Chapter 1

Introduction

Auctions have been used since ancient times for selling numerous kinds of objects. According to Herodotus, auctions were used in Babylon even as early as 500 B.C. In contemporary times, both the range and the values of objects offered for sale through auctions have grown to astonishing proportions.

Presently various kinds of commodities ranging from art objects, antiques, tobacco, fish and fresh flowers to scrap metal and gold bullion are put up for sales through auctions. Bond issues by public utilities generally are auctioned off to investment banking syndicates. To finance the borrowing needs of the government, weekly auctions conducted by the U.S. Treasury are used to sell long-term securities. The use of auctions has very important contributions in facilitating the transfer of assets from public to private hands throughout the world. These include the sale of industrial enterprises in Eastern Europe and the former Soviet Union as well as the transportation systems in Britain and Scandinavia. Conventionally, auctions have been used for selling the rights to use natural resources from public property - such as timber rights and off-shore oil leases. In today's world, auctions of rights to use the electromagnetic spectrum for communication are also a worldwide phenomenon.

There has been a fabulous growth in the number of Internet auction websites, where individuals can put up items for sale under common auction rules. The volume of goods sold in such auctions has grown exponentially. The process of procurement via competitive bidding is no different from an auction except that in this case the bidders compete for the right to sell their products or services. Billions of dollars of government purchases are almost exclusively made in this way, and the practice is widespread, if not endemic, in business.

This PhD thesis focuses on two specific types of auctions viz. 'Contests' and 'Standard Auctions'.
CHAPTER 1. INTRODUCTION

Contest is a widely used phenomenon for allocating prize(s). Prizes can be anything, viz. different objects, money, services etc. In any contest there are two types of agents - the contest designer, who sets the rules of the contest and the contestants who participate and compete for the prize(s).

The sequence of actions taken by these two types of agents of a contest is the following: first the contest designer sets the rules for the contest, which become common knowledge to all the agents. Then all the contestants decide to put in some effort for winning the contest. Finally, they perform in the contest and the winner(s) are decided by the rules of the contest. Note that the cost that any contestant bears due to the efforts undertaken by her is non-refundable, i.e. even if a contestant fails to win the contest, she will not get back the cost she has incurred due to effort. Therefore, contests are similar to all pay auctions where all the bidders pay before the winner is announced and don’t get back the payment they have made. On the other hand, contests and all pay auctions also share some dis-similarities. For example, in an all pay auction the auctioneer sets the payment rule of the auction, who doesn’t know the valuations (or the type) of any bidder. Therefore, the payment of any bidder in an all pay auction solely depends on the bid and not the valuation (type) of that bidder. In case of a contest, the abilities (or the types) of all the contestants are set by nature, and the cost functions are given to the contestants. Therefore, the payment that a contestant incurs (in terms of cost of efforts), may depend on her ability (type) as well.

Many commonly known auctions fall in the category of standard auction, for example first price sealed bid auction, second price sealed bid auction etc. The most important characteristic of a standard auction is that, here the highest bidder always wins the auction in the cases where the buyers are bidding for single objects. In case of procurement auctions, where the sellers are bidding, the rule of standard auction dictates that the seller with the lowest bid should win the auction. Regardless of its type, an auction consists of three parties, viz. the buyers, the sellers and the mechanism designer. The mechanism designer sets the rules of the auction. In most of the cases, either the buyers or the sellers play the role of the mechanism designer. For example in case of government procurement most of the times the government itself sets the rules of the procurement auction. In auctions any party (buyers and/or sellers) can bid depending on the rules of auction. In first price sealed bid auction only buyers can bid, in standard procurement auction only sellers can bid and finally in double auctions both the buyers and the sellers can bid.

This thesis is organized in two parts. The first part consists of three chapters on the theory of contest, which analyze contests from the perspectives of the contest designer as well as the contestants. The second part contains two chapters on standard auction and double auction. We now summarize our main findings.
CHAPTER 1. INTRODUCTION

In chapter 2, we analyze the optimal-prize structure in multi-prize contests where risk-neutral players have private information about their abilities. We analyze such contests with the assumption of non-linear cost and non-linear performance functions. Here the objective of the contest designer is to maximize the sum of expected performances of all the contestants. We also study the multi-prize contests where in the performance function there is a fixed component as well as a variable component (which depends on the effort undertaken by the contestant).

