Chapter 5

Double Auction

5.1 Introduction

Suppose that there is a seller with a privately known cost \( C \in [c, \bar{c}] \) of producing a single indivisible good. Suppose also that there is a buyer with a privately known value \( V \in [\underline{v}, \bar{v}] \) of consuming the good. The cost \( C \) and \( V \) are independently distributed, and the prior distributions are commonly known and have full support on the respective intervals. Thus, there is incomplete information on both sides of the market. Finally, suppose that \( \underline{v} < \bar{c} \) and \( \bar{v} > \underline{c} \), so that the supports overlap and sometimes it is efficient not to trade. Is there some way to guarantee that the trade will take place whenever it should? To answer this question, it is natural to adopt a mechanism design perspective.

A mechanism decides whether the good is traded or not. It also decides the amount \( P \) the buyer pays for the good and the amount \( R \) the seller receives. If the good is traded, the net gain to the buyer is \( V - P \), and the net gain to the seller is \( R - C \). At the moment, we do not restrict \( P \) or \( R \) to be positive or negative, nor do we assume that the budget is balanced - that is \( P = R \).

A mechanism is efficient if whenever \( V > C \), the object is produced and allocated to the buyer. The next impossibility theorem is due to Myerson and Satterthwaite (1983, JET).

**Proposition 5.1.1.** In double auction problem, there is no mechanism that is efficient, incentive compatible, individually rational, and at the same time balanced budget.
5.2 The Large Double Auction

In this section we will revisit the double auction problem. First we will show the above impossibility theorem will hold even if we assume weakly balanced budget\(^1\) rather than balanced budget. Next we will design a mechanism that ensures efficiency, incentive compatibility, individual rationality and weakly balanced budget condition under “sufficiently” large competition. That is if the number of buyers are “sufficiently” large, then the mechanism ensures the four conditions stated above.

**Proposition 5.2.1.** In a double auction problem, there is no mechanism that is efficient, incentive compatible, individually rational, and at the same time weakly balanced budget.

**Proof:**

First consider a VCG mechanism, whose operation in this context is as follows:

The buyer announces a valuation \(v\) and the seller announces a cost \(c\).

1. If \(v \leq c\), then the object is not exchanged and no payments are made.
2. If \(v > c\), then the object is exchanged. The buyer who bids \(v\) pays \(\max\{c, v\}\) and the seller receives \(\min\{v, c\}\).

It is routine to verify that it is a weakly dominant strategy for the buyer to announce their true valuations and the seller to announce the true cost. This mechanism is efficient since in equilibrium, the object is transferred whenever \(v > c\).

A buyer with valuation \(y\) has an expected payoff of zero, and any buyer with valuation \(v > y\) has a positive expected payoff. In the same way a seller with a cost \(\bar{c}\) has an expected payoff zero, and any seller with a cost \(c < \bar{c}\) has a positive expected payoff. Thus the mechanism is individually rational.

Whenever \(v > c\), we have trade. The fact that \(y < \bar{c}\) implies that the amount the seller receives, i.e. \(R = \min\{v, \bar{c}\}\), is greater than the amount the buyer pays, i.e. \(P = \max\{c, y\}\). Then in the context the VCG mechanism always runs in deficit.

\(^1\)A mechanism satisfies weakly balanced budget condition if and only if the expected gain of the mechanism designer is non-negative in that mechanism.
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Now suppose we have some other mechanism that is incentive compatible and efficient. By the revenue equivalence theorem, there is a constant $K$ such that the expected payment for any buyer with value $v$ under this mechanism differs from his expected payment under the VCG mechanism by exactly $K$. Similarly there is a constant $L$ such that the expected receipts of any seller with the cost $c$ under this mechanism differ from her expected receipts under the VCG mechanism by exactly $L$.

