CHAPTER-I

Introduction and Motivation
1.1 INTRODUCTION

In today’s modern Industry, Business, Agriculture, Handloom and textile, Information Technology and so on requires optimal decisions to the complex problems like improving the quality and reliability of the products produced and minimizing the expected losses, maximizing the profits under certain conditions. Thus, the manufacturers have to make sure that their products have to meet the expected quality and reliability. The maintenance actions are very essential to meet the required quality and reliability. The growing importance of maintenance in the evolving industrial scenario and the technological advancements of the recent years attracted many researchers which have yielded the development of modern maintenance strategies such as the corrective (unscheduled) maintenance or preventive (scheduled) maintenance. The actions include the combination of all technical and corresponding administrative, managerial and supervision actions. Maintenance can significantly affect the quality and reliability of products when they are produced.

Maintenance actions performed on a repairable system can be categorized in two ways. It may be a corrective (unscheduled) maintenance or preventive (scheduled) maintenance. Corrective maintenance actions are unscheduled actions intended to restore a system from a failed state into an operating state. The actions involve repair or replacement of all failed parts and components necessary for successful operation of the system. Since a component’s lifetime is not known a priori, corrective maintenance is performed at unpredictable intervals. Its main objective is to restore the system to a satisfactory operating condition within the shortest possible time. Preventive maintenance actions are scheduled actions carried out to improve equipment life and avoid any unplanned maintenance activity. It includes lubrication, testing, cleaning, adjusting, and minor component replacement to extend the life of equipment and facilities. Preventive maintenance is used to avoid costly effects of equipment breakdowns. The primary objective of
preventive maintenance is to prevent the failure of equipment before it actually occurs. Improved system reliability, decreased cost of replacement and decreased system downtime are the benefits of preventive maintenance. The study of repairable components and systems strongly depend on the model of repair or renewal involved in the maintenance process.

For a repairable system, the life cycle can be described by a sequence of up and down states. Initially the system operates until the first failure occurs and then it is repaired and restored to its original operating state. It will fail again after some random time of operation, get repaired again, and this process of failure and repair will repeat. Now the sequences of failure and repair times can be considered a sequence of independent and non-negative random variables constituting a renewal process. Each time a unit fails and is restored to operating condition, a renewal is said to have occurred. This type of renewal process is known as an *alternating renewal process* because the state of the component alternates between an *operating state* and a *repair state*. One of the main assumptions in renewal theory is that the failed components are replaced with new ones or repaired so that they are ‘as good as new’.

Not only the *corrective* (unscheduled) *maintenance* or *preventive* (scheduled) *maintenance* actions but also redundancy is *one of the techniques* which are extensively used to enhance the system reliability and its mean life. Here redundancy is the provision of alternative means or parallel paths between sub-parts of the system. Another type of redundancy used in engineering design is called standby redundancy. In this case, only one component is active in the system. One or more additional components may be placed in the system, but in the standby condition. There are three types of standby: hot standby, warm standby and cold standby. A cold standby component has zero failure rates. In other words it does not fail while in cold standby.
At the initial stage of research in **Repair Replacement Problems (RRP)**, a repaired system is assumed to be ‘as good as new system’ and this kind of repair is called perfect repair. The repair of a component or replacement with a new one can be considered to be a perfect repair. In practice, most repairs are not perfect. Consequently, the system after repair cannot be ‘as good as new’. In the beginning, Lotka [41] introduced replacement model for a repairable system under the assumption that the system after repair can’t be ‘as good as new’.

Barlow and Hunter [4] developed a minimal repair model in which the repair activities do not alter the rate of failure of the system. Brown and Proschan [10] investigated an imperfect repair model in which the repair will be perfect repair with probability ‘p’ or minimal repair with probability 1-p, where 0<p<1. For a deteriorating repairable system, the successive working time of the system after repair may become shorter and shorter, whereas the successive repair time of the system may become longer and longer. Conversely, the system may neither work nor repair any more. To model such a deteriorating system Lam [26,27] first introduced a Geometric Process Repair model. Using this model, he analyzed two kinds of replacement policies- one called policy ‘T’, based on the working age of the system the other, called policy ‘N’, based on the cumulative number of failures of the system.

Zhang [72] generalized Lam’s work by a bi-variate replacement policy (T, N) under which the system is replaced at the working age T or at the time of the $N^{th}$ failure, whichever occurs first.

However, much research work has been carried out by many researchers using the geometric processes for modeling repair time, working time and to estimate reliability of the system, Braun et.al [11] presented that both the increasing geometric process and the $\alpha$-series process have a finite first moment under certain general conditions. Thus the decreasing $\alpha$-series process may be more appropriate for modeling system working times while...
the increasing geometric process is more suitable for modeling repair times of the system.

At present, it is perhaps the most extensively used parametric family of failure distribution is an exponential failure distribution. Also two monotone process approaches is considered to be more relevant, realistic and direct to the modeling of deteriorating system and maintenance problems that are encountered in most cases other than perfect or minimal repair.

Thus, we are motivated to develop optimal replacement models for single unit cold standby system, two unit cold standby systems, Two dissimilar component cold standby system with priority in use, Two dissimilar component cold standby system with priority in use and repair and two component dissimilar series system by using two monotone processes exposing to exponential failure law.

In the next section, concentration is focused on some preliminaries relating to reliability theory and replacement models for a ready reference.
In this section, some preliminaries relevant to the theory of Reliability and its optimization are discussed in the present investigation. Generally, the system designer designs the system keeping in mind the customer’s taste, attitude, requirements, the constraints involved in designing the system such as cost, time, material etc, and performance of its intended functions. The system’s performance in view of attaining the objectives is measured by the concept of reliability.

Reliability theory is the foundation for the **reliability engineering**. Reliability engineers rely heavily on statistics, probability theory and reliability theory. Many statistical tools and techniques are used in reliability engineering, such as reliability prediction, Weibulanalysis, thermal management, reliability life testing and accelerated life testing, and so on.

**RELIABILITY:**

The term, **reliability** is defined to be the probability that a unit or system can perform its intended function adequately over a specified period of time under stated operating conditions. That is, the reliability of a system is a function of the reliability of its components. In mathematical terms, the reliability of a component or a system at time ‘t’ is defined as:

Let T represents the life time of a device then reliability function is denoted by R(t) and is given by:

\[ R(t) = P(T > t) = 1 - P(T \leq t) \]

\[ = 1 - \int_{0}^{t} f(x)dx \]

\[ = \int_{t}^{\infty} f(x)dx \quad (1.2.1) \]
In other words, the **reliability** is the probability that the device will survive beyond time $t$. It is clear that $R(0)=1$ and $R(\infty) = 0$, thus the function $R(t)$ is a non-increasing function of time $t$ in the interval $[0,1]$.

**FAILURE TIME DISTRIBUTION:**

Suppose that $T$ is a random variable, denoting life time of a device, which takes non-negative values, then the probability that the device will fail at most time $t$ is usually called the probability distribution function denoted by $F(t)$ and is given by:

$$F(t) = P(T \leq t) = \int_0^t f(x)dx$$  \hspace{1cm} (1.2.2)

From the definition of reliability in (1.2.1), we may observe that:

$$R(t) = 1-F(t).$$  \hspace{1cm} (1.2.3)

Partial derivative with respect to ‘$t$’ on both sides of (1.2.3), we have:

$$f(t) = -\frac{d}{dt}R(t)$$  \hspace{1cm} (1.2.4)

Where $f(t)$ is the probability density function (p.d.f) of $F(t)$.

