2. PERTURBATION BASED LOAD FLOWS

2.1 INTRODUCTION

Almost all load flow techniques [1]-[9], [42], [43] found in literature use a very familiar solutions structure. Starting from an initial guess, active and reactive power mismatches at the system buses are determined. Either the bus power equations or their derivatives are then used to compute a correction vector that is used to update the problem variables. Load flow problem diverges when the estimated corrections are grossly erroneous. As, the corrections are estimated directly by solving a set of equations, one actually reaches a point of no return when the load flow starts diverging. So, it can be said that the very cause of the divergence lies in the method adopted for the solution. To overcome the drawback an altogether different approach has been used in this chapter to develop two simple load flow techniques. Rather than computing the corrections required to reach the final solution, the current solutions are perturbed around their present values. Only those perturbations which help the present solutions to move towards the convergence are accepted. This procedure helps avoiding divergence of the problem, though the methods in general are slow. The proposed methods are therefore useful when the normal load flow methods fail to converge or the load flow is started with arbitrary variables values which may be far away from the actual solutions. Such situations arise when load flow problems are solved using evolutionary computing techniques where random initializations of the variables at a population of points are to be done.

The proposed load flows are actually developed to be used along with the evolutionary techniques, though the developed methods are independently complete. Two algorithms have been proposed. In the first the acceptance or rejection of the perturbations are decided based upon their performance on a localized network while in the second it is judged on the total network. The most positive feature of these methods is that they never diverge. In the following methods are first described, their performance on standard test systems are then presented along with the comparative analysis with other load flow techniques.

2.2. LOAD FLOW BASED ON LOCAL SEARCH TECHNIQUE

In the load flow problem voltages and phase angles of the system buses are determined using some iterative techniques. Assuming flat start values of the variables these iterative techniques compute the correction required so as to minimize the nodal power mismatches. An alternative approach using local search may be used where a test correction is generated first and the effect of this correction is used to make further corrections. Voltages and phase angles of the system buses are perturbed one at a time around their present values.

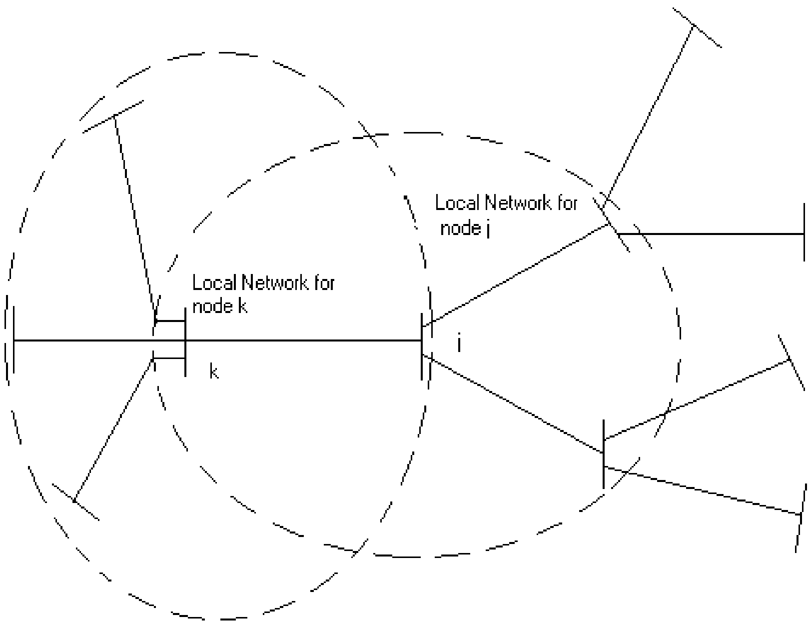

Fig. 2.1: The constitution of Local Network

The effect of the perturbation is judged on a local network which is formed taking the branches and buses directly connected to the bus whose variables have been perturbed. Fig. 2.1 may be viewed as a section of any typical power system. The encircled sub-network around bus k is the local network for bus k when perturbations are applied on the voltage and phase angles of bus k. Similarly the encircled sub-network around bus i represents the local network for bus i. Assuming that perturbation has been applied on the variables of node i, it is assumed that complex voltages of all nodes except node i remain constant. Voltage and phase angle of node i in the local network are now corrected with a very small step. While correcting V_i (or δ_i) a random number is generated between 0 and $\Delta V_{i,\max}$ (or $\Delta\delta_{i,\max}$). The $\Delta\delta_i$ or ΔV_i such generated is either subtracted or added to δ_i or V_i depending upon whether ΔP_i or ΔQ_i are negative or positive. Sum of the squares of the mismatches and maximum mismatch in the local network are determined. If both the sum square mismatch and maximum mismatch decrease, the perturbed solution is accepted and the method is repeated on the same bus. Such corrections are applied to all nodes of the systems. Since we assume constant voltage in the network outside the concerned node, efforts involved in the computations of the mismatches are very little as we need to compute the changes in the flows of the lines connected to the node only.

Active and reactive power mismatches of node i and also of those buses directly connected to node i are now calculated to include the impact of the perturbation. As the voltage/phase angle of bus i only has been changed keeping all other bus voltage and phase angles constant, ΔP , ΔQ of the buses outside the local network remain unchanged.

2.2.1. THE ALGORITHM OF THE LOCAL SEARCH

The algorithm is explained in the following steps:

I. Assume normal starting values for the variables (V , δ) and compute the active and reactive power mismatches (ΔP , ΔQ) of the nodes.

