Chapter 7

Trade in Education and Educational Quality: A Theoretical Analysis

Section 7.1: Introduction

Theoretical queries on the subject of international trade have in general been concerned about the directions, welfare and distributional implications of international flows of goods and services under different cost and market conditions. Cost advantages have frequently been identified as a reason for countries to export goods and services. Countries importing those goods and services are expected to benefit particularly through a rising consumer surplus due to lower import prices. Availability of a newer variety, or a different quality of a product in a foreign country has also been pointed as reason for international flows of goods and services. Theorisations in international trade have assumed, possibly without any exception, that both domestic and foreign producers are profit earners.

The case of international trade in education services is distinguished from the case of trade in other goods and services in primarily two ways. Firstly, education providers around the world are not necessarily profit earners. Educational institutes frequently operate on subsidy or on donated fund and provide education at a below average-cost price. Yet, trade of education services is not likely to occur without the motive of earning profit; therefore below-cost pricing is not likely in case of imported education services. Hence the question of lowering prices through import does not seem a likely scenario. Secondly, availability of a different variety or quality of education in a foreign country may nevertheless motivate trade in education services. But the perception of quality of education also differs from that of most other goods and services. Unlike most other goods and services education is a service whose quality depends, at
least partially, on the quality of its consumers. Therefore, availability of a superior quality of education in a foreign country does not ensure that importing education from such a country will imply that the foreign supplier will provide a superior quality of education in the importing country as well. These distinctions of the education service make it interesting to explore the implications of trade liberalisation in education services.

Let us now turn to the concerns over trade liberalisation in education; and such concerns for importing countries are many. Trade in education services, especially through Mode 3 (Commercial Presence) instigates debates more than what supply of traded education services through any other Mode (Consumption Abroad, Cross Border Supply or through Movement of Natural Persons) does. The variety of conditions applied on market access under Mode 3 by various WTO Members (discussed in Chapter 5) reveal the variety in the nature of these concerns. Under Mode 3 education is exported by universities / educational institutes\(^1\) by investing directly in setting up Branch Campuses or Affiliate Universities in foreign countries. The question of quality of education expected from a Branch Campus or an Affiliate University appears to be a significant source of concern for importing countries, especially the developing countries. On one hand, there are expectations that affiliates of reputed universities will provide a superior quality of education than that available in most developing countries. On the other hand, there are also apprehensions that cost minimising 'exporters' might offer a poor quality of education. Another concern related to import of education service, especially via Mode 3, is about whether such provision of education will substitute the existing suppliers of education in importing countries or compliment them. In either event alterations are possible in the access to education to different classes of students. In view of the fact that primary, secondary and tertiary education in many Member countries of the WTO are publicly provided with subsidised prices and that imported education is not expected to be subsidised, access to such education will be limited to higher income groups.

The objective of this chapter is to develop a theoretical model to analyse the impacts of trade liberalisation in education, with particular reference to Mode 3 of supply of traded education services. The specific issues that we attempt to explore are (1) quality of education a Branch Campus or an Affiliate University is likely to offer to its prospective students; and (2) alterations in the access to education as result of trade in education. We choose to analyse the above-mentioned issues for an importing country with a subsidised system of education where, in the autarkic situation, access to education is restricted to relatively higher ability students irrespective of their economic background. The model shows that in such a circumstance quality of education provided by the Affiliate University depends on the quality of education

\(^1\) 'University' and 'educational institute' are interchangeably used here to mean 'supplier of education service'
that the domestic university offers. Alterations in the access to education in the importing country under this liberal trade regime are determined by the distribution of wealth.

The arguments over implications of trade liberalisation (through Mode 3) on the quality of education resemble the arguments over implications of privatisation on the quality of education. As we have discussed in the previous chapter, the literature on education acknowledges that quality of education is a tricky issue and there are several dimensions to it. The quality of any product depends on how well the product serves the purpose for which it is produced. Consumers, prospective employers and governments may have different perceptions about the purpose of education and hence about its quality. Theoretical literature in economics on the issue of privatisation of education commonly has considered quality of education, provided by a school or institute, either as the average quality of students attending the school or as the quality of teaching provided by the school. Educational resources (size of library, teacher-student ratio etc.) has also been identified as another aspect of quality of education in the literature in education.