We now focus our attention on some of the failure probability distributions that are useful in the present study.

**EXPONENTIAL DISTRIBUTION:**

Let $T$ be a random variable representing the life time of a device, is said to have an Exponential distribution with parameter ‘$\lambda$’ and its probability density function $f(t)$ is given by:

$$f(t) = \lambda \exp(-\lambda t) \hspace{0.5cm} t > 0$$  \hspace{1cm} (1.2.5)

We list some of the characteristics of the distribution:
\[ R(t) = e^{-\lambda t}, t > 0 \] \hspace{1cm} (1.2.6)

\[ F(t) = 1 - e^{-\lambda t}, t > 0, \] \hspace{1cm} (1.2.7)

\[ H(t) = \lambda t, t > 0, \] \hspace{1cm} (1.2.8)

Mean = \( E(t) = \lambda t, t > 0, \) \hspace{1cm} (1.2.9)

**GAMMA DISTRIBUTION:**

A random variable \( T \) is said to follow gamma distribution with scale parameter \( \lambda \) and shape parameter \( \beta \) if and only if its probability density function \( f(t) \) is given by:

\[ f(t) = \frac{\lambda^{\beta} e^{-\lambda t} t^{\beta-1}}{\Gamma(\beta)}, t \geq 0, \lambda > 0, \beta > 0 \] \hspace{1cm} (1.2.10)

Some of the characteristics of the distribution are:

\[ R(t) = \frac{\lambda^{\beta}}{\Gamma(\beta)} \int_{t}^{\infty} x^{\beta-1} e^{-\lambda x} dx, t \geq 0, \] \hspace{1cm} (1.2.11)

\[ F(t) = \frac{\lambda^{\beta}}{\Gamma(\beta)} \int_{0}^{t} x^{\beta-1} e^{-\lambda x} dx, t \geq 0, \] \hspace{1cm} (1.2.12)

\[ h(t) = \frac{t^{\beta-1} e^{-\lambda t}}{\int_{0}^{t} x^{\beta-1} e^{-\lambda x} dx}, \] \hspace{1cm} (1.2.13)

Mean = \( \frac{\beta}{\lambda} \). \hspace{1cm} (1.2.14)

**Remark:** When \( \beta = 1 \) Gamma distribution reduces to negative exponential distribution with parameter \( \lambda > 0. \)

**CONVOLUTION:**
Consider the two cumulative distribution functions $G(x)$ and $H(y)$ of the random variables $X$ and $Y$ respectively. If $Z$ be the another random variable which define $Z=X+Y$ then cumulative distribution function (cdf) of sum of the random variables $U(Z)$ can be expressed as:

$$U(Z) = P[Z \leq z] = P[x + y \leq z] = \int_{-\infty}^{+\infty} P(x + y \leq z/Y = y) dH(y)$$

$$= \int_{-\infty}^{+\infty} P(x \leq z - y) dH(y)$$

$$= \int_{-\infty}^{+\infty} G(z - y) dH(y)$$

$$= (G*H)(Z)$$

(1.2.16)

is called convolution of the functions $G$ and $H$. The cdf of sum of two independent random variables is equal to the convolution of the cumulative distribution function of the two variables.

HAZARD RATE FUNCTION:

The hazard rate function or failure rate function is denoted by $h(t)$ and is defined as the probability that a device will fail in the next time unit, given that it has been working properly up to time $t$ and is given by:

$$h(t) = \lim_{\delta t \to 0} \frac{P(t \leq T \leq t + \delta t)}{P(T > t)} = \frac{f(t)}{R(t)}$$

(1.2.17)

The cumulative hazard function is defined as:
\[ h(t) = \int_0^t h(x) \, dx. \quad (1.2.18) \]

The failure rate function is used to indicate health condition of working component. A high failure rate indicates a bad health condition because the probability that the equipment will fail in the next instant time is high. It is noticed that one of the functions \( F(t), f(t), R(t), H(t) \) or \( h(t) \) is enough to describe completely the life time distribution of a system.

**MEAN TIME TO FAILURE (MTTF):**

The expected value or the mean of the life time \( T \) is called the mean time to failure. The mathematical expectation of the life time \( T \) is denoted by \( E(T) \) and is given by:

\[ MTTF = E(T) = \int_0^\infty t f(t) \, dt. \]

On Integration of the above equation, we have:

\[ = \int_0^\infty R(t) \, dt \quad (1.2.19) \]

The devices which go through several failures before scrapped are usually called repairable devices. For repairable devices the MTTF represents the mean time to the first failure. After it is repaired and put into operation again, the average time to the next failure is called mean time before failure (MTBF). If each repair restores the device to “as good as new” condition, then the repair is called perfect repair. Under perfect repair MTTF and MTBF are equal.

The average amount of time needed to repair a failed device is called mean time to repair (MTTR).

**INDICATOR FUNCTION:**
An event $A$ partitions the sample space $S$ into two mutually exclusive and collectively exhaustive subsets $A$ and $\overline{A}$. The indicator of the event $A$ is a random variable $I_A$ and defined by:

$$I_A(S) = 1, \text{ if } s \in A,$$

$$= 0, \text{ if } s \in \overline{A},$$

then event $A$ occurs if and only if $I_A = 1$. The probability mass function of $I_A$ is given by

$$P_{I_A}(0) = P(\overline{A}) = 1 - P(A), \quad (1.2.20)$$

$$P_{I_A}(1) = P(A). \quad (1.2.21)$$

The concept of indicator function in a certain cases allows us to make efficient computations without detailed knowledge of distribution function. This is quite useful, particularly in case, where the distribution is difficult to calculate.

We now discuss more details about standby systems, which are used in the present study, particularly in chapter-IV and chapter-V.

**STANDBY SYSTEMS:**

Standby systems can be classified into two categories namely:

a) Simple standby systems.

b) $k$-out-of-$m$ standby systems.

**a) Simple standby systems**

The system in which only one component or unit is active and all other components or units are in standby is called simple standby system. In other words parallel system, consisting of $m$-units, is said to be simple standby system if and only if one component works and all other components ($m-1$) are in standby. In this case, only one component is active in the system. One or more additional components may be placed in the system, but in standby condition. A sensing and
switching mechanism is used to monitor the operation of the active component. Whenever the active component is failed, standby component is immediately switched into active operation. The following shows the diagrammatic representation of two standby components.

![Diagram of two standby components]

Fig: 1.1 Simple Standby System

SSM: Sensing and Switching Mechanism

Simple standby systems can be classified into three types of standby systems namely:

i) Hot standby  
ii) Warm standby  
iii) Cold standby

A system in which parallel components or redundant components have the same failure rate as active components is called **hot standby system** i.e., hot standby components have the same failure rate as the active components. A parallel system in which the redundant components or standby components have a failure rate that is in between ‘0’ and ‘1’ active components failure rate is called **warm standby system** and the components which are in standby are called warm standby.
components i.e., A warm standby system may include hot standby components or cold standby components.