II. Identify the local network around bus i (Fig. 2.1), as the network directly connected to the bus and apply the following steps

- Compute active and reactive power mismatches of the nodes of the local network
- Identify the maximum of these mismatches ΔPQ_{old}
- Find the sum of the squares of the mismatches $\Delta PQ_{sum}^{old} = \sum_j (\Delta P_j^2 + \Delta Q_j^2)$ where j includes of the nodes of the local network.
- Keeping all other variables fixed at their old values, perturb only the voltage magnitude of bus i, by a small value.
- Compute the impact of the perturbation as

$$\Delta P_i^{inc} + j\Delta Q_i^{inc} = \frac{\Delta V_i}{V_i}(P_i + jQ_i) + Y_{ii}^* \Delta V_i (V_i + \Delta V_i) \tag{2.1}$$

$$\Delta P_j^{inc} + j\Delta Q_j^{inc} = V_j Y_{ji}^* \Delta V_i e^{-j\delta_i} \tag{2.2}$$

- Find maximum mismatch ΔPQ_{new} as $\max(\Delta P_i, \Delta Q_i, (\Delta P_j, \Delta Q_j)_{\text{for all j}})$
- Compute also the sum of the square of the mismatches ΔPQ_{sum}^{new} as $\sum_j (\Delta P_j^2 + \Delta Q_j^2)$
- If $\Delta PQ_{new} < \Delta PQ_{old}$ & $\Delta PQ_{sum}^{new} < \Delta PQ_{sum}^{old}$ repeat the above steps for node i, else go to next step
- Keeping all other variables fixed at their old values, perturb only the phase angle of bus i, by a small value.
- Compute the effect of the perturbation as

$$\Delta P_i^{inc} + j\Delta Q_i^{inc} = (P_i + jQ_i)(e^{j\Delta\delta_i} - 1) + V_i^2 Y_{ii}^* e^{j\Delta\delta_i}(e^{-j\Delta\delta_i} - 1) \tag{2.3}$$

$$\Delta P_j^{inc} + j\Delta Q_j^{inc} = V_i^* V_j Y_{ji} (e^{-j\Delta\delta_i} - 1) \quad (2.4)$$

Above equations are derived in the appendix.

- Again, If $\Delta PQ_{new} < \Delta PQ_{old}$ & $\Delta PQ_{sum}^{new} < \Delta PQ_{sum}^{old}$ repeat the above steps for node i, else go to next step

III. Apply step II for all buses of the network.

IV. Iteration=iteration+1

V. Stop in case of convergence; otherwise repeat the process from step II.

2.2.2. PERFORMANCE OF THE LOCAL SEARCH BASED LOAD FLOW

Local search based load flow employs simple updating technique and subsequent power mismatch computation which also is simple. Solution time per iteration is therefore very small. The flat start has been taken like conventional load flows. It has converged for all the test cases including those of critical loading conditions and ill conditioned systems. The local search based load flow needs large number of iterations to converge. The mismatches however decrease monotonically. Table 2.1 shows some of the test results for different standard [41] as well as ill conditioned systems.

Table 2.1
Test results of Local Search Based Load Flow

Test system	05-bus	11-bus	IEEE 14-bus	IEEE 30-bus	IEEE 57-bus	IEEE 118-bus
Number of iterations	31	58	62	125	207	358

The variations of the maximum errors with the number of iterations have been shown through Fig. 2.2 to Fig. 2.5 for the local search based load flow. As the error reduces with the increase in the number of iterations, convergence is guaranteed, but definitely at slow pace.

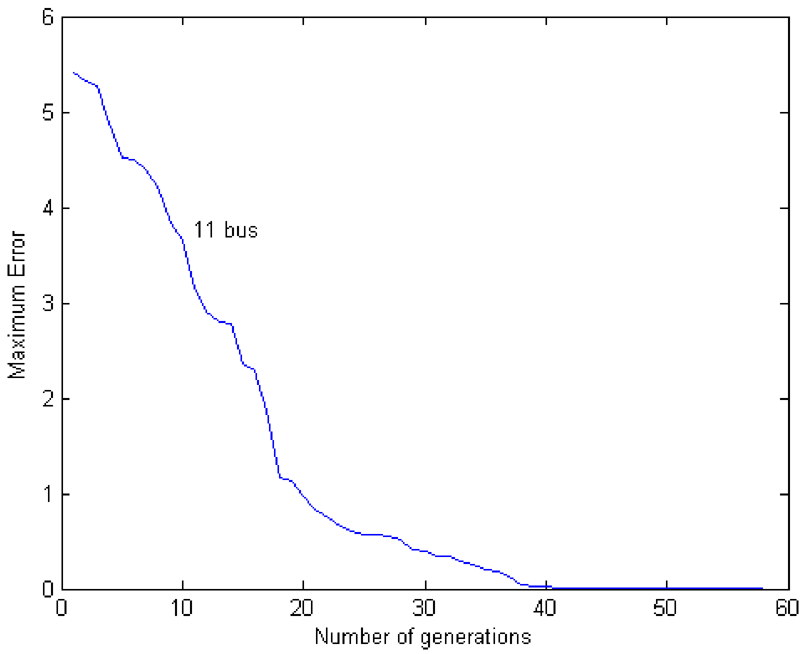

Fig. 2.2: Convergence characteristics of 11-bus test system for local search based load flow