Suppose the other mechanism is individually rational. Since in the VCG mechanism, a buyer with value $v$ gets an expected payoff of zero, we must have that $K \leq 0$. Similarly, since a seller with a cost $\bar{c}$ gets an expected payoff of zero, we have $L \geq 0$. The expected deficit under the other mechanism is just the expected deficit under the VCG mechanism plus $L - K \geq 0$. But since the VCG mechanism runs in deficit, every other mechanism also runs in deficit\(^2\). Thus, there does not exist an efficient mechanism that is incentive compatible, individually rational and simultaneously weakly balanced budget. Therefore, the impossibility result still holds even if we relax the balanced budget assumption.\(^\dagger\)

Now we formulate a mechanism that will ensure efficiency, incentive compatibility, individual rationality and weakly balanced budget conditions under sufficiently large number of buyers. This is a two stage mechanism where in the first stage each buyer bids for the object and pays his own bid. The mechanism designer selects the highest bidder. In the second stage, the highest bidder in the first stage and the seller bid. The buyer gets a particular amount from the mechanism designer depending on his/her bid. If in the second stage the bid of the buyer is more than that of the seller then only trade takes place, where the buyer pays what he/she bids to the seller and gets the object.

The idea behind such a mechanism is to compensate for the deficit of the mechanism designer by charging all the bidders in the first stage. Due to revenue equivalence theorem we know that whether in the first stage we charge the highest bidder only or all the bidder the expected revenue of the mechanism designer would be the same. Therefore to increase the expected revenue of the mechanism designer, we introduce a reserve price. And we conclude that even after introducing the reserve price if there are sufficiently large number of bidders, then the mechanism designer can fully collect the deficit from the buyers, and therefore ensures efficiency, incentive compatibility and individual rationality. Formally the mechanism is discussed below. For simplicity we assume that both the seller's and the buyers' valuations are distributed on the same interval, i.e. $\bar{v} = c$ and $\bar{v} = \bar{c}$.

\(^2\)See Auction Theory by Vijay Krishna (2002), page 77, proposition 5.5
5.2.1 Assumptions:

- There is a mechanism designer.
- The valuation of the mechanism designer is zero.
- Mechanism designer wants to allocate the object efficiently by maintaining incentive compatibility, individual rationality and “weakly balanced budget condition”.
- There is a single seller and \( N \) buyers, where \( N > 1 \).
- Seller has an object and wants to sell that object.
- The valuation of the seller is \( V_0 \). Valuation of the \( i^{th} \) buyer is \( V_i \) for all \( i = 1, \ldots, N \).
- All the buyers’ valuations and the seller’s valuation lie in the interval \([0, \omega]\) i.e. \( \forall j = 0, 1, \ldots, N \ V_j \in [0, \omega] \). The valuations are distributed with a distribution function \( F(.) \) which is increasing. \( F(.) \) admits a continuous density \( f(.) = F’(.) \) and has full support.
- Both the seller and the buyers are risk neutral i.e. they want to maximize their respective expected profits.
- Buyers are not subject to any liquidity constraint.
- All components of the model other than the valuations are assumed to be commonly known to all the bidders, the seller and the mechanism designer.

5.2.2 The Mechanism

Stage I:

- The mechanism designer sets a reserve price of amount \( R \), for the good, all the buyers have to submit a sealed bid more than or equal to that reserve price.
- The mechanism designer collects the bids from all the bidders (it is an all-pay auction with reserve price).
- The mechanism designer locates the highest bidder, and all the bidders except the highest bidder are out of the game.
Stage II:

- The highest bidder bids again for the object. Let he/she bid $b_{II}^I$ in this stage.
- The mechanism designer pays the amount $\int_0^{b_{II}^I} F(t)dt$ to the highest bidder.
- The mechanism designer asks the seller that whether the seller is interested in selling the object at a price $b_{II}^I$. If the seller says “yes” then the highest bidder pays the seller an amount $b_{II}^I$ and takes the object from the seller, otherwise no transaction takes place.

We will first derive the expected payoffs of the bidders, the objective of each bidder is to maximize his/her own payoffs. We are interested in deriving Perfect Bayesian Nash Equilibrium of the game. We will, then, derive the equilibrium bidding strategy for the seller, and finally we will derive the equilibrium bidding strategies of each bidder (Note that, because this is a two stage game where a bidder's strategy is a complete plan of actions in each stage, i.e. we have to specify a strategy for each of Stage I as well as Stage II).