Previously Moldovanu and Sela (2001) analyzed such a contest with non-linear cost functions only. They assume that performance is equal to the effort put in by a contestant and show that it may be optimal to offer multiple prizes if the cost function is sufficiently convex. The results they derived, show that all contestants, except the contestant with the lowest possible ability, will put in a strictly positive effort irrespective of the amount of prize money given.

In this chapter, the results shown are different from their results in two major respects. First, we assume that performance \( (P) \) is a function of effort and ability (they are not just equal), so that performance can now take any functional form viz. concave, convex or linear, and we show that the equilibrium performance of any contestant not only depends on the cost function, but also on the performance function. Therefore, whether it is optimal for the contest designer to offer multiple prizes, doesn’t depend on cost \( (C) \) (which is also a function of effort and ability) alone. At equilibrium, effort is a function of ability and therefore, performance and cost are functions of ability. We have shown that whether it is optimal for the contest designer to offer multiple prizes or not depends on the function \( \Omega \) where, \( \Omega = C(P^{-1}) \). In particular, we have shown that if \( \Omega \) is sufficiently convex then only it is optimal for the contest designer to offer multiple prizes. This shows that even if the cost function is linear, if the performance function is sufficiently concave then it may be optimal for the contest designer to offer multiple prizes. This result is also an extension of a result obtained by Moldovanu and Sela which states that with a linear cost function it is optimal to offer single prizes.

In the same chapter, we analyze a case where performance function is additively separable in effort and ability, i.e. it has a fixed component. We show that in this case the number of contestants putting in positive efforts may vary depending on the absolute amount of money given as prizes. That is, for any amount of prize money given, there exists a \( \tilde{C} \), such that a contestant with ability less than or equal to \( \tilde{C} \) will put in zero effort at equilibrium and all the contestants whose ability is greater than \( \tilde{C} \) will put in positive efforts. This means if the absolute amount of prize money is large then \( \tilde{C} \) is small and if the prize money is small then \( \tilde{C} \) is large. This is quite obvious because a large prize money works as an incentive for a contestant to perform better and therefore a contestant will try to put in more effort at equilibrium and these efforts result in better overall
performances. We will illustrate an example where a very low prize money can lead to a very high \( C \) such that in equilibrium no contestant will put in positive efforts. If the contest designer wants positive efforts from the participating contestants then she should give a prize money greater than a particular threshold value.

Finally we have shown that at equilibrium the cost function may become a strictly increasing function of ability, i.e. more able contestant will put more effort in equilibrium. This may seem to be a counter-intuitive result because a participant whose ability is very high knows that the probability of winning the contest for her is also very high, therefore she can afford to invest less effort. One possible explanation for this paradox could be that if there is very high competition then her probability of winning becomes relatively less, so she puts in more effort at equilibrium. However, this explanation may not be true; we have shown in an example that even if there is very small competition (where the number of contestants is just three), the result stated above still holds true. Finally we have studied the effect of entry fees on contests.

Note that in chapter 2 we focus on a single contest. That is, we do not allow the contest designer to design multiple contests. In chapter 3 we analyze multiple contests. Multiple contests can assume mainly two forms, viz. simultaneous contests and sequential contests. In simultaneous contests, two or more contests will run simultaneously. Here two possibilities arise, viz. either contest designer distribute the set of contestants in different contests or the contestants choose which contest to participate. The first situation is termed as multiple contest with known number of contestants and the second situation is termed as multiple contest with unknown number of participants. We have analyzed both these cases in detail.

First, we analyze simultaneous contests and compare simultaneous contests with a single contest. A contest designer can either organize a single contest or decide to split the number of contestants in two parts and organize two simultaneous contests. Here we show that if the ability of the contestants are distributed uniformly then it is always better to design a single unified contest rather than designing two simultaneous contests (this result can easily be extended for \( K \) simultaneous contest where \( K \geq 2 \)). Next we have analyzed a two stage sequential contest. Here also we compare a single stage contest with a two stage sequential contest. In a two stage sequential contest, all the contestants will compete for the prize offered in the first stage, then all the contestants, except the winner of stage one, will compete for the prize offered in the second stage. We assume that the value of prize offered in the first stage is greater than or equal to the value of the prize offered in the second stage. The result we find suggests that it is better for the contest designer to design a single stage contest than a two stage sequential contest if the abilities of the contestants are distributed uniformly. Combining these two results we propose that if the abilities of the...
contestants are distributed uniformly then a unified single stage contest yields a higher aggregate expected performance than a two stage sequential contest and two simultaneous contests. This whole analysis is done by assuming linear cost and performance functions.