A parallel system in which the standby components have zero failure rate is called **cold standby system** and the components in standby are called cold standby components.

Cold standby system is very useful to increase the system reliability. In a cold standby system, standby components do not fail, thus we should concentrate on active component and sensing and switching mechanism. Consider the case where there are only two components in the system and sensing, switching mechanism is perfect under the assumption that the failed components may not be repaired. Suppose that the component 1 is active operation and component 2 is standby. When component 1 fails the standby component becomes active operation. The system life is over, when component 2 fails. For example, bulbs in head light of a two wheeler, where two units are two filaments in the bulbs. Then system life time is equal to the sum of the life times of components 1 and 2.

\[ T_s = T_1 + T_2. \]  \hspace{1cm} (1.2.22)

Then reliability of the system is given by:

\[ R_s(t) = P[T_s > t] = R_1(t) + \int_0^t f_1(x)R_2(t-x)dx \]  \hspace{1cm} (1.2.23)

and the MTTF of the system is given by

\[ \text{MTTF}_s = E[T_s] = E[T_1] + E[T_2]. \]  \hspace{1cm} (1.2.24)

In a standby system the standby component not in operation from the beginning and is put into operation only when the basic component fails. Therefore, standby equipment does not age at all during standby and also does not fail during standby. This is intact adds to the system reliability.
b) k-out-of-m standby system

A system with m-parallel units of which K units are in active operation and m-k units are in standby is called k-out-of-m standby system. In other words a system where more than one of its parallel components are in active state to meet the demand, is called k-out-of-m standby system. For example, a four engine aircraft where only two engines are required for successful operation or two out of four generators in a generating station may be necessary to supply the required power to the customers.

In k-out-of-m standby system components are independent then binomial distribution can be used to find the system reliability. If \( p \) is the probability of success of each component then the probability of exactly \( k \) out of \( m \) components are successful is given by:

\[
P(m, k) = \binom{m}{k} p^k q^{m-k}.
\]

Where \( k \) is the minimum number of components required for successful operation of the system, then the system will be successful if \( k, k+1, k+2, \ldots, m \) components are successful. Thus, the system reliability is the sum of the binomial probabilities.

\[
R = \sum_{i=k}^{m} B(m, i) p^i q^{m-i}.
\]

Now we discuss some monotone processes, which are used in the present study to analyze the system reliability and maintenance problems.

**REPLACEMENT POLICIES:**

Often, the units are replaced as and when they fail all of sudden or work badly. But it is not always feasible to replace the components with a new one without considering some aspects like cost, space, time and so on. Thus, some components are to be considered with ‘as good as new’ condition after the repair. The general replacement policies are:
i) Replacement of item that deteriorates with time without considering the value of money

ii) Replacement of item that deteriorates with time whose value of money changes with a constant rate.

iii) Replacement of items that fails completely.

iv) Individual replacement policies.

v) Group replacement policies.

The manufacturer or a experimenter may choose any one of the above models which suits well according to his constraints.

**GEOMETRIC PROCESS (GP)**

A stochastic process \( X_n, n=1,2,... \) is a geometric process (GP), if there exists some \( a > 0 \) such the \( \{a^{n-1} X_n, n=1,2,...\} \) form a renewal process. The number ‘\( a \)’ is called the ratio of the geometric process. Clearly a geometric process is stochastically increasing if the ratio \( 0 < a < 1 \), it is stochastically decreasing of the ratio \( a > 1 \). A GP will become a renewal process of the ratio \( a=1 \). Thus GP is simple monotone process and is a generalization of renewal process.

Let \( E(X_1) = \lambda \) and \( V(X_1) = \sigma^2 \), then

\[
E(X_n) = \frac{\lambda}{a^{n-1}}, \quad V(X_n) = \frac{\sigma^2}{a^{2(n-1)}}.
\]

Therefore, \( a, \lambda \) and \( \sigma^2 \) are three important parameters in the GP.

**ALPHA SERIES PROCESS (\( \alpha \))**:

Assume that stochastic process \( \{X_n, n=1,2,...\} \), is a sequence of independent non-negative random variables. If the distribution function of \( X_n \) is \( F_n(t) = F(k^{\alpha} t) \) for some \( \alpha > 0 \) and all \( n=1, 2, 3... \) then \( \{X_n, n=1, 2,...\} \) is called a \( \alpha \) series process, \( \alpha \) is called exponent of the process. Braun et al [11].

Obviously:
i) if $\alpha > 0$, then $\{X_n, n=1,2,\ldots\}$ is stochastically decreasing, i.e., $X_n >_{st} X_{n+1}, n=1,2,\ldots$

ii) if $\alpha < 0$, then $\{X_n, n=1,2,\ldots\}$ is stochastically increasing, i.e., $X_n <_{st} X_{n+1}, n=1,2,\ldots$

Where st represents ‘stochastically’.

iii) if $\alpha = 0$, then the $\alpha$ series process becomes a renewal process.

A stochastic process which involves both geometric process and alpha series process is called two monotone processes.

RENEWAL-REWARD THEOREM:

$$\lim_{t \to \infty} \frac{E(R_t)}{E(C_t)} = \frac{E(R_1)}{E(C_1)},$$

with probability 1.

In other words, for almost any realization of the process, the long-run average reward per time unit is equal to the expected reward earned during one cycle divided by the expected length of one cycle. $R_n =$ the total reward earned in the $n$th renewal cycle, $n=1,2,\ldots$. It is assumed that $R_1, R_2,$ are independent and identically distributed random variables. In applications $R_n$ typically depends on $C_n$ where $C_n$ is length in the $n$th renewal cycle. In case $R_n$ can take on both positive and negative values, it is assumed that $E(|R_1|) < \infty$. Let $R(t) =$ the cumulative reward earned up to time $t$.

The process $\{R(t), t \geq 0\}$ is called a renewal-reward process.

POISSON PROCESS

Consider a random Event such as (i) incoming telephone calls (at a switchboard), (ii) arrival of customers for service (at a counter), (iii) occurrence of accidents (at a certain place), (iv) failures of units in a system, (iv) failure of bulbs in a function hall etc. Let us consider the total number $N(t)$ of occurrences of the event $E$ in an interval $[0,t]$ of duration $t$, i.e., if we start from an initial epoch $t=0$, $N(t)$ will denote the number of occurrences up to the epoch $t$. For example, if an event actually occurs at instants of
time $t_1, t_2, t_3, \ldots \ldots$ then $N(t)$ jumps abruptly from 0 to 1, at $t = t_1$ from 1 to 2 at $t = t_2$ and so on, the situation can be graphically represented as:

![Graph showing jumps in $N(t)$](image)

**Fig. 1.1.2** Number $N(t)$ of occurrences of an event in an interval of duration $[0,t]$

The values of $N(t)$ given here are observed values of the random variable $N(t)$. Let $p_n(t)$ be the probability that random variable $N(t)$ assumes the values $n$. i.e.,

$$P_n(t) = Pr \{N(t) = n\} \quad (1.2.27)$$

This probability is a function of the time $t$. Since the only possible values of $n$ are $n=0, 1, 2, 3, \ldots \ldots, \infty$. Further we know that:

$$\sum_{n=0}^{\infty} P_n(t) = 1 \quad (1.2.28)$$

Thus, $\{P_n(t)\}$ represents the probability distribution of random variable $N(t)$ for every value of $t$. The family of random variables $\{N(t), t \geq 0\}$ is a stochastic process. Here
the time $t$ is continuous, the state space of $N(t)$ is discrete and the integral-valued and
the process is integral-valued. One of the most important integral-valued processes is
Poisson process. This serves as a mathematical model for a wide range of empirical
phenomena with remarkable accuracy. The justification for this is based on the concept
of rare events. Under certain conditions, $N(t)$ follows Poisson distribution with mean $\lambda t$
($\lambda$ being a constant). In case of many empirical phenomena, these conditions are
approximately true and the corresponding stochastic process $\{N(t)\}$ follows the Poisson
Process under the following postulates.