In this model our attempt has been to incorporate both quality of fellow students and educational facilities provided by the institute as determinants of the students perception of quality of education. We assume that students' perception of quality of education provided by any institute positively depends on the cut-off set by the university on marks / grade obtained by the students in the previous level (indicating its peer quality) and average educational expenditure incurred by the institute. Given the set up of the model, another distinction can be observed between the model and the existing literature on the issue. The volume of production or the size of schools, as found in the literature, is fixed by the fixed 'establishment' costs incurred by the school, as is usual in many manufacturing sectors. Our attempt in this regard has been to show that the size of a school can be limited even when the establishment cost is variable, or is an endogenous choice of the institute. That is to say that when quality of the product (education) as perceived by the consumers (students) depends on the quality of other consumers (peer quality) and all consumers (students) are not of identical qualities, the volume of production can be finite in order to maintain a certain level of product (educational) quality even if all costs are variable.

As opposed to the students' perception of quality, the governments or the accreditation agencies assess quality of education provided by any particular institute as measured by the educational facilities or resources it offers. This distinction between the students' (consumers involved in the production and consumption of the service) and the

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3 Adam Smith and J. S. Mill – see discussion in Section 6.3.a of the Chapter 6.
accreditation authority's (a third party not involved in the production or consumption of the service but responsible for assessing its quality) perception about educational quality is important with respect to the model. Our aim here is to find out the conditions, under which, the Affiliate University offers a superior educational quality, than the domestic university, measured in terms of the amount of educational resources used per student. We find that the Affiliate University offers a quality of education (measured by the amount of educational resources incurred per student) that is relatively superior than the domestic university of the importing country if and only if (1) it can charge a price beyond some threshold value for a superior educational quality and (2) the domestic university maintains a cut-off merit index that is neither too high or too low. In all other situations the Affiliate University does not improve upon the domestic university in terms of quality (measured by the amount of educational resources incurred per student).

Section 7.2: The Model

7.2.a: Definitions and Assumptions of the Model:

Two countries, Home and Foreign are considering trade in higher education. Each country has a domestic university. Let us call the domestic universities of the Home and the Foreign country Home University (HU) and Foreign University (FU) respectively. FU considers exporting education to Home country by setting up an Affiliate University in the Home country. HU, unlike FU, does not consider exporting its services. Whether or not FU is a profit earning university, the Affiliate University (AU) is set up in order to earn profit. In this model, our interests are (1) quality of education and educational facilities that the affiliate university is expected to provide in the Home country and (2) the alterations in access to education that might occur as a result of the setting up of the affiliate university.

A 1: The population of students, willing to pursue and qualified for higher education, in the Home Country (country that hosts the Affiliate University), is divided into two categories – the rich and the poor, according to the amount they can afford to spend on higher education. Proportion of the rich in the population is $\beta$ and that of the poor is $(1 - \beta)$.

A 2: All individuals belonging to rich group can afford the price $a_r$ and all those belonging to poor group can afford the price $a_p$ for higher education, where, $a_r > a_p > 0$.

A 3: Each student can be represented by a merit index. The merit index of any student, irrespective of what price she can afford, can take any value in $[0, 1]$. Merit indices of poor
students, $s_p$, is uniformly distributed in $[0, 1]$ with density $(1-\beta)$. [See left panel of figure 1.]

Merit indices of rich students, $s_r$, is uniformly distributed in $[0, 1]$ with density $\beta$. [See right panel of figure 1.]

**Figure 1**

A 4: Quality of education, as perceived by the students, in a university campus, depends positively on the quality of the students selected by the university and the expenditure on educational facilities, per student, incurred by the campus. The quality of students in a university is determined only after students have enrolled. Thus, prospective applicants cannot exactly judge the quality of the student body of a university, ex ante, before making a choice as to which university to join. The cut-off merit index (marks / grades indexed in $[0, 1]$) accepted by a university works as an indicator of its ex post quality of the student body. Prospective applicants thus deduce the quality of educational service of a university $i$ as

$$Q_i = s_i E_i$$  \hspace{1cm} (1)

Where, $s_i$ is the cut-off merit index, $s_i \in [0, 1]$ and $E_i$, the expenditure on educational facilities, per student, incurred by the university $i$.

This particular functional form indicates that peer-group (decided by the cut-off merit index) and educational facilities (decided by expenditure incurred by the university) are perceived as imperfect substitutes by the students.

A 5: Expenditure on educational facilities, per student, can take two discrete values – $E_L$ (or low expenditure) and $E_H$ (or high expenditure). If any university has to improve facility from $E_L$, it has to spend $E_H$ and no less, on each student. The relative value of $E_L$, with respect to $E_H$ is given by,

$$E^* = E_L / E_H$$  \hspace{1cm} (2)

Since $E_L < E_H$, $0 < E^* < 1$. 
A 6: We have specified the population of students, in Assumption 1, as those willing to pursue higher education; that is, each student has a positive gain from educational investment. Each of these students prefers receiving education to not receiving it. For investing in education, students have a fixed amount of money given by what they can afford. A student is willing to invest the entire investable fund (that is the amount she can afford) on the institute of her choice. When a student has to choose between two universities, she chooses between them in the same way an investor would choose between two investment opportunities; she chooses the one with the higher expected return. Students prefer the university that offers them higher quality, since a superior quality of education is expected to generate a higher return on educational investment.