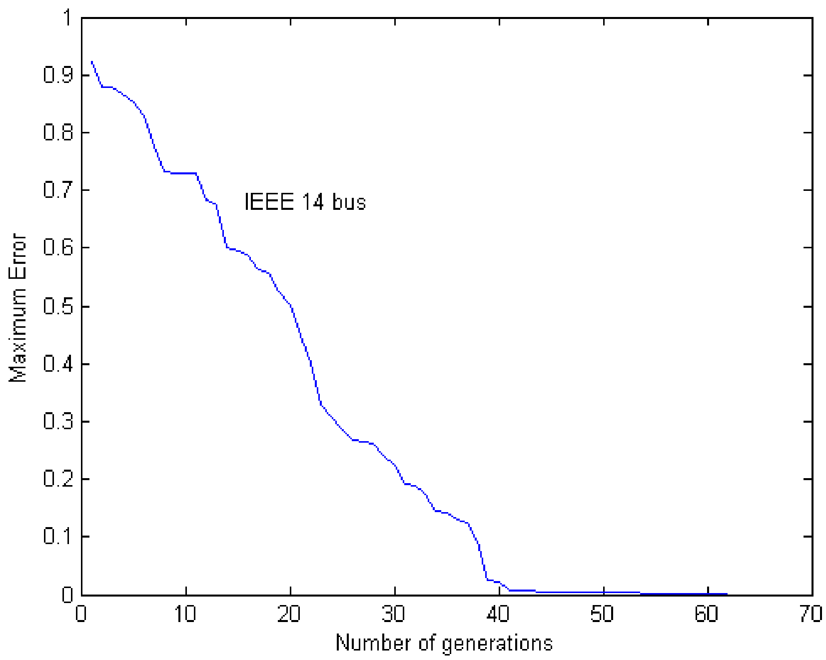

Fig. 2.3: Convergence characteristics of IEEE 14 bus test system for local search based load flow

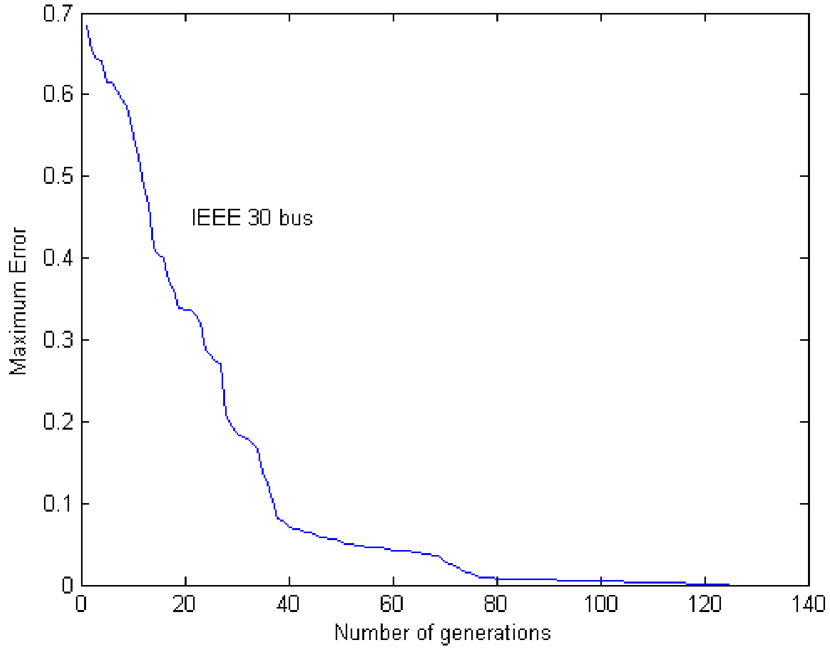

Fig. 2.4: Convergence characteristics of IEEE 30 bus test system for local search based load flow

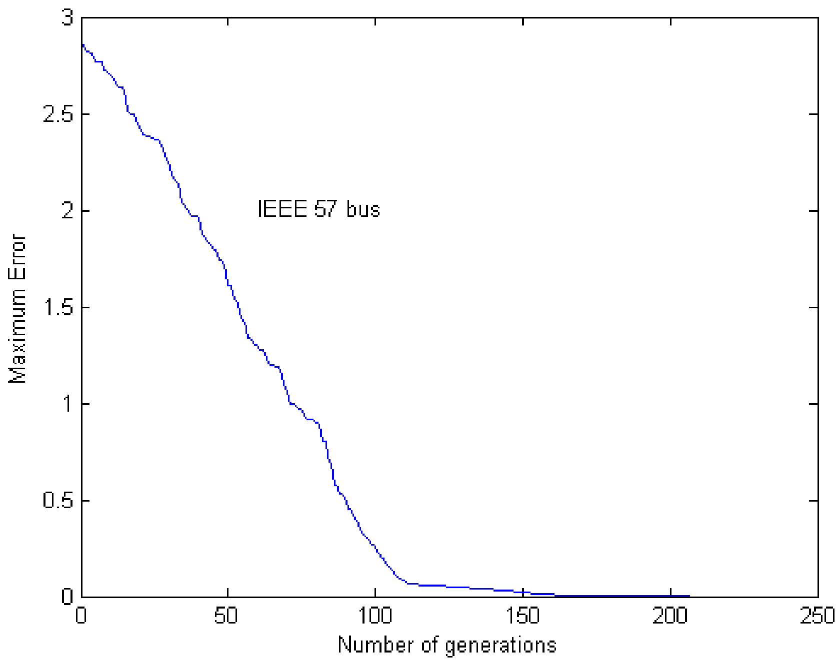

Fig. 2.5: Convergence characteristics of IEEE 57 bus test system for local search based load flow

2.3. LOAD FLOW BASED ON LINEAR PERTURBATION

In the local solution technique variables are perturbed in small steps around their current values. An alternative perturbation technique where the perturbation step size is determined on the basis of the bus power mismatches is proposed in the present section.