THE POSTULATES OF POISSON PROCESS:

(i) The probability of occurring one failure in a small interval $(t, t + \delta t)$ is $\lambda \delta t$.
    Where $\lambda$ is a positive constant and $\delta t$ denotes a small increment in time $t$.

(ii) The probability of occurring more than one failure in a small interval
    $(t, t + \delta t)$ is zero.

(iii) The probability of occurring any failure in a small interval $(t, t + \delta t)$ is
    independent of actual time $t$ and also of all previous failures.
    There are many types of Poisson process which are explained as follows.

HOMOGENEOUS POISSON PROCESS:

A counting process $\{N(t), t \geq 0\}$ is called a homogeneous Poisson process with
parameter $\lambda > 0$ if the following conditions are satisfied:

a) $N(0)=0$.

b) The process has stationary and independent increments.

c) The probability for an event to occur in the interval $(t, t+\delta t)$ may be written as

$\lambda \delta t + o\delta t$

\[ d) \] The probability for more than one event to occur in the interval $(t, t+\delta t)$ is $o\delta t$

which is negligible for small $\delta t$. 
In a homogeneous Poisson Process, the number of events in any interval of length \( t \) follows the Poisson distribution with the parameter \( \lambda t \), i.e., for all \( s > 0 \) and \( t > 0 \)

\[
P( N(s+t) - N(s) = n ) = P( N(t) = n ) = \frac{e^{-\lambda t} (\lambda t)^n}{n!}, \text{ for } n=0,1,2,... \tag{1.2.29}
\]

In this case

\[
E[N(t)] = \lambda t \\
V[N(t)] = \lambda t 
\tag{1.2.30}
\]

Where \( \lambda \) is called rate of occurrence of events and is denoted by \( h(t) \), i.e.,

\[
h(t) = \frac{d}{dt} E[N(t)] = \lambda \tag{1.2.31}
\]

**NON-HOMOGENEOUS POISSON PROCESS:**

The counting process \( \{N(t), t \geq 0\} \) is called the non-homogeneous Poisson process with an intensity function \( \lambda(t) > 0 \) if:

i) \( N(0) = 0 \)

ii) \( \{N(t), t > 0\} \) has independent increments

iii) \( P[N(t+\delta t)-N(t)=1] = \lambda(t) + o(\delta t) \)

iv) \( P[N(t+\delta t)-N(t) > 2] = o(\delta t) \)

The number of events that occurs in the interval \( (s, s+t) \) follows Poisson distribution with parameter \( \int_s^{s+t} s(x)dx \) i.e.,

\[
P[N(s+t) - N(s) = n] = \frac{e^{\int_s^{s+t} s(x)dx} \left( \int_s^{s+t} s(x)dx \right)^n}{n!}, \text{ for } n=0,1,2,... \tag{1.2.32}
\]

**RENEWAL PROCESS:**
A counting process \( \{N(t), t \geq 0\} \) is called a renewal process if the following conditions are satisfied.

1) \( N(0)=0. \)
2) The process has stationary and independent increments.
3) The probability for an event to occur in the interval \((t, t+\delta t)\) is finite.
4) The probability for more than one event to occur in the interval \((t, t+\delta t)\) is \(o(\delta t)\) which is negligible for small \(\delta t\) and
5) The inter arrival times to be i.i.d random variables having an arbitrary distribution.

RENEWAL REWARD PROCESS:

Assume that a renewal process \( \{N(T), t \geq 0\} \) with i.i.d inter arrival times \( T_n \) having with CDF \( F(t) \) for \( n \geq 1 \). There is a reward denoted by \( C_n \) (\( n \geq 1 \)), when an \( n^{th} \) renewal occurs. If this reward is negative, it would mean a penalty. It is assumed that \( C_n \) for \( n \geq 1 \) are i.i.d. random variables. The reward \( C_n \) at the \( n^{th} \) renewal may be dependent on \( T_n \). However each pair of \( (T_n, C_n) \) is independent of the pair of \( (T_i, C_i) \) for all \( i < n \). Let \( C(t) \) represent the total reward earned by time \( t \). Then, we have

\[
C(t) = \sum_{i=1}^{N(t)} C_i
\]

(1.2.33)
1.3 LITERATURE REVIEW

In this section, some notable contributions made by several researchers in the area of reliability and its optimization are provided. These works are essential for understanding and developing the present study.

The earliest attempt in the direction of optimization techniques, theory of reliability was due to Lotka [41]. He investigated a model in which the system is subjected to random failures and presented a replacement policy.

An excellent account of the growth and developments in the theory of Reliability was presented by Barlow and Proschan [5, 6] Kuo.W and Zuo M. J [22] and, Balaguru Swamy [3], Allesandro Birolini [2] and so on.

Barlow and Hunter [4] considered two types of preventive maintenance policies. By using elementary renewal theory, they defined a policy to be optimum if it maximizes limiting efficiency. In each case, unique solutions are obtained for certain integral equations depending on the failure distribution. Further, they compared two optimum policies under certain constraints.

Brown and Proschan [10] proposed an imperfect repair model in which the repair is perfect with the probability p or minimal with probability 1-p. Here repair takes negligible time. If the interval between successive good-as-new states in terms of underlying life distribution F and showed that if F is in any of life distribution classes such as Increasing Failure Rate (IFR), Decreasing Failure Rate (DFR), Constant Failure Rate (CFR), Average Increasing Failure Rate (AIFR), Average Decreasing Failure Rate (ADFR),
(ADFR) and so on. F is in the same classes. The derived results are very interesting in the context of stochastic processes. Finally, they obtained a number of monotonic properties for various parameters and random variables of the stochastic process.

Geol and Gupta [16] established a system having two identical units, one operative and the other cold standby and each unit of the system having three modes of failures namely normal, partial and total failure. The replacement time of a failed unit is a random variable. They considered failure-time distributions of units as exponential, where the repair-time distributions are arbitrary and evaluated several reliability characteristics of interest to system designers and operation managers using theory of regeneration point technique.

Rakesh Gupta [49] studied the probabilistic analysis of a two-unit cold standby system with two-phase repair and preventive maintenance. The whole of the repair process is divided into two phases. Phase I may be covered by a repairman who is not trained. This is less expensive. Phase II may be covered by more skilled repairman i.e. preventive maintenance of an operative unit is made after a fixed amount of time, which is required only when the other unit is in normal mode. He analyzed the system by using the regeneration points in Markov renewal process and obtained various economic reliability measures for the system effectiveness.