If the merit index of a student is such that she is eligible to get admission into both universities and she can afford to pay the price charged by either university, then she chooses to enroll in the university, the quality of education in which she perceives to be superior, (that is, the one with a higher $Q_i$ as per equation (1)). In case the quality of education is perceived to be identical in both universities, such a student chooses to enroll in the university that charges a lesser price. In case the quality provided as well as the prices charged by each university are identical, such a student can choose to enroll in either of the two universities.

In case either the merit index of the student is such that she is eligible to get admission into only one of the universities and she is able to pay the price charged by that university, she has no choice but to enroll in that very university. In all other cases, where the student is not meritorious to get admission in any of the university or cannot pay the price charged by either of the universities, she has no choice but to discontinue education.

A 7: The Home University (HU) is a subsidised university and charges a uniform price that is affordable for everyone, i.e., HU charges a fee of $a_p$. We assume that $a_r > E_H > E_L > a_p$. (3) We also assume that HU provides an expenditure on educational facilities of $E_L$ per student. Given that HU aims to make higher education available to all students irrespective of economic background, it spends only $E_L$ per student. Since the poorer students cannot afford even a relatively lower expenditure of $E_L$ per student (as $E_L > a_p$) it requires a subsidy. In view of the fact that we are concerned about whether or not the Affiliate University (AU) offers improved educational facilities $E_H$, we assume that educational facility, provided by HU is $E_L$ and AU is free to provide a better facility if it wants to.

Though HU, by charging $a_p$, a price affordable to everyone, does not discriminate between students on the basis of their economic class, yet, it admits a student if and only if her merit index is equal to or greater than a cut-off of $y \in [0,1]$. The volume of subsidy received by HU is enough to admit students, seeking higher education, with quality in $[y, 1]$. 

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A8: HU is non-strategic. Any action of AU or the students do not alter any of HU's offers (fees charged, cut off merit index y or expenditure per student Eₐ).

A9: The Affiliate University is a profit maximiser. Profit of AU is given by,

$$\pi_A = (P_A - E_A) n_A$$  \hspace{1cm} (4)

where, $P_A$ is the fee charged by AU from each student.

$E_A$ is the educational expenditure, per student, offered by AU and $E_A \in \{E_L, E_H\}$,

$n_A$ is the enrolment in AU.

Any university, profit maximising or otherwise, can be concerned about the quality of education it offers. The profit maximising university would be concerned about the quality of education it offers since the quality of a university relative to that of the other university determines the size of its pool of applicants and hence the size of enrolment and consequently its profit. Students' perception of quality of education in an institute, as stated in A4, depends positively on quality of enrolled students as well as on the educational expenditure per student.

For this reason, a profit maximising educational institute might not admit all applicants, that is, all those who are willing to buy its service. In this respect, the profit maximising educational institute, acts unlike the profit maximising firm of many other goods and services sectors. In case of products, such as education services, where the quality of consumers determine the quality of the product, volume of production and sales can be limited by the choice of quality that the supplier wants to offer, even if there is no fixed cost involved. In case of most other products, where quality of the product is independent of the quality of the consumers, usually it is the fixed costs that limit the volume of production. The Affiliate University AU, a profit maximiser thus selects students by setting a cut-off student merit $x \in [0, 1]$ and admitting all applicants in $[x, 1]$.

7.2. b The Game:

We assumed as per (A7) that HU is a subsidised university that charges a fee affordable for everyone, i.e., HU charges a fee of $a_p$, sets the cut-off merit index at $y$ and it incurs an educational expenditure of $E_L$ per student. We also assumed (A8) that HU is non-strategic, i.e., irrespective of the action taken by AU and that of the potential students, HU provides ($a_p$, $y$, $E_L$).

The players in this game are (1) AU and (2) the students who can afford education at AU and are good enough to get admission to both HU and AU, i.e., all students who can afford $P_A$ and are indexed in $[\max \{x, y\}, 1]$ according to their merit.