This method does not fully avoid the use of the Jacobian matrix; rather the diagonals of the Jacobian are used to have an estimate of the correction element. The decoupling property of the power system variables is not only exploited but overestimated in developing the method.

The range of perturbation $\Delta\delta_{\max}$ and $\Delta V_{\max}$ are fixed by the maximum mismatch $\Delta P_{\max}$ and $\Delta Q_{\max}$ of the solution and the mismatch of the individual node ΔP_i and ΔQ_i. For a given network, the maximum perturbation of phase angle and voltages are determined as

$$\Delta\delta_{\max} = \frac{|\Delta P_{\max}|}{|\partial P/\partial\delta|_{\max}} \tag{2.5}$$

and $$\Delta V_{\max} = \frac{|\Delta Q_{\max}|}{|\partial Q/\partial V|_{\max}} \tag{2.6}$$

The actual perturbation for individual node is determined assuming a linear relationship as

$$\Delta\delta_i = \Delta\delta_{\max}\frac{\Delta P_i}{|\Delta P_{\max}|} \tag{2.7}$$

and $$\Delta V_i = \Delta V_{\max}\frac{\Delta Q_i}{|\Delta Q_{\max}|} \tag{2.8}$$

The $\Delta\delta_i$ or ΔV_i such generated is added to δ_i or V_i. After all δ 's and V 's are perturbed in this way, the active and reactive bus mismatches are determined with the updated voltage and phase angles and the maximum mismatch is determined. If both the maximum mismatch and the sum square error are more than that of the old values, the process is repeated, otherwise go to the next iteration.

2.3.1. THE ALGORITHM OF THE LINEAR PERTURBATION

I. Initialize voltage magnitude and phase angles based on flat start.

II. Calculate active and reactive power mismatches.

III. Find out the maximum mismatch and the sum of the squares of the mismatches.

IV. Update voltage magnitudes and phase angles using equation 2.5, 2.6, 2.7 and 2.8.

V. Determine the power mismatches for the updated solution.

VI. Determine the sum of the squares and the maximum mismatch of the updated solution.

VII. If both the sum square mismatch and maximum mismatch are more than the old values, repeat the process from step III.

VIII. Iteration = iteration + 1.

IX. Stop in case of convergence, otherwise repeat the process from step II.

2.3.2. TEST RESULTS OF THE LINEAR PERTURBATION

The linear perturbation based load flow has been tested on different standard as well as ill conditioned systems. The test results for flat start of the variables are shown in Table 2.2. If the starting values are generated randomly instead of flat start, the desired convergence is obtained but at the cost of more number of iterations.

Table 2.2
Test results of linear perturbation based load flow

Test system	05-bus	11-bus	IEEE 14-bus	IEEE 30-bus	IEEE 57-bus	IEEE 118-bus
Number of iterations	24	45	49	78	164	212

The convergence characteristics of the different test systems are shown in Fig. 2.6 to 2.9. It may be noticed that error decreases monotonically as the iteration progresses. This ensures that the linear perturbation, like the local search technique, has a very strong convergence property.

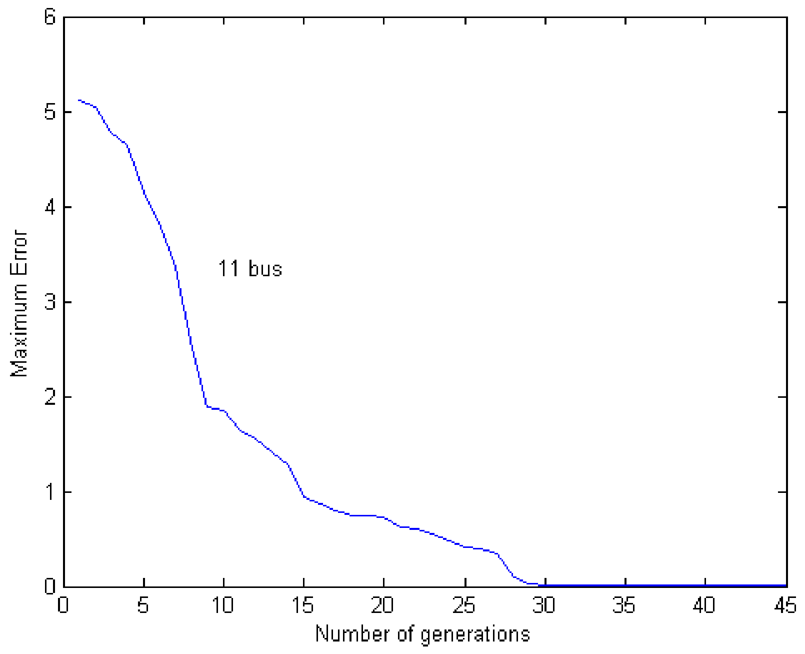

Fig. 2.6: Convergence characteristics of 11-bus test system for linear perturbation based load flow