Singh and Srinivasu [55] considered the stochastic analysis of a two unit cold standby system, in which the failed unit has to wait for repair due to preparation time for repair. Preparation for repair is the arrangement of the tools and other materials, which are important for repair. They carried out the analysis under the supposition that failure time distributions are negative exponential where as the inspection, preparation time for repair and repair time distributions are arbitrary and independent each other. They obtained several parameters of interest by regenerative point process technique.
Block et.al [8] developed a stochastic model to describe the operation in time to maintain system setting. They put a device in operation at time $t=0$. Each time it fails, a maintenance action is taken with probability $P(t)$ is a complete repair or with probability $Q(t) = 1 - P(t)$ is a minimal repair where $t$ is the age of the equipment in use at the failure time. They assumed that complete repair restores the device to its good condition, which minimal repair restores the equipment to its condition just prior to failure and maintenance action takes negligible time.

Mahmoud [42] developed two-unit standby with random switching and imperfect switchover. He obtained the mean time First System Failure (FSF) and Last System Failure (LSF) of the distribution. Further, he considered same system taking into account preventive maintenance and proved that the mean time FSF with type (b)-imperfect switching is greater than or equal to the mean time FSF with type (a)-perfect switching. He found the mean time FSF by adopting preventive maintenance (PM) under the type (a)-perfect switching and the type (b)-imperfect switching conditions.

Lam [26] introduced the geometric process which is a sequence of independent non-negative random variables $X_1, X_2...$ such that the distribution function of $X_n$ is $F(a^{n-1}x)$ where ‘$a$’ is a positive constant. Here if $a>1$, then it is a decreasing geometric process and $0< a<1$, it is a increasing geometric process. He considered a replacement model as successive survival times of the system after repair form a decreasing geometric process and the consecutive repair times of the system constitute an increasing geometric process. Besides the replacement policy based on the working age of the system, he considered a new kind of replacement policy which is determined by the number of failures. The explicit expressions of the long-run average cost per unit time under each replacement policy are then evaluated.

Lam [27] considered the geometric process(GP) model and developed a replacement model at the successive survival times of the system after repair form a decreasing GP while the consecutive repair time of the system constitute an increasing GP. He considered two kinds of replacement policies, one based on the working age of
the system and the other is determined by the number of failures. He calculated an explicit expression for long-run average cost per unit time under each replacement policy and obtained corresponding optimal replacement policies numerically.

Singh et.al [56] investigated the cost benefit analysis of a two unit-standby redundant system in which the repairman acts the triple role of inspection, preparation for repair and repair. They carried out the analysis under the supposition that the failure time distribution is negative exponential, where as the inspection, preparation for repair and repair time distributions are arbitrary. They obtained several parameters to study the stochastic behavior of the system by using regeneration point technique.

Kijima [20] developed general repair model for a repairable system by using the virtual age process of the system with an aim to construct stochastic models of the operation in time of a repairable system which is maintained by a general repair and analyzed the related stochastic process.

Gupta et.al [18] studied stochastic behavior of two-unit cold standby system in which each unit works in three different modes. In this paper, they included the idea of administrative delay and no priority in repair with two similar units and three modes of failures. They derived various reliability parameters of interest to system designers.

Stadje and Zukerman [57] considered the problem of finding optimal replacement strategies for some repair replacement models in which the system can be repaired upon failure, but the length of the operating intervals decrease in some sense, where as the duration of the repairs increases. Under some assumptions they developed several optimal models for replacement strategies, the reward and cost structure.

Goel et.al [17] discussed a system having two-unit dissimilar cold-standby system with an imperfect switch. Initially one unit is operative and is called the priority unit and the other is cold-standby unit. When the priority unit fails, the standby unit is switched to operate with the help of switching device. The switch may be available at
the time of need with known probability $P$. In this paper, they considered the distributions of random variables time to failure and time to repair are to be arbitrary. Further, they derived explicit results for exponential distribution in each particular case. They analyzed the system by using the regenerative point’s technique and obtained several reliability characteristics.

Rakesh Gupta et.al [50] examined two-unit standby system with two modes of failure each unit having normal and total failure by treating one unit in operative and the other is kept as a warm standby. The warm standby becomes cold standby after a random time and vice versa. Upon failure of the operative unit, if the standby unit is warm standby it starts to operate instantaneously, otherwise the system goes down until the cold standby starts to operate. When both units failed totally, then the system failure occurs. He obtained the expressions for reliability and mean time to system failure by using regenerative point technique with Markov renewal process and also he studied graphical behavior of time to system failure in a simple case.

Stadje and Zukerman [58] discussed two maintenance models for repairable systems. They assumed general repair model in which the contractor may select degree of repair in the range, between minimal and perfect repair. In model I- Minimal Repair Model (MRM), they restrict an attention to maintenance strategies in which only minimal or perfect repair can be employed upon failure. In model II-Perfect Repair Model (PRM), one can select between $N$ different repairs proportions. Further, they determined optimal maintenance strategies for some age dependent cost functions.

Lam [29] investigated a geometric process replacement model, in which the successive survival of the system form a non-increasing geometric process while the consecutive repair times of the system constitute a non-decreasing geometric process and the system is replaced at the time of $N^{th}$ failures occurs. Based on the above assumption, he derived the explicit expression for the long-run average cost per unit
time to determine the optimal replacement policy $N^*$ and also, he showed the uniqueness and monotony of the policy $N^*$.

Lam [30] proposed the geometric process replacement model as the successive survival times of the system form a non-increasing geometric process while the consecutive repair times of the system constitute a non-decreasing geometric process, and the system is replaced at the time of the $N^{th}$ failure after its installation. Based on the average cost rate, Lam determined the optimal replacement policy $N^*$.

Agnihotri et.al [1] investigated a stochastic model for a two-unit warm standby system with single repair facility. In this paper they considered a failed unit sent for fault detection to decide whether it failed due to machine defect or critical human error. The probabilities of these two causes have been fixed. They obtained different measures of system effectiveness by using the regenerative point technique with Markov renewal process.

Rander et.al [51] studied a two unit cold standby system with major and minor failures and preparation time for repair, in case of major failure. They analyzed the system by using regenerative point technique.

Stanley [59] proposed a repair replacement model for a deteriorating system in which the successive survival times of the system are stochastically non-increasing and form a geometric process and the magnitude of each shock at each failure and the consecutive repair times after failure are also constitute geometric process and are stochastically non-decreasing. Under these assumptions, he obtained an explicit expression for the long-run average cost per unit time and introduced a new repair replacement policy under working age of the system.

Makies and Jardinne [43] investigated a replacement model with general repair which brings the state of the system to a certain better state and obtained an optimal
replacement policy by minimizing expected average cost per unit time by illustrating
with numerical examples.

Zhang [72] generalized Lam’s earlier work [26,27] by a bivariate policy \((T, N)\) under which the system is replaced at working age \(T\) or at the time of \(N^{th}\) failure, whichever occurs first assuming that the system after repair is not ‘as good as new’ and developed an optimal replacement policy \((T,N)^*\) by minimizing the long-run average cost per unit time. Further, under some mild conditions he also proved that an optimal policy \((T, N)^*\) is better than the optimal policy \(N^*\) or \(T^*\).