The strategies of the players are as follows.
In **stage 1** of the game, AU chooses the vector \((P_A, x, E_A)\), such that price \(P_A \in \mathbb{R}^+\), cut off \(x \in [0, 1]\) and educational expenditure per student \(E_A \in \{E_L, E_H\}\), i.e., AU specifies its fee, cut-off merit of the student and expenditure on educational facilities that would be incurred per student. Given the offer made by HU (non-strategic and exogenous to the game here) and the action chosen by AU, the students can determine whether they are in a position to choose or not. It is important to note that all students are not in a position to choose. Only those students who are indexed in \(\max \{x, y\}, 1\) according to their merit and can afford the price \(P_A\) charged by AU are in a position to choose between universities.

In **stage 2** these students choose between enrolling in AU and HU.

**Proposition 1**: (a) *The Affiliate University will not provide education to the economically poor students and (b) will charge a price of \(a_r\).*

**Proof of part (a)**

The profit function of AU is \(\pi_A = (P_A - E_A) n_A\) \hspace{1cm} (4)

This implies that the AU will make positive profit only if \(P_A > E_A\).

Since, \(E_A \in \{E_L, E_H\}\), and \(E_L < E_H\), the minimum \(E_A\) is \(E_L\).

Given that the AU does not get any subsidy it cannot make a positive profit if \(P_A \leq E_L\).

On the other hand, if \(P_A > a_r\), no student in the Home country can afford to study in AU.

Hence, in equation (4) \(n_A\) will be 0 and in turn, \(\pi_A\) will also be zero if \(P_A > a_r\).

Therefore, AU does not earn a positive profit if \(P_A \leq E_L\) or \(P_A > a_r\).

So, AU has to charge \(P_A > E_L\) and \(P_A \leq a_r\) in order to earn a positive profit.

The students belonging to poor backgrounds can afford to study in AU only if \(P_A \leq a_p\).

By our assumption (equation 3, in A7), \(E_L > a_p\).

Since \(P_A > E_L\) and \(E_L > a_p\), therefore, \(P_A > a_p\).

So, the poor students cannot afford to study in AU and are not in a position to choose between AU education and HU education. The poor students with merit \(s_p \in [y, 1]\) will get admission in HU and those with merit \(s_p \in [0, y]\) will discontinue education.

**Proof of part (b):**

Given that AU sets a cut-off merit index at \(x\), first, we identify the set of students who can actually make a choice between universities. Then, given AU's offer of \(E_A\), we proceed to find the enrolment and profit levels of AU. Finally, we find the price choice of AU that maximises its profit.
We have shown above that AU will choose $P_A \in (E_L, a_i)$. So, only the rich students can afford to study in AU. But that does not mean that all rich students can get admission in AU. The rich students with merit $s_r \in [x, 1]$ will be offered admission in AU.

If $x > y$, the rich students with merit $s_r \in [x, 1]$ are in a position to choose between AU and HU, those with merit $s_r \in [y, x)$ will get admission only in HU and those rich students with merit $s_r \in [0, y)$ will discontinue education. (See Figure 2)

Figure 2

If $x = y$, the rich students with merit $s_r \in [x, 1]$ are in a position to choose between AU and HU, and those with merit $s_r \in [0, x)$ will discontinue education. (See Figure 3)

Figure 3

If $x < y$, the rich students with merit $s_r \in [y, 1]$ are in a position to choose between AU and HU, those with merit $s_r \in [x, y)$ will get admission only in AU and those with merit $s_r \in [0, x)$ will discontinue education. (See Figure 4)
Now that the students who can choose between universities are identified, let us find the possible enrolment volumes and profit levels of AU.

Given this stage 1 offer of cut-off merit index $x$, in stage 2, the rich students who with merit $s_r \in [\max \{x, y\}, 1]$ decide whether to join AU or HU, and that determines the enrolment and the profit levels of AU. Note that all the students who can choose between AU and HU are equally informed and decide in an identical manner. All students, who can choose, apply to the university, which provides a quality of education that is perceived superior and if the qualities of educational service offered by the two universities are same they apply to the university with a lower price (as per assumption A6). We know that AU chooses $P_A \in (E_L, a]$. HU on the other hand charges a fee $a_p$. By assumption A7, $E_L > a_p$. So, AU charges a fee that is higher than that charged by HU and hence, if $Q_H \geq Q_A$, i.e., $yE_L \geq xE_A$, the students who can choose, get admitted to HU. But if $Q_H < Q_A$, i.e., $yE_L < xE_A$, they get admitted to AU.