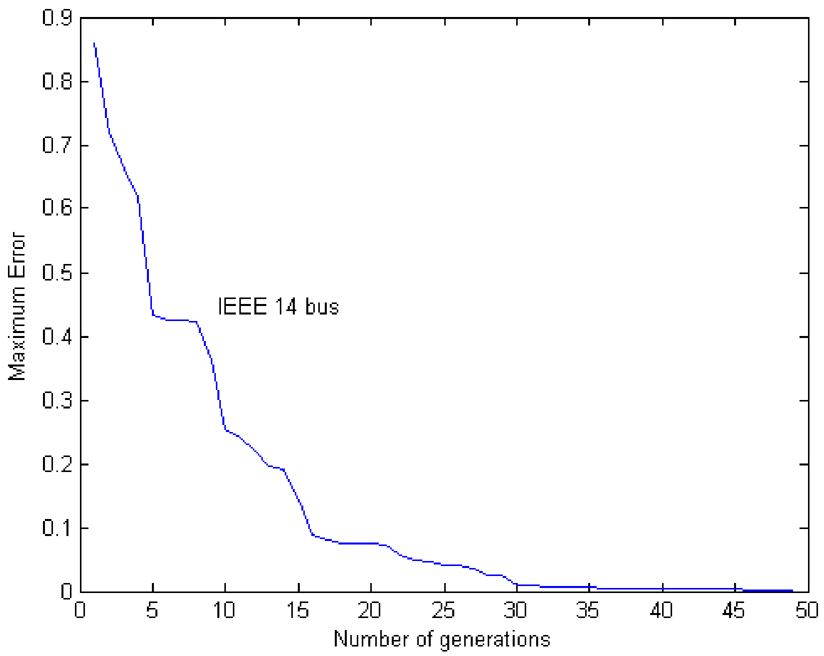

Fig. 2.7: Convergence characteristics of IEEE 14 bus test system for linear perturbation based load flow

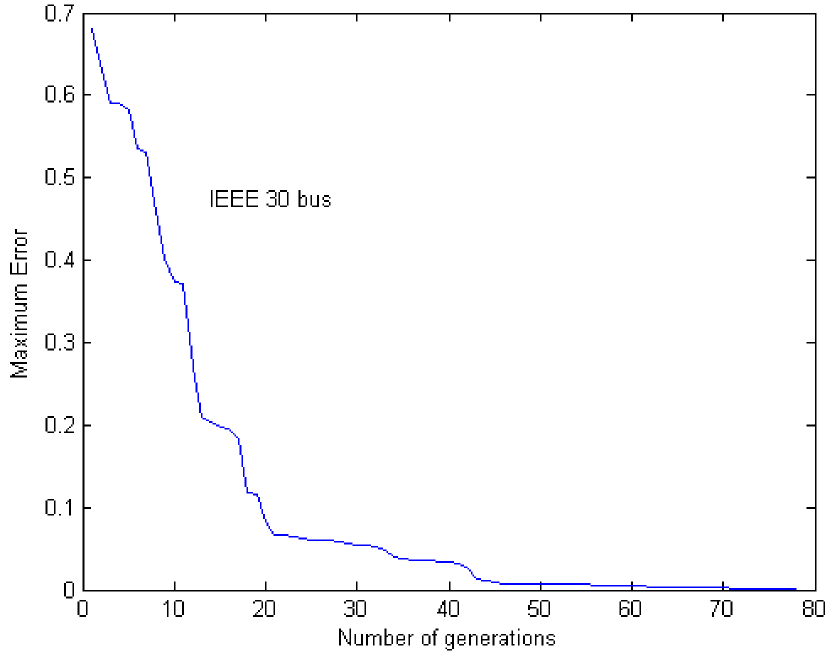

Fig. 2.8: Convergence characteristics of IEEE 30 bus test system for linear perturbation based load flow

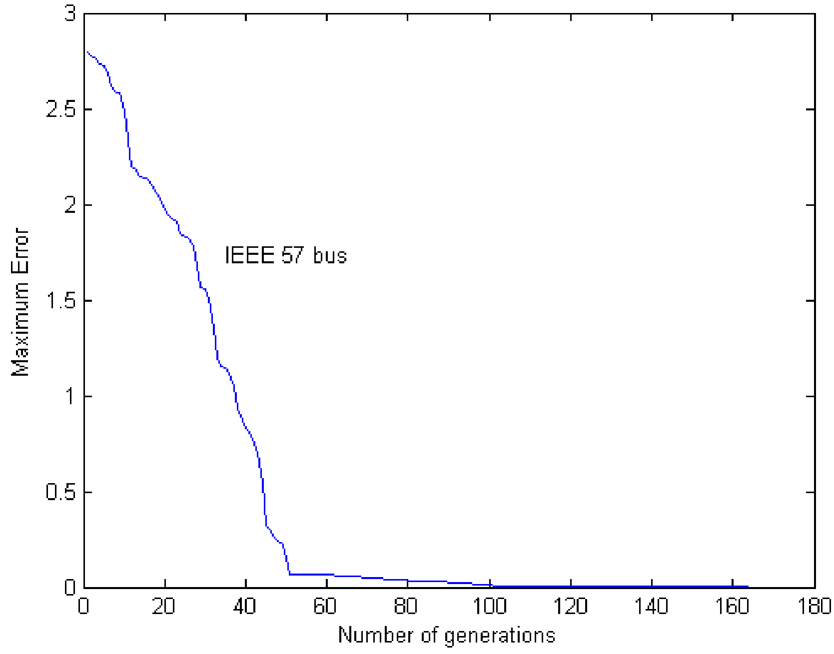

Fig. 2.9: Convergence characteristics of IEEE 57 bus test system for linear perturbation based load flow

2.4. TEST RESULTS FOR RANDOM INITIALIZATION

Instead of flat start if the starting values are generated randomly within a certain range, then also the proposed methods show convergence. The voltage magnitudes are generated randomly from 0.7 to 1.2 p.u. and phase angles are initialized between 0.1 to –2.5 radian. The test results with random initialization are given in Table 2.3 & 2.4 for local search and linear perturbation based load flow algorithms respectively.