Lam and Zhang [24] discussed a geometric process model for the analysis of a two component series system with one repairman. For each component, the successive operating times form a decreasing geometric process, whereas the consecutive repair times constitute an increasing geometric process with exponential distribution, but the replacement times forms a renewal process. They derived a set of partial differential equations by introducing two supplementary variables and these equations are solved analytically.

Zhang [73] considered a repairable system of two identical components, namely component 1 and component 2, and one repairman. By using a Geometric process, he considered a replacement policy \(N\) based on the number of repairs of component 1 and derived an explicit expression of the long-run expected reward per unit time by providing a numerical example.

Leung [38] introduced new repair-replacement models for a deteriorating system, in which the successive operating times of the system form an arithmetico geometric process and are stochastically non-increasing, while the successive repair times after failure also constitute an arithmetico-geometric process but are stochastically non-decreasing. Two kinds of replacement policies are considered, one based on the working age and the other determined by the number of failures of the system. Applying the well known results of renewal reward process, the author derives
expressions for the long-run average cost per unit time at good operating condition, under the two kinds of policies proposed.

Zhang et.al [77] investigated a deteriorating simple repairable system with (K+1) states (K ≥ 2) including K failure states and one working state and studied a replacement policy called policy N based on the failure number of the system. The explicit expression of long-run expected profit per unit time is derived for a general monotone process model which includes the geometric process repair model as special case.

Wang and Zhang [62] considered a shock model for a repairable system with two types of failures by assuming that two kinds of shock in a sequence of random shocks will make the system failed, one based on the inter arrival time between two consecutive shocks less than given positive δ and the other based on the shock magnitude of single shock more than a given positive value (Ψ). Under this assumption they obtained some reliability indices of the shock model such as the system reliability and the mean working time before system failure. Also determined the replacement policy N based on the number of failures of the system by minimizing the long-run average cost per unit time. Further, it is also established through numerically.

Zhang [74] studied a deteriorating repairable system with three states including two failure states and one working state. A replacement policy N based on the failure number of the system is adopted under which the system will be replaced at the time of Nth failure and determined an optimal replacement policy N* by minimizing the average cost rate (ACR) and derived an explicit expression for average cost rate.

Lam et.al [35] developed a monotone process model for one-component degenerative system with K+1 states (K failure states and one working state) and they showed that this model is equivalent to a geometric process model of a two state one
component system such that both systems have the same average cost rate and the same optimal policy.

Lam and Zhang [25] presented a geometric process maintenance model for a deteriorating system under random Environment. Whenever a random shock arrives, the system operating time is reduced. Assume that the consecutive repair times of the system after failures, form an increasing geometric process, the successive operating times of the system after repairs constitute a decreasing geometric process. They adopted a replacement policy N, by which the system is replaced at the time of the failure N. They derived an explicit expression for the long-run average cost and they determined numerically optimal replacement policy.

Lam and Zhang [36] proposed a shock model for the maintenance problem of a repairable system. For a deteriorating system, they assume that the successive threshold values are geometrically non-decreasing after repair, and the consecutive repair times after failure form an increasing geometric process and they determined optimal policy N* for minimizing the long-run average cost per unit time.

Braun et.al [11] established some properties of the geometric process along with those of a related process proposed them as $\alpha$-series process may be viewed as complementary to one another. Further, they showed that the increasing geometric process might be appropriate for modeling machine down times, while the $\alpha$-series process can be used for modeling machine up times and considered applications in reliability and stochastic scheduling in order to demonstrate versatility of the alternative proposed model.

Lam [33] discussed a monotone process model for a one component multi state system with K+L states, viz K working states and L failure states by making different assumptions and he applied the model to a multi state deteriorating system as well as to a multi state improving system and showed the monotone process model for a multi state system is equivalent to a geometric process model for a two state system.
Leung [39] examined an arithmetico-geometric process (AGP) approach which is considered to be relevant, realistic and appropriate to modeling of a deteriorating system maintenance problem. The expression for the long-run average cost per unit time developed by him is so worthy than the expression developed by Zhang [68].

Zhang et.al [78] studied a cold standby repairable system consisting of two identical components and one repairman. In this they considered two kinds of replacement policy, one based on the working age T and other based on the failure number N of component 1. Here, the problem is to choose optimal replacement policies $T^*$ and $N^*$ respectively such that the long-run average cost per unit time of the system is minimized and also they proved under some mild conditions that the optimal policy $N^*$ is better than the optimal policy $T^*$. Finally, a numerical example for policy N is discussed.

Wang and Zhang [65] generalized the Lam’s work by introducing a bivariate replacement policy $(L, N)$ based on the fixed length interval length of the Preventive Repair (PR) and the PR-number of the system respectively. The main objective of this paper is to determine an optimal preventive replacement policy $(L,N)^*$ such that the average cost rate per unit time is minimized. An explicit expression for the long-run average cost per unit time is derived. Finally, they put forward a numerical example with Weibull distributed operating times.

Wang and Zhang [64] investigated a series repairable system consisting of two non-identical components and one repairer. It is assumed that each component after repair in the system is not ‘as good as new’. Under this assumption, by using a geometric process repair model, a replacement policy $(M, N)$ based on the number of failures of component 1 and component 2 respectively is considered. They determine an optimal replacement policy $(M^*, N^*)$ such that the long run expected cost per unit time is minimized. The explicit expression for the long run expected cost per unit time is
derived and the corresponding optimal replacement policy can be determined empirically.

Zhang and Wang [80] proposed a geometric process repair model for a K-dissimilar-component series repairable system with one repairman. For each component, the successive operating times form a decreasing geometric process where as the consecutive repair times constitute an increasing geometric process. Under these assumptions, they considered a replacement policy \( M = (N_1, N_2, \ldots, N_K) \) based on the number of failures of component 1, component 2, ..., and component \( k \) respectively. Here the problem is to determine an optimal replacement policy \( M^* = (N_1^*, N_2^*, \ldots, N_K^*) \) such that the average cost rate is minimized. The explicit expression of the average cost rate is derived and the corresponding optimal replacement policy \( M^* \) is determined analytically.

Zhang et al. [71] developed a deteriorating simple repairable system with \( (k+1) \) states, including \( K \) failure states and one working state. They considered a bi-variate replacement policy \( (T, N) \) in which the system is replaced when its working age has reached \( T \) or the number of failures has reached \( N \), whichever occurs first. Their objective was to determine an optimal replacement policy \( (T, N) \) such that the long-run expected profit per unit time is maximized. The explicit expression of the long-run expected profit per unit time is derived and the corresponding optimal replacement policy is determined numerically and also they proved that for a multistate repairable system the optimal policy \( (T, N)^* \) is better than the optimal policy \( N^* \).

