 5.225 | 0.04368 | $6.0 \times 10^{-3}$ | 88:12
U9 | 1.8 | 5.301 | 0.04461 | $5.8 \times 10^{-3}$ | 88:12
U10 | 2.0 | 6.20 | 0.03225 | $5.7 \times 10^{-3}$ | 85:15

Table-3.4 Results of the half CT Specimen **CT12_2C** by Nodal Release technique

Here, $\frac{du}{da} = 2.178$
Crack extension $\lambda$, in each step, is equal to 0.2 mm & $B_N = 9.88$ mm.

$\Delta E_{pl} = 0.04368$ J

Thus,

$G_f = 241$ Mpa

### 3.5.3 Small CT Specimen: CT08_2C

**Geometry:**

![Diagram of CT08_2C specimen](image)

- $B = 8.12$
- $B_N = 6.7$
- $W = 24.96$
- $a/W = 0.289$
- $a = 7.21$

*All dimensions are in mm*

**Fig.-3.10 Half CT Specimen CT08_2C**

![Graph of load vs. CMOD](image)

- Experimental curve for CT08_2C
- FE generated curve
- Simulated points

**Fig.-3.11 FEM generated curve superimposed on the experimental curve for CT08_2C**
Table 3.5 Results of the small CT Specimen CT08_2C by Nodal Release technique

<table>
<thead>
<tr>
<th>After Unloading No</th>
<th>Crack growth (mm) (experimental)</th>
<th>CMOD (mm)</th>
<th>∆E_{pl} (J)</th>
<th>∆E_{sur} (J)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>0.2</td>
<td>2.487</td>
<td>0.0237</td>
<td>4.6 x 10^{-3}</td>
<td>84 :16</td>
</tr>
<tr>
<td>U2</td>
<td>0.4</td>
<td>2.846</td>
<td>0.0258</td>
<td>4.1 x 10^{-3}</td>
<td>86 :14</td>
</tr>
<tr>
<td>U3</td>
<td>0.6</td>
<td>3.609</td>
<td>0.0297</td>
<td>5.1 x 10^{-3}</td>
<td>85 :15</td>
</tr>
<tr>
<td>U4</td>
<td>0.8</td>
<td>4.135</td>
<td>0.0312</td>
<td>5.2 x 10^{-3}</td>
<td>86 :14</td>
</tr>
<tr>
<td>U5</td>
<td>1.0</td>
<td>4.667</td>
<td>0.0325</td>
<td>5.6 x 10^{-3}</td>
<td>85 :14</td>
</tr>
<tr>
<td>U6</td>
<td>1.2</td>
<td>5.059</td>
<td>0.0321</td>
<td>5.8 x 10^{-3}</td>
<td>85 :15</td>
</tr>
<tr>
<td>U7</td>
<td>1.4</td>
<td>5.191</td>
<td>0.03175</td>
<td>6.1 x 10^{-3}</td>
<td>84 :16</td>
</tr>
</tbody>
</table>

Crack extension $\lambda$, in each step, is equal to 0.2 mm & $BN = 6.7$ mm.

$\Delta E_{pl} = 0.0321$ J

Thus,

$G_{fr} = 259$ Mpa
3.6 DISCUSSION

Following observation can be made from the results above:

- Though the specimen thickness varies in wide range, from 20 mm to 6.7 mm (net thickness), the Gfr-value is confined within a narrow range, from 230 to 259 MPa, i.e., variation is within 13% with respect to the full CT specimen. This highlights the nature of the new crack growth parameter Gfr, which is claimed to be independent of geometry.

- By this method the energy release rate for creating the new crack surface (“surface energy”) can be estimated. Separating this energy from the total dissipated energy required for ductile crack propagation is always a challenging task for the researchers in this particular area. Also, as it appears in the above results the above energy is about 8-15%, of the total dissipated energy. A similar observation is reported in by Sun & Wang (2002).

- Though the trends of the results, as mentioned above, are clear, the results are slightly crippled by the incompleteness of the experimental data, for example, data up to only 2.5 mm crack growth is available for the full CT specimen, CT25_5C. To confirm the material intrinsic nature of Gfr, it is felt that crack growth must be continued to 4 to 5 mm.

3.7 Limitation of nodal release method

The nodal release technique has to be implemented very carefully since there is a chance of divergence in FE computation. Sometimes large number of iterative steps are required. This technique makes the crack growth as continuous process. The separation of trailing plasticity becomes difficult. By shifting the control volume at the beginning of each unloading process (which is a discrete phenomena) may not separate the trailing plasticity completely.