Table 2.3
Test results of local search based load flow with random initialization

Test system	05-bus	11-bus	IEEE 14-bus	IEEE 30-bus	IEEE 57-bus	IEEE 118-bus
Number of iterations	74	119	121	263	367	824

Table 2.4
Test results of linear perturbation based load flow with random initialization

Test system	05-bus	11-bus	IEEE 14-bus	IEEE 30-bus	IEEE 57-bus	IEEE 118-bus
Number of iterations	62	115	102	197	372	613

From the above test results it is noticed that the number of generations are increased when starting values are initialized randomly in place of flat-start.

2.5. TIME COMPARISON

Both the methods showed convergence for all the test cases even when ill conditioning were introduced in the test cases. The program developed and run on Pentium IV CPU 2.40 GHz, 256 MB of RAM, 40 GB hard disc computer using MATLAB 6.5 version. The comparisons of time per iteration for the proposed methods are shown in Table 2.5 and the total convergence time of the proposed methods are given in Table 2.6.

TABLE 2.5
Comparison of time per iteration in second

Method	5 bus	11 bus	IEEE 14 bus	IEEE 30 bus	IEEE 57 bus	IEEE 118 bus
Local search	0.0470	0.0620	0.0790	0.1250	0.2500	0.7040
Linear perturbation	0.0630	0.0790	0.1090	0.2340	0.4690	1.1250

TABLE 2.6
Comparison of total convergence time in second

Method	5 bus	11 bus	IEEE 14 bus	IEEE 30 bus	IEEE 57 bus	IEEE 118 bus
Local search	1.4570	3.5960	4.8980	15.6250	51.75	252.0320
Linear perturbation	1.5120	3.5550	5.3410	18.2520	76.9160	238.5000

From Table 2.5 it is clear that in local search technique the time per iteration is less than the linear perturbation technique for every type of test system. But in Table 2.6 it is observed that in most of the test systems the total convergence time for the local search based load flow is slightly less than that of the linear perturbation based load flow.

In both the perturbation based load flow methods the time per iteration remains same but the total convergence time has increased when random-start is used as the number of generations are increased. The total convergence time of the proposed methods with random-start initialization are given in Table 2.7.

TABLE 2.7
Comparison of total convergence time in second for random initialization

Method	5 bus	11 bus	IEEE 14 bus	IEEE 30 bus	IEEE 57 bus	IEEE 118 bus
Local search	3.4780	7.3780	9.5590	32.8750	91.7500	580.0960
Linear perturbation	3.9060	9.0850	11.1180	46.0980	174.4680	689.6250

2.6. APPLICATIONS OF THE PERTURBATION BASED LOAD FLOWS

The proposed perturbation based load flow can have some interesting applications:

i. Along with the conventional Newton-Raphson power flow, it can be used to determine the critical loading conditions of the power system.
ii. Along with the evolutionary based power flows, it can help to determine the low voltage solutions.
iii. Hybrid of evolutionary and perturbation method can find the normal power flow solutions faster.

The last two applications are elaborated in the forthcoming chapters while the fast one are discussed below.

The Newton-Raphson power flow suffers from the singularity problem at or near the maximum loadability condition. The perturbation based power flow is applied in such conditions with starting values of the variables set at the last converged solutions of the Newton-Raphson power flow. The perturbation based power flow had been successful in determining the critical loading solutions in all the test cases conducted. Table 2.8 shows the result obtained for standard test systems. The last converged solution of Newton-Raphson power flow and that for the critical loading condition are reported in the Table 2.9.

TABLE 2.8

Test results of Newton–Raphson load flow method for maximum load multiplier

Test System	Load Multiplier	Solutions (Voltage magnitudes in p.u. & phase angles in degree)
5 bus	3.0631.	V=[1.0600 0.7992 0.6693 0.6545 0.5849] Delta=[0 -12.6782 -24.8959 -27.4500 -35.3876]
14 bus	4.0078	V=[1.0600 1.0450 1.0100 0.7089 0.6804 1.0700 0.8099 1.0900 0.7237 0.7432 0.8868 0.9779 0.9300 0.7001] Delta=[0 -42.0913 -91.9330 -76.6392 -65.6839 -110.9485 -99.1857 -99.1857 -110.8437 -112.8055 -112.2714 -114.9088 -114.9606 -120.5280]

TABLE 2.9

Test results of Perturbation based load flow upto the critical loading condition

Test System	Load Multiplier	Solutions (Voltage magnitudes in p.u. & phase angles in degree)
5 bus	3.0887	V=[1.0600 0.7992 0.6415 0.6264 0.5477] Delta=[0 -14.3782 -27.7187 -30.6634 -40.0921]
14 bus	4.0115	V=[1.0600 1.0450 1.0100 0.7067 0.6778 1.0700 0.8082 1.0900 0.7218 0.7415 0.8859 0.9777 0.9295 0.6985] Delta=[0 -42.2734 -92.3019 -76.9851 -65.9778 -111.5109 -99.6575 -99.6575 -111.3740 -113.3506 -112.8257 -115.4759 -115.5257 -121.1089]

2.6.1. P-V/Q-V CURVE USING PERTURBATION BASED TECHNIQUES

Optimum multiplier based load flow [16] proposed by Iba K., Suzuki H., Egawa M., and Watanabe T. can find the low voltage solutions nearest to the normal solution. Thus the optimal power flow based load flow can be used along with the proposed perturbation based load flow to draw the P-V/Q-V curves of a power system. Such curves are very useful for voltage stability analysis. The P-V/Q-V curves obtained for 5 bus and 14 bus test systems are shown in Fig. 2.10 and 2.11 respectively.