Wang and Zhang [63] presented a more reasonable assumption for a geometric process repair model based on Zhang [74] and Wang and Zhang [65]. They adopted a new assumption that the interval of the preventive repair (i.e. the corresponding preventive repair point) will depend on the reliability of the system instead of the assumption of the fixed length of the preventive repair in Zhang [74] and Wang and Zhang [65]. When the reliability of the system drops to a constant \( R \) which is called a
critical reliability of the system, the system is closed and preventive repair is executed at once. The critical reliability $R$ and the failure-number $N$ of the system will form bivariate mixed policy $(R,N)$. The explicit expression of the average cost rate of the system could be derived and the corresponding optimal mixed policy $(R,N)^*$ could be determined analytically or numerically. Zhang and Wang [69] applied the geometric process repair model to a two-dissimilar-component cold standby repairable system with one repairman and priority in use and repair. It is assumed that either component after repair is not “as good as new” and follows a geometric process repair, and component 1 has priority in use and repair. Under these assumptions, they considered a replacement policy $N$ based on the number of repairs of component 1 under which the system is replaced when the repair number of component 1 reaches $N$. They determined an optimal replacement policy $N^*$ such that the average cost rate of the system is minimized. The explicit equation of the average cost rate of the system is derived and the corresponding optimal replacement policy $N^*$ can be determined analytically or numerically.

Zhang and Wang [81] studied a degenerative simple system (i.e. a degenerative one-component system with one repairman) with $k + 1$ states, including $k$ failure states and one working state by assuming that system after repair is not “as good as new”, and the degeneration of the system is stochastic. Under these assumptions, they considered a new replacement policy $T$ based on the system age determined an optimal replacement policy $T^*$ such that the average cost rate (i.e. the long-run average cost per unit time) of the system is minimized. Further, they proved that the repair model for the multistate system forms a general monotone process repair model which includes the geometric process repair model as a special case.

Yu-Hung Chine [67] presented a model for determining the optimal number of minimal repairs before ordering spare for preventive replacement. By introducing the costs of ordering, repair, downtime, replacement, and the salvage value of an un-failed system, the expected long-term cost rates and cost effectiveness are derived. It is shown that, under certain conditions, the optimal number of minimal repairs, which minimizes
the cost rate or maximizes the cost effectiveness, is given by a unique solution of an equation. A numerical example is also provided for the proposed model.

Venkata Ramudu and Krishna Reddy [61] studied a single unit cold standby repairable system consisting of two identical components with one repairman. It is assumed that the successive operating times of each component form a decreasing arithmetico - geometric process while the consecutive repair times form an increasing arithmetico-geometric process and each component after repair is not ‘as good as new’. Under these assumptions, by using arithmetico-geometric process, a repair replacement policy N is studied based on the number of failures of the component 1. An explicit expression for the long –run average cost per unit time is derived and corresponding optimal replacement policy N* is determined such that the long –run average cost per unit time is minimized. Finally, numerical results are provided to highlight the theoretical results.

Venkata Ramudu and Krishna Reddy [60] considered a two unit cold standby repairable system consisting of two identical components with one repairman. It is assumed that the successive operating times of each component form a decreasing arithmetico - geometric process while the consecutive repair times form an increasing arithmetico-geometric process and each component after repair is not ‘as good as new’. Under these assumptions, a repair replacement policy N is studied based on the number of failures of the component 1. An explicit expression for the long –run average cost per unit time is derived and corresponding optimal replacement policy N* is determined such that the long –run average cost per unit time is minimized. Finally, numerical results are provided to highlight the theoretical results.

Zhang and Wang [79] applied the geometric process repair model to a two-component cold standby repairable system with one repairman by assuming that each component after repair is not “as good as new” and follows a geometric process repair, and component 1 has use priority. It is considered that a repair–replacement policy N based on the number of failures of component 1 under which the system is replaced when the failure number of component 1 reaches N. Our purpose is to determine an
optimal replacement policy \( N^* \) such that the average cost rate of the system is minimized. The explicit expression of the average cost rate of the system is derived and the corresponding optimal replacement policy \( N^* \) can be determined analytically or numerically. Finally, a numerical example is given to illustrate some theoretical results included the uniqueness of the optimal replacement policy \( N^* \), the sensitivity analysis, i.e. the influence of the ratio of the geometric process on the optimal solution, and the comparison of the optimal solutions for the model with and without use priority.

Li Yuan, JianXu [40] studied a cold standby repairable system with two different components and one repairman who can take multiple vacations. If there is a component which fails and the repairman is on vacation, the failed component will wait for repair until the repairman is available. In the system, assume that component 1 has priority in use. After repair, component 1 follows a geometric process repair, while component 2 can be repaired as good as new after failures. Under these assumptions, a replacement policy \( N \) based on the failed times of component 1 is studied. The system will be replaced if the failure times of component 1 reach \( N \). The explicit expression of the expected cost rate is given, so that the optimal replacement time \( N_n \) is determined. Finally, a numerical example is given to illustrate the theoretical results of the model.

Kit Nam Francis Leung et.al [21] a cold standby repairable system consisting of two dissimilar components and one repairman is studied. Assume that working time distributions and repair time distributions of the two components are both exponential, and Component 1 has repair priority when both components are broken down. After repair, Component 1 follows a geometric process repair while Component 2 obeys a perfect repair. Under these assumptions, using the perfect repair model, the geometric process repair model and the supplementary variable technique, by not only studying some important reliability indices, but also considering a replacement policy \( T \), under which the system is replaced when the working age of Component 1 reaches \( T \). The problem is to determine an optimal policy \( T_n \) such that the long-run average loss per unit time (i.e. average loss rate) of the system is minimized. The explicit expression for the average loss rate of the system is derived, and the corresponding optimal
replacement policy $T_n$ can be found numerically. Finally, a numerical example for replacement policy $T$ is given to illustrate some theoretical results and the model’s applicability.

Chin-Chih Chang, Shey-Huei Sheu, Yen-Luan Chen, Zhe George Zhang [12] A system is subject to shocks that arrive according to a non-homogeneous Poisson process. As these shocks occur, the system experiences one of two types of failures: a type-I failure (minor), rectified by a minimal repair; or a type-II failure (catastrophic) that calls for a replacement. In this study, they considered a multi-criteria replacement policy based on system age, nature of failure, and entire repair-cost history. Under such a policy, the system is replaced at planned life time $T$, or at the $n$th type-I failure, or at the $k$th type-I failure ($k < n$) at which the accumulated repair cost exceeds the pre-determined limit, or at the first type-II failure, whichever occurs first. An optimal policy over the control parameters is studied analytically by showing its existence, uniqueness, and structural properties. This model is a generalization of several existing models in the literature. Some numerical examples are presented to show several useful insights.