Let the mechanism designer set a reserve price $R$. Let $i^{th}$ bidder be the highest bidder. Also let he/she bid $b_{II}^I$ in Stage I and $b_{II}^I$ in Stage II. The payoff functions of the bidder is

Payoff function in Stage II:

$$\Pi_i^{II} = \begin{cases} 
(V_i - b_{II}^I) + \int_0^{b_{II}^I} F(t)dt & \text{if seller says YES} \\
\int_0^{b_{II}^I} F(t)dt & \text{Otherwise}
\end{cases}$$

Payoff function in Stage I:

$$\Pi_i^I = \begin{cases} 
(V_i - b_{II}^I) + \int_0^{b_{II}^I} F, & \text{sys YES in Stage II} \\
\int_0^{b_{II}^I} F(t)dt - b_i & \text{sys NO in Stage II} \\
-b_i & \text{Otherwise}
\end{cases}$$

The expected payoffs of the bidder is given by.
Expected payoff in Stage II: 
\[(V_i - b^{II}_i)F(b^{II}_i) + \int_0^{b^{II}_i} F(t)dt\]

Expected payoff in Stage I: 
\[\left[(V_i - b^{II}_i)F(b^{II}_i) + \int_0^{b^{II}_i} F(t)dt\right] G(\beta^{-1}(b'_i)) - b'_i\]

Note that \(G(\beta^{-1}(b'_i)) = F(\beta^{-1}(b'_i))^{-1}\), i.e. the probability that \(b'_i\) is the highest bid at Stage I.

Let us assume, \(V_R\) is a valuation, such that no bidder with a valuation less than \(V_R\) will participate in this auction. We will derive \(V_R\) formally after we calculate the equilibrium bidding strategies of a buyer in both the stage.

We will derive three propositions below, with proposition I telling us the equilibrium bidding strategy of the seller in stage II, proposition II focusing on the equilibrium bidding strategy for the highest bidder of Stage I in Stage II, and finally the proposition III deriving the equilibrium bidding strategy for each buyer in Stage I.

**Proposition 5.2.2.** In the Stage II the equilibrium strategy of the seller is to accept the offer of the mechanism designer if \(V_0 \leq b^{II}_H\) and reject the offer otherwise.

**Proposition 5.2.3.** It is always optimal for the highest bidder of Stage I, to bid his/her true valuation in the Stage II.

Proof:

As we derived above the expected payoff of the highest bidder of Stage I, in Stage II is given by:

\[(V_i - b^{II}_H)F(b^{II}_H) + \int_0^{b^{II}_H} F(t)dt \tag{5.2.1}\]

The objective of the highest bidder is to maximize this expected payoff by choosing \(b^{II}_H\). Below we derive the first order condition for the maximization. Differentiating the equation 5.2.1 with respect to \(b^{II}_H\) and setting that is equal to zero we get.

\[
(V_i - b^{II}_H)f(b^{II}_H) - F(b^{II}_H) + F(b^{II}_H) = 0
\]
\[
(V_i - b^{II}_H)f(b^{II}_H) = 0
\]
\[
V_i = b^{II}_H
\]
The second order condition will be satisfied because,

\[ (V_i - b_{iH}^{IL}) f'(b_{iH}^{IL}) - f(b_{iH}^{IL}) \]
\[ = -f(b_{iH}^{IL}) \]
\[ < 0 \]

Therefore, the expected payoff of the highest bidder of the Stage I, is maximized at his/her own valuation at Stage II. ♦

Therefore, the expected payoff of the highest bidder of the Stage I, is \( \int_0^{V_i} F(t) dt \) at Stage II, where let \( i^{th} \) bidder be the highest bidder in Stage I. As we stated earlier that we define \( V_R \) in such a way that any bidder whose valuation is less than \( V_R \) will not enter into the auction and because the equilibrium profit function of a bidder is continuous, we must have

\[
\left[ \int_0^{V_R} F(t) dt \right] G(V_R) - R = 0 \quad (5.2.2)
\]

In other words we must have \( \beta(V_R) = R \) and \( V_R > 0 \). From the equation 5.2.2 one can calculate \( V_R \), in order to show that this \( V_R \) is unique, the only thing left to prove is that the equilibrium payoff function is increasing in valuations, which we will show after we derive the equilibrium bidding strategy of a buyer in Stage I.

In stage I, therefore, at equilibrium the expected payoff of the \( i^{th} \) bidder is

\[
\left[ \int_0^{b_i^{IL}} F(t) dt \right] G(\beta^{-1}(b_i^{IL})) - b_i^l
\]

The objective of the \( i^{th} \) bidder is, therefore, to maximize this expected payoff by choosing \( b_i^l \) such that \( b_i^l \geq R \). We already know that at the equilibrium if \( i^{th} \) bidder is the highest bidder then \( b_i^{II} = V_i \). Therefore at Stage I, the objective of the \( i^{th} \) bidder is to maximize \[ \left[ \int_0^{V_i} F(t) dt \right] G(\beta^{-1}(b_i^l)) - b_i^l \] such that \( b_i^l \geq R \) holds.