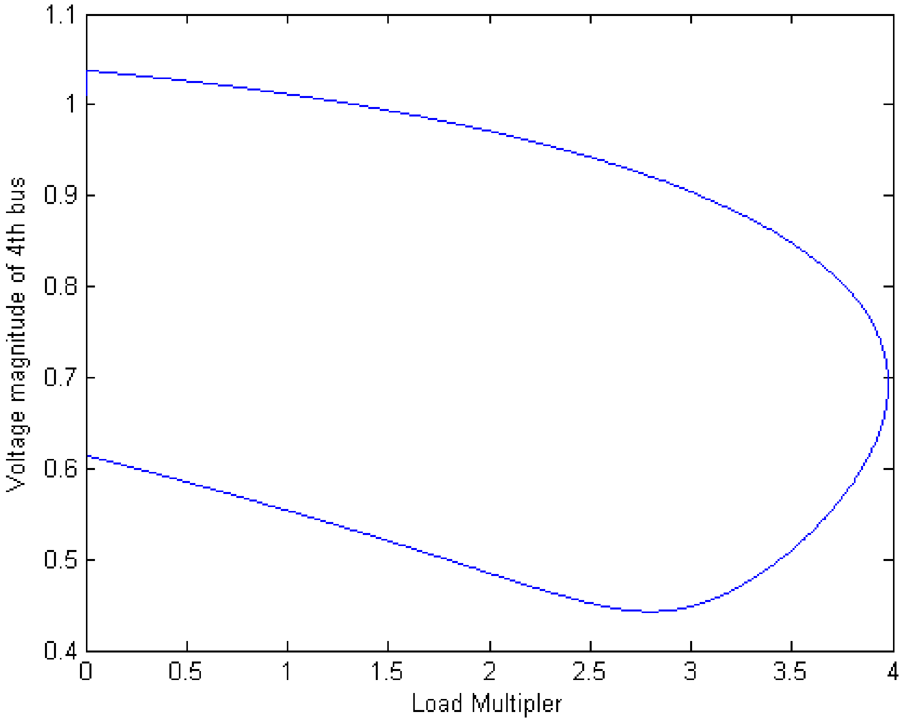

Fig. 2.10: PV curve of 4th bus of IEEE 14 bus test system

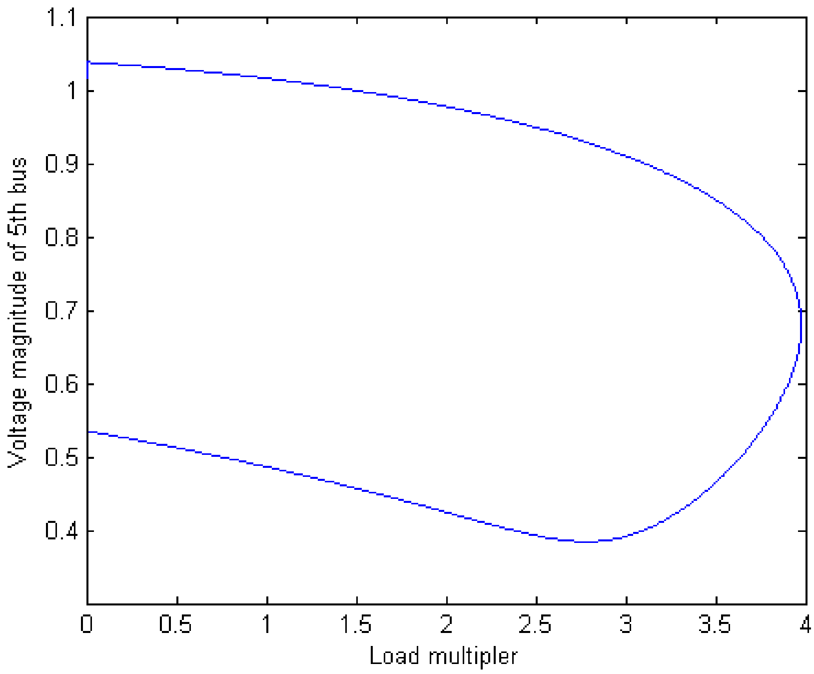

Fig. 2.11: PV curve of 5th bus of IEEE 14 bus test system

2.7 CONCLUSION

Two new power flow methods based on perturbation techniques are proposed in this chapter. The methods, though a little bit slow, are quite reliable in terms of convergence. Proposed power flows are useful in finding the critical loading conditions, in situations when conventional methods fail and also along with the evolutionary methods, particularly in finding multiple power flow solutions.

APPENDIX A2

A2.1. TERMS OF POWER MISMATCHES FOR THE CHANGES IN BUS

Complex power of bus i in terms of the bus voltages V, and admittance matrix Y, is given by

$$P_i + jQ_i = V_i I_i^*$$

$$= V_i \sum_{m=1}^{n} Y_{im}^* V_m^* \quad \text{(A2.1)}$$

For a change in the magnitude of ith bus voltage by ΔV_i , changes in the mismatches of bus i and j may be computed as:

$$\Delta P_i + j\Delta Q_i = \frac{\Delta V_i}{V_i}(P_i + jQ_i) + Y_{ii}^* \Delta V_i (V_i + \Delta V_i) \quad \text{(A2.2)}$$

$$\Delta P_j + j\Delta Q_j = Y_{ji}^* \Delta V_i e^{-j\delta_i} \quad \text{(A2.3)}$$