Ming Xu, Tao Chen, Xianhui Yang [45] investigated replacement scheduling for non-repairable safety-related systems (SRS) with multiple components and states. The aim is to determine the cost-minimizing time for replacing SRS while meeting the required safety. Traditionally, such scheduling decisions are made without considering the interaction between the SRS and the production system under protection, the interaction being essential to formulate the expected cost to be minimized. In this paper, the SRS is represented by a non-homogeneous continuous time Markov model, and its state distribution is evaluated with the aid of the universal generating function. Moreover, a structure function of SRS with recursive property is developed to evaluate the state distribution efficiently. These methods form the basis to derive an explicit expression of the expected system cost per unit time, and to determine the optimal time to replace the SRS. The proposed methodology is demonstrated through an illustrative example.
Wang and Zhang [66] discussed the optimal replacement problem for a system with two types of failures. One type of failure is repairable, which is conducted by a repairman when it occurs, and the other is un-repairable, which leads to a replacement of the system at once. The repair of the system is not ‘as good as new’. The consecutive operating times of the system after repair form a decreasing geometric process, while the repair times after failure are assumed to be independent and identically distributed. Replacement policy N is adopted, where N is the number of repairable failures. The system will be replaced at the N\textsuperscript{th} repairable failure or at the un repairable failure, whichever occurs first. Two replacement models are considered, one is based on the limiting availability and the other based on the long-run average cost rate of the system. They derived an explicit expression for the limiting availability and the long-run average cost rate of the system under policy N, respectively. By maximizing the limiting availability \(A(N)\) and minimizing the long-run average cost rate \(C(N)\), They theoretically obtain the optimal replacement policies \(N^*\) in both the cases.

Shey-Huei Sheu et.al [54] studied the optimal replacement policy with general repairs for an operating system subject to shocks occurring to a non-homogeneous pure birth process (NHPBP). A shock causes that the system experiences one of two types of failures: type-I failure (minor failure) is rectified by a general repair, or type-II failure (catastrophic failure) is removed by an unplanned replacement. The probabilities of these two types of failures depend on the number of shocks since the last replacement. They considered a bivariate replacement policy \((N,T)\) under which the system is replaced at planned life age \(T\), or at the nth type-I failure, or at any type-II failure, whichever occurs first. The optimal replacement schedule which minimizes the expected cost rate model is derived analytically and discussed numerically.

Chin-Chih Chang et.al [13] presented a replacement model with age dependent failure type based on a cumulative repair-cost limit policy, whose concept uses the information of all repair costs to decide whether the system is repaired or replaced. As failures occur, the system experiences one of the two types of failures: a type-I failure (minor), rectified by a minimal repair; or a type-II failure (catastrophic) that calls for a
replacement. A critical type-I failure means a minor failure at which the accumulated repair cost exceeds the pre-determined limit for the first time. The system is replaced at the nth type-I failure, or at a critical type-I failure, or at first type-II failure, whichever occurs first. The optimal number of minimal repairs before replacement which minimizes the mean cost rate is derived and studied in terms of its existence and uniqueness. Several classical models in maintenance literature are special cases of our model.

1.4 INITIATION AND INVESTIGATION

In most of the studies the maintenance problems are developed based on the Geometric Process Models (GPM). Obviously, in this direction Lam [26, 27], Stadje and Zukerman [57,58], Stanley [59], Leung and Lee [37], Wang and Zhang [62] and others studied repair replacement models under the GPM. However, the Geometric Process Models are not free from certain disadvantages such as insufficient/erroneous data on
expected number of failures at an arbitrary time. Braun et.al [11] introduced an alternative model, the \( \alpha \)-series process, which supports these characteristics. Furthermore, they explained the increasing geometric process grows at most logarithmically in time, while the decreasing geometric process is almost certain to have a time of explosion. The \( \alpha \)-series process grows either as a polynomial in time or exponential in time. It also noted that the geometric process doesn’t satisfy a central limit theorem, while the \( \alpha \)-series process does. Braun et.al [11] also presented that both the increasing geometric process and the \( \alpha \)-series process have a finite first moment under certain general conditions. However the decreasing geometric process usually has an infinite first moment under certain conditions. Thus the decreasing \( \alpha \)-series process may be more appropriate for modeling system working times while the increasing geometric process is more suitable for modeling repair times of the systems.

These processes are considered together with other performance measures like profit, cost, down time etc., based on this understanding two monotone process approaches is considered to be more relevant, realistic, and direct to the modeling of the deteriorating system maintenance problems that are encountered in most situations other than perfect or minimal repair.

Based on this understanding, this study is intended to develop a single unit cold standby repairable system and two unit cold standby repairable systems, two dissimilar components with priority in use, two dissimilar components with priority in use and repair and two dissimilar components series system.

The layout of the thesis is as follows:

**Chapter I**, discusses a brief review of preliminary needs for the present work, and an overview of the earlier research works. It also explains the factors related to the present topics of the thesis. Finally, the chapter is provided layout of the thesis at the end.
Chapter II, deals with an optimal replacement policy for a single unit cold standby repairable system using two monotone process focusing on exponential failure law. In this model, it is assumed that each component after repair is not ‘as good as new’ and also the successive working times form a decreasing $\alpha$-series process, the successive repair times form an increasing geometric process and both the processes are exposing to exponential failure law. Under these assumptions, we develop an explicit expression for the long-run average cost per unit time in such a way that it is minimized. Further, we determined an optimal replacement policy $N^*$ and to demonstrate the theoretical results, the numerical results are presented and graphs of relevant processes are provided.

Chapter III, studies an optimal replacement policy for a two unit cold standby repairable system using two monotone processes exposing to exponential distribution. We study an optimal replacement policy $N$ for a cold standby system consisting of two identical components with one repairman. Assume that each component after repair is not ‘as good as new’ and the successive working times form a decreasing $\alpha$-series process, the successive repair time’s form an increasing geometric process and both the processes are exposing to exponential failure law. Under these assumptions, we derived an explicit expression for the long-run average cost per unit time and corresponding optimal replacement policy $N^*$ is determined. Finally, we provide numerical results to highlight the theoretical results and also provide the results through the graphs wherever it required. Finally, a brief summary and further scope for research on the lines of the present work is discussed.

Chapter IV, provides the two monotone processes repair model for a two-dissimilar-component cold standby repairable system with one repairman and priority in use and repair. It is assumed that either component after repair is not “as good as new” and follows a two monotone process repair model, and component 1 has priority in use and repair. Under these assumptions, we considered a replacement policy $N$ based on the number of repairs of component 1 under which the system is replaced when the repair
number of component 1 reaches $N$. We determined an optimal replacement policy $N^*$ such that the average cost rate of the system is minimized. The explicit equation of the average cost rate of the system is derived and the corresponding optimal replacement policy $N^*$ can be determined analytically or numerically.

Chapter V, develops the two monotone process repair model to a two-dissimilar-component cold standby repairable system with one repairman and priority in use. It assumed that either component after repair is not "as good as new" and follows a two monotone process repair model, and component 1 has priority in use. Under these assumptions, considered a replacement policy $N$ based on the number of repairs of component 1 under which the system is replaced when the repair number of component 1 reaches $N$. Also determined an optimal replacement policy $N^*$ such that the average cost rate of the system is minimized. The explicit equation of the average cost rate of the system is derived and the corresponding optimal replacement policy $N^*$ can be determined analytically or numerically.

Chapter VI studies a series repairable system consisting of two-non identical components with one repairman. It is assumed that the successive operating times of each component form a decreasing $\alpha$-series process, while the consecutive repair times form an increasing geometric process and each component after repair is not 'as good as new'. Under these assumptions, by using two monotone processes, a repair replacement policy $(N_1, N_2)$ is studied based on the number of failures of the component 1 and component 2 and derived an explicit expression for the long run average cost per unit time. An optimal replacement policy $(N_1, N_2)^*$ is determined such that the average cost rate is minimum.

The thesis is appended with a list of References and publications of this work in the end.