Maximizing this with respect to \( b_i^l \) yields the first order condition:
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\[
\left[ \int_0^{V_i} F(t) dt \right] \frac{g(\beta^{-1}(b_i^l))}{\beta'(\beta^{-1}(b_i^l))} - 1 = 0
\]

where \( g \equiv G' \).

At a symmetric equilibrium, \( b_i^l = \beta(V_i) \) and as we assume (for the time being) that \( \beta \) is a strictly increasing function, we have \( \beta^{-1}(b_i^l) = V_i \), and thus the above equation yields the following differential equation

\[
\left[ \int_0^{V_i} F(t) dt \right] g(V_i) = \beta'(V_i)
\]

Or equivalently,

\[
\left[ \int_0^{V_i} F(t) dt \right] g(V_i) = \frac{d}{dV_i} \beta(V_i)
\]

and since \( \beta(V_R) = R \), we have

\[
\beta(V_i) = R + \int_{V_i}^{V_R} \int_0^y F(t) g(y) dt dy
\]

The derivation of \( \beta \) is only heuristic because the above equation is merely a necessary condition - we have not formally established that if the other \( N - 1 \) bidders follow \( \beta \), then it is indeed optimal for a bidder with value \( V_i \) to bid \( \beta(V_i) \). Next we will show that this is indeed correct.

Let us assume that all but bidder \( i \) follow the strategy \( \beta \) given by the equation above. We will argue that in that case it is optimal for bidder \( i \) to follow \( \beta \) also. First, notice that \( \beta \) is an increasing and continuous function, in particular \( \left[ \int_0^{V_i} F(t) dt \right] g(V_i) = \beta'(V_i) \), note that \( g(V_i) > 0 \) and \( \int_0^{V_i} F(t) dt > 0 \) for all \( V_i > 0 \), therefore \( \beta''(V_i) > 0 \). Thus, in equilibrium the bidder with the highest valuation submits the highest bid and wins the auction. It is not optimal for the bidder \( i \) to bid a \( b_i^l > \beta(\omega) \). The expected payoff of bidder \( i \) with valuation \( V_i \) if he/she bids an amount \( b_i^l \leq \beta(\omega) \) is calculated as follows. Denote by \( z = \beta^{-1}(b_i^l) \) the valuation for which \( b_i^l \) is the equilibrium bid - that is, \( \beta(z) = b_i^l \). Then we can write bidder \( i \)'s expected payoff from bidding \( \beta(z) \), when his/her valuation is \( V_i \), as follows
\[ \Pi(b_i^l, V_i) = G(z) \left[ \int_0^{V_i} F(t) dt \right] - \beta(z) \]
\[ = G(z) \left[ \int_0^{V_i} F(t) dt \right] - \int_z^{V_i} \left[ \int_0^y F(t) dt \right] g(y) dy - R \]
\[ = G(z) \left[ \int_0^{V_i} F(t) dt \right] - G(z) \left[ \int_0^z F(t) dt \right] + G(V_R) \left[ \int_0^{V_R} F(t) dt \right] + \int_z^{V_R} F(y) G(y) dy - R \]
\[ = G(z) \left[ \int_z^{V_i} F(t) dt \right] + G(V_R) \left[ \int_0^{V_R} F(t) dt \right] + \int_z^{V_R} F(y) G(y) dy - R \]
\[ = G(z) \left[ \int_z^{V_i} F(t) dt \right] + \int_{V_R}^{V_i} F(t) dt \]

where the third equality is obtained as a result of integration by parts and the last equality is obtained by noting equation 5.2.2. We thus obtained that

\[ \Pi(\beta(V_i), V_i) = \int_{V_i}^{V_R} F(y) G(y) dy \]

and therefore,

\[ \Pi(\beta(V_i), V_i) - \Pi(b_i^l, V_i) = \int_z^{V_i} F(y) G(y) dy - G(z) \left[ \int_z^{V_i} F(t) dt \right] \]

Note that to prove our claim we have to show that \( \Pi(\beta(V_i), V_i) - \Pi(b_i^l, V_i) \geq 0 \) regardless of whether \( z \geq V_i \) or \( z \leq V_i \). To show this we will minimize the expression \( \Pi(\beta(V_i), V_i) - \Pi(b_i^l, V_i) \) with respect to \( z \) and we will show that this expression will be minimized at \( z = V_i \) and the minimum value of this expression is zero.