Similarly for a change in the phase angle of bus i by $\Delta\delta_i$, changes in the mismatches of bus i and j are computed as:

$$\Delta P_j + j\Delta Q_j = V_i^* V_j Y_{ji} (e^{-j\Delta\delta_i} - 1) \quad \text{(A2.4)}$$

$$\Delta P_i + j\Delta Q_i = (P_i + jQ_i)(e^{j\Delta\delta_i} - 1) + V_i^2 Y_{ii}^* e^{j\Delta\delta_i} (e^{-j\Delta\delta_i} - 1) \quad \text{(A2.5)}$$

A2.2. CALCULATION OF POWER MISMATCHES FOR THE CHANGE IN VOLTAGE MAGNITUDE

If ith bus voltage magnitude is incremented by ΔV_i keeping the phase angle constant, the modified bus power may be derived as:

For ith bus:

$$P_i' + jQ_i' = (V_i + \Delta V_i) e^{j\delta_i} [\sum_{\substack{m=1 \\ \neq i}}^{n} Y_{im}^* V_{im}^* + Y_{ii}^* (V_i + \Delta V_i) e^{-j\delta_i}] \quad \text{(A2.6)}$$

$$= (V_i + \Delta V_i).e^{j\delta_i}.[\sum_{m=1}^{n} Y_{im}^*.V_{im}^* + Y_{ii}^* \Delta V_i .e^{-j\delta_i}]$$

$$= V_i.e^{j\delta_i}.\sum_{m=1}^{n} Y_{im}^*.V_{im}^* + \Delta V_i.e^{j\delta_i} \sum_{m=1}^{n} Y_{im}^*.V_{im}^*$$

$$+V_i.e^{j\delta_i} Y_{ii}^* \Delta V_i.e^{-j\delta_i} + \Delta V_i.e^{j\delta_i} Y_{ii}^* \Delta V_i.e^{-j\delta_i}$$

$$= P_i + jQ_i + \frac{\Delta V_i}{V_i}(P_i + jQ_i) \ +V_i \Delta V_i Y_{ii}^* + \Delta V_i Y_{ii}^* \Delta V_i$$

$$\Delta P_i + j\Delta Q_i = \frac{\Delta V_i}{V_i}(P_i + jQ_i) + Y_{ii}^* \Delta V_i (V_i + \Delta V_i)$$

For j^th^ bus:

$$P_j^{'} + jQ_j^{'} = V_j I_j^* \tag{A2.7}$$

$$= \boldsymbol{V_j \sum Y_{jk}^* V_k}$$

$$= V_j [\sum Y_{jk}^* V_{V_k}^* + Y_{ji}^* V_i^*]$$

$$= V_j [S^{'} + Y_{ji}^* (V_i + \Delta V_i) e^{-j\delta_i}]$$

$$= S^{'} + Y_{ji}^* V_i e^{-j\delta_i} + Y_{ji}^* \Delta V_i e^{-j\delta_i}$$

$$= P_j + jQ_j + Y_{ji}^* \Delta V_i e^{-j\delta_i}$$

$$\Delta P_j + j\Delta Q_j = Y_{ji}^* \Delta V_i e^{-j\delta_i}$$

A.2.3. CALCULATION OF POWER MISMATCHES FOR THE CHANGE IN PHASE ANGLE

If the phase angle of bus i is incremented by $\Delta\delta_i$, keeping the voltage magnitude constant, the expression for the modified bus power is derived as follows:

For i^th^ bus:

$$P_i^{''} + jQ_i^{''} = V_i e^{j(\delta_i + j\Delta\delta_i)} [\sum_{\substack{m=1 \\ \neq i}}^{n} Y_{im}^* V_m^* + Y_{ii}^* V_i e^{-j(\delta_i + \Delta\delta_i)}] \tag{A2.8}$$

$$= V_i e^{j\delta_i} e^{j\Delta\delta_i} [\sum_{\substack{m=1 \\ \neq i}}^{n} Y_{im}^* V_m^* + Y_{ii}^* V_i^* \; - Y_{ii}^* V_i e^{-j\delta_i} + Y_{ii}^* V_i e^{-j\delta_i} e^{-j\Delta\delta_i}]$$

$$= (P_i + jQ_i) e^{j\Delta\delta_i} + V_i e^{j\delta_i} e^{j\Delta\delta_i} Y_{ii}^* V_i e^{-j\delta_i} (e^{-j\Delta\delta_i} - 1)$$

$$\Delta P_i + j\Delta Q_i = (P_i + jQ_i)(e^{j\Delta\delta_i} - 1) \; + V_i^2 Y_{ii}^* e^{j\Delta\delta_i} (e^{-j\Delta\delta_i} - 1)$$

For j^th bus:

$$P_j^{"} + jQ_j^{"} = V_j [\sum Y_{ji}^* V_k^* + Y_{ji}^* V_i e^{-j\delta_i} e^{-j\Delta\delta_i}] \qquad \text{(A2.9)}$$

$$= V_j [\sum Y_{ji}^* V_k^* + Y_{ji}^* V_i^* - Y_{ji}^* V_i e^{-j\delta_i} + Y_{ji}^* V_i e^{-j\delta_i} e^{-j\Delta\delta_i}]$$

$$= P_j + jQ_j + V_j Y_{ji}^* V_i e^{-j\delta_i} (e^{-j\Delta\delta_i} - 1)$$

$$\Delta P_j + \Delta Q_j = V_i^* V_j Y_{ji} (e^{-j\Delta\delta_i} - 1)$$