Finally in this chapter we study multiple simultaneous contests with unknown number of bidders. We have derived a condition by which a contestant will decide in which contest she will participate.

In chapter 4 we study single contests with interdependent valuations. Valuations are interdependent when the valuation of any contestant is not only dependent on the signal about the object she gets but also on the valuations of all the other contestants. The previous two chapters consider money as prizes. But here we consider goods as prizes and we assume pure common value, where the valuation of any good is the same for all the contestants. This also ensures that the ranking of the prizes are the same for all the contestants (which is necessary for our model). We begin our analysis by assuming linear cost and performance functions and also that the mechanism designer can offer at most two prizes, for simplicity. But later we relax both the assumptions and provide a more generalized framework where cost function may be concave or convex and the contest designer can offer any number of prizes.

The model begins by computing the symmetric increasing equilibrium bidding strategy for a contestant. We compare two cases; in the first case there are two prizes, viz. a first prize ($P_1$) and a second prize ($P_2$) and a contestant (let’s say contestant $i$) who’s performance is second highest wins $P_2$. Let the valuation of $P_2$ for contestant $i$ be $v_2$. In the second case there is a single prize, viz. $P$, which is a bundle of goods consisting of $P_1$ and $P_2$. If the contestant $i$ wins the contest he gets $P$, then let’s assume that the valuation of $P_2$ for her is $\bar{v}_2$. Our analysis shows that if $v_2 > \bar{v}_2$ then it may better to offer multiple prizes rather than a single first prize. This result is a generalized version of the previous result we have shown in the previous two chapters. Note that in case of independent valuation $v_2 = \bar{v}_2$, so offering a single prize is optimal (which we have shown in chapter 2).

Then we have derived the equilibrium bidding strategy of a contestant where the cost function can be concave or convex in nature. Finally we derive the equilibrium bidding strategy for a contestant where the contest has more than two prizes.

---

1. Here we assume goods because goods are generally indivisible in nature unlike money, which is crucial for our analysis.

2. For example, suppose in a contest there are two prizes. The first prize is a Television set ($P_1$) and the second prize is one year free cable connection ($P_2$). Also assume that there is another contest, which has only a single first prize. Here the first prize is a Television set and one year free cable connection ($P$). The valuation of one year free cable connection for her may be different when she gets a Television set with one year free cable connection than when she only gets one year free cable connection (without the Television set).
The second part of the thesis deals with double auction and standard auction. In any standard auction the rule of the auction dictates that the highest bidder wins the auction (in case of procurement auction the lowest bidder wins). Many commonly known auctions fall in the category of standard auction viz. first prize sealed bid auction, second price sealed bid auction, all pay auction, Dutch auction, English auction etc. In a standard auction either the buyers or the sellers submit bids. Double auction is different. In a double auction, generally there is only one seller and a single buyer. Both the seller and the buyer bid simultaneously. If the bid of the seller is higher than that of the buyer then there is no trade. But if the bid of the buyer is higher than that of the seller then trade takes place at a price which is a linear combination of the bids of the buyer and the seller, so that both the agents gain from trade. In such types of double auction there is no mechanism that can ensure incentive compatibility, individual rationality, efficiency and balanced budget condition together\(^3\).

In the first chapter of this part, we extend the double auction by allowing the participation of more than one buyers. We conclude that if there is a sufficiently large number of buyers then there exists a two stage mechanism which is incentive compatible, individually rational, efficient and at the same time weakly balances the budget. Finally, we provide an example of such a mechanism.

In the last chapter, we find out the necessary and sufficient conditions for the existence of an increasing, symmetric perfect Bayesian Nash equilibrium. We have found out that if the bidding function, derived from the first order condition of the expected utility (profit) maximization problem, is strictly increasing throughout the domain of the valuations then that bidding strategy constitutes a symmetric increasing Bayesian Nash equilibrium of that auction. We also illustrate our finding with some examples.

\(^3\)See Myerson, Roger B. and Satterthwaite, M, 1983, JET