Consider the first order condition of the minimization problem

\[ -F(z) G(z) + F(z) G(z) - g(z) \int_z^{V_i} F(t) dt = 0 \]
\[ \Rightarrow -g(z) \int_z^{V_i} F(t) dt = 0 \]
\[ \Rightarrow z = V_i \]
The second order condition for the minimization can be checked routinely because

\[-g'(z) \int_2^{V_i} F(t) dt + F(z)g(z) \mid z = V_i = F(V_i)g(V_i) > 0\]

We have thus argued that if all the other bidders are following the strategy \( \beta \), a bidder with a value \( V_i \geq V_R \) cannot benefit by bidding anything other than \( \beta(V_i) \); and this implies that \( \beta \) is a symmetric increasing equilibrium bidding strategy.

Now we will show that the equilibrium payoff function is increasing in valuations. The equilibrium payoff function of a buyer whose valuation is \( V_i \), is given by

\[\Pi(\beta(V_i), V_i) = \int_{V_R}^{V_i} F(y)G(y) dy\]

Note that,

\[\Pi'(V_i) = F(V_i)G(V_i) > 0 \text{ as } V_i \geq V_R > 0\]

![Figure 5.1: The equilibrium payoff function](image)
Therefore, we have shown that $V_R$ is unique (see Figure 5.1).

Therefore, if the number of bidders is sufficiently large then the mechanism designer can fully recover the loss he incurs in Stage II. Formally, the expected gain of the mechanism designer is given by the expression below

$$N \int_{V_R}^{\infty} \beta(z) f(z) dz$$

$$= N \int_{V_R}^{\infty} \left[ R + \int_{V_R}^{\infty} F(t) g(y) dt dy \right] f(z) dz$$

$$= N \int_{V_R}^{\infty} R f(y) dy + \int_{V_R}^{\infty} \int_{V_R}^{\infty} F(t) g(y) dt dy f(z) dz$$

$$= N \int_{V_R}^{\infty} \left[ R f(y) dy + \int_{V_R}^{\infty} \int_{V_R}^{\infty} F(t) g(y) dt dy f(z) dz \right]$$

The expected loss of the mechanism designer is given by the expression below

$$\int_{V_R}^{\infty} \int_{0}^{t} F(y) dy f(t) dt$$

The incentive compatibility of the mechanism designer will be satisfied if the condition below holds.

$$N \left[ R \left[ 1 - F(V_R) \right] + \int_{V_R}^{\infty} \int_{V_R}^{\infty} F(t) g(y) dt dy f(z) dz \right] \geq \int_{V_R}^{\infty} \int_{0}^{t} F(y) dy f(t) dt$$

Therefore, for sufficiently large number of bidders, the mechanism designer can fully compensate for her loss in Stage II by the gain she makes in Stage I. Therefore, the expected loss of the mechanism designer will always be compensated for by the expected gain whenever the inequality below will be satisfied.

$$N \geq \frac{\int_{V_R}^{\infty} \int_{0}^{t} F(y) dy f(t) dt}{\left[ R \left[ 1 - F(V_R) \right] + \int_{V_R}^{\infty} \int_{V_R}^{\infty} F(t) g(y) dt dy f(z) dz \right]}$$

(5.2.3)
Below we are going to give an example, where we assume that the valuations are distributed uniformly over the interval \([0, \omega]\).

Example: Let us now assume that all the buyers' valuations and the seller's valuation is uniformly distributed over the interval \([0, \omega]\). We for the time being assume that there are \(N\) buyers where \(N \geq 2\). We will derive the required number of buyer's so that a mechanism designer can run our mechanism. Let us also assume that the reserve price is \(R\) where \(R > 0\). First we will derive the equilibrium bidding strategy of a buyer in Stage I when her valuation is \(V_i\).

\[
\int_0^{V_R} F(t) \, dt G(V_R) = R
\]

\[
\implies \int_0^{V_R} \frac{t}{\omega} \, dt \left[ \frac{V_R}{\omega} \right]^{N-1} = R
\]

\[
\frac{V_R^{N+1}}{2\omega^N} = R
\]

\[
V_R = \sqrt[2]{2R\omega^N}
\]

The above expression is showing the relationship between reservation price and minimum valuation of a bidder who can participate in this auction. Below we derive the expression for an equilibrium bidding strategy.

\[
\beta(V_i) = R + \int_{V_R}^{V_i} \int_0^y F(t) g(y) \, dt \, dy
\]

\[
= R + \int_{V_R}^{V_i} \int_0^y \frac{t}{\omega} \left( \frac{y}{\omega} \right)^{N-2} \, dt \, dy
\]

\[
= R + \frac{(N-1)}{\omega^N} \int_{V_R}^{V_i} y \left( \frac{y}{\omega} \right)^{N-2} \, dy
\]

\[
= R + \frac{(N-1)}{\omega^N} \frac{1}{2} \int_{V_R}^{V_i} y^N \, dy
\]

\[
= R + \frac{(N-1) (V_i - V_R)^{N+1}}{2(N+1)\omega^N}
\]

Note that in this case, \(f(t) = \frac{1}{\omega^2}\), \(F(t) = \frac{t}{\omega}\), \(G(t) = \left[ \frac{t}{\omega} \right]^{N-1}\), \(g(t) = (N-1) \left[ \frac{t}{\omega} \right]^{N-2}\) and \(V_i \geq V_R\). The equilibrium bidding strategy of the buyer in Stage I is given by the equation above.
Let us now calculate \( R \left[ 1 - F(V_R) \right] + \int_{V_R}^{\omega} \int_{V_R}^{y} F(t)g(y)dt dyf(z)dz \), which is the expected gain of the mechanism designer from this auction.

\[
R \left[ 1 - F(V_R) \right] + \int_{V_R}^{\omega} \int_{V_R}^{y} F(t)g(y)dt dyf(z)dz = R \left[ 1 - F(V_R) \right] + \int_{V_R}^{\omega} \frac{(N - 1)(z - V_R)^{N+1}}{2(N + 1)\omega^N} \frac{1}{\omega} dz \\
= R \left[ 1 - F(V_R) \right] + \frac{(N - 1)}{2(N + 1)\omega^{N+1}} \int_{V_R}^{\omega} (z - V_R)^{N+1} dz \\
= R \left[ 1 - \frac{V_R}{\omega} \right] + \frac{(N - 1)(\omega - V_R)^{N+2}}{2(N + 1)(N + 2)\omega^{N+1}}
\]

The expected loss of the mechanism designer is given by the equation below-

\[
\int_{V_R}^{\omega} \int_{0}^{t} F(y)d(y)dt = \int_{V_R}^{\omega} \frac{y}{\omega} \frac{1}{\omega} dt \\
= \frac{1}{2\omega^2} \int_{V_R}^{\omega} t^2 dt \\
= \frac{\omega^3 - V_R^3}{6\omega^2}
\]

Therefore, the required number of buyer \( N \) must satisfy the following condition-

\[
N \geq \frac{\omega^3 - V_R^3}{6\omega^2} \frac{R \left[ 1 - \frac{V_R}{\omega} \right] + \frac{(N - 1)(\omega - V_R)^{N+2}}{2(N + 1)(N + 2)\omega^N + 1}}{\omega^2 + \omega V_R + V_R^2} \\
= \frac{\omega^2 + \omega V_R + V_R^2}{R + \frac{(N - 1)(\omega - V_R)^{N+1}}{2(N + 1)(N + 2)\omega^N}} \frac{\omega^2 + \omega V_R + V_R^2}{6R\omega + \frac{3(N - 1)(\omega - V_R)^{N+1}}{(N + 1)(N + 2)\omega^N - 1}}
\]

Note that if the above condition holds then the mechanism will satisfy all the four conditions stated above. For example let \( \omega = 1 \) and \( R = 0.1 \) so for any \( N \geq 4 \) the above condition is satisfied.
5.3 Conclusion

In this short chapter we first show that even if we consider a weakly balanced budget condition as against a balanced budget condition, and if there is a single buyer and a single seller, no mechanism can guarantee efficiency, incentive compatibility, individual rationality and weakly balanced budget. Finally we design a mechanism which sufficiently intense competition (i.e. if the number of buyers is sufficiently large) ensures efficiency along with incentive compatibility, individual rationality and weakly balanced budget.