Case (i): $Q_A \leq Q_H$ and $x \leq y$

Suppose $x$ and $E_A$ are such that $xE_A \leq yE_L$, i.e., $Q_A \leq Q_H$ and $x \leq y$. In this case, all the students who can choose between AU and HU, i.e., rich students with merit $s_r \in [y, 1]$, will join HU. All rich students with merit $s_r \in [x, y)$, will join AU. We assumed (as per A1) that proportion of rich in the population is $\beta$. So the enrolment in AU is

$$n_A = \beta (y - x)$$

and hence the profit of AU is given as:

$$\pi_A = (P_A - E_A) \beta (y - x) \quad (5)$$

Case (ii): $Q_A \leq Q_H$ and $x > y$

Here, $Q_A \leq Q_H$. That is, $xE_A \leq yE_L$. If $x > y$ then $E_A < E_L$, which is impossible (by A5). Hence this particular case can be ruled out.

Case (iii): $Q_A > Q_H$ and $x \leq y$
Suppose $x$ and $E_A$ are such that $x \in E_A > y \in E_A$ (i.e., $Q_A > Q_H$) and $x \leq y$. So all the students who can choose between AU and HU, i.e., rich students with merit $s_r \in [y, 1]$, will join AU. The rich students with merit $s_r \in [x, y)$ do not have any choice but to join AU. So the enrolment in AU is $n_A = \beta (1 - x)$ and hence the profit of AU is given as: $\pi_A = (P_A - E_A) \beta (1 - x)$ (6)

Case (iv): $Q_A > Q_H$ and $x > y$

Suppose $x$ and $E_A$ are such that $x \in E_A > y \in E_A$, i.e., $(Q_A > Q_H)$ and $x > y$. So all the students who can choose between AU and HU, i.e., rich students with merit $s_r \in [x, 1]$, will join AU. The rich students with merit $s_r \in [y, x)$ do not have any choice but to join HU. So the enrolment in AU is $n_A = \beta (1 - x)$ and hence the profit of AU is given as: $\pi_A = (P_A - E_A) \beta (1 - x)$ (7)

The profit of AU, in all the above-mentioned possible cases, is either 0 or is given by either (5) or (6). Equations (6) and (7) are identical. Clearly, both in (5) and (6), $\pi_A$ is increasing in $P_A$. We already know that the AU provides education only to the rich students and $P_A \in (E_L, a)$. Clearly, $\pi_A$ is maximised at $P_A = a_r$. So, $P_A = a_r$ (weakly) dominates all $P_A \in (E_L, a)$. Proposition I shows that AU will charge a fee $a_r$, irrespective of its choice regarding $x$ and $E_A$.

Now, let us find out which cut-off merit index and educational expenditure will AU choose to offer. For this, we first have to find the non-dominated strategies of AU.

Proposition II  

*Given that the Home University's cut-off merit index is $y$, the following three strategies of the Affiliate University are non-dominated - $(E_A, x_1), (E_H, x_2)$ and $(E_A, x_3)$, where $x_1 = 0$, $x_2 = E^*y$ and $x_3 = y$.*

Proof: In order to figure out the non-dominated strategies of AU, we proceed systematically as follows. Given HU's cut-off merit index $y$, we shall consider AU's choices of cut-off $x \in [0, y)$, $x = y$ and $x \in (y, 1]$. For each set of cut-off, we shall consider AU's offer of educational expenditure $E_H$ and $E_L$.

Case (i): Suppose AU chooses some cut-off merit index $x \in (y, 1]$.

Prospective students will determine AU's quality of education depending on the educational expenditure AU chooses to make.
Sub-case (ia): Suppose that AU offers educational expenditure $E_L$ along with cut-off $x \in (y, 1]$.

In this case $Q_A = x E_L$. Since $Q_H = y E_L$ and $x > y$, therefore $Q_A > Q_H$ thus all rich students with merit $s_r \in [x, 1]$, will join AU. The rich students with merit $s_r \in [y, x)$ do not have any choice but to join HU. The poor students with merit $s_A \in [y, 1]$ will join HU. The remaining students are forced to discontinue education. (See Figure 5)

![Figure 5](image)

So the enrolment in AU is $n_A = \beta (1 - x)$, where $x \in (y, 1]$

and hence the profit of AU is given as: $\pi_A = (a_r - E_L) \beta (1 - x)$  \hspace{1cm} (8)

Sub-case (ib): Suppose that AU offers educational expenditure $E_H$ along with cut-off $x \in (y, 1]$.

Since $Q_A = x E_H$, $Q_H = y E_L$, $E_H > E_L$ and $x > y$, therefore $Q_A > Q_H$ and all rich students with merit $s_r \in [x, 1]$, will join AU. The rich students with merit $s_r \in [y, x)$ and the poor students with merit $s_A \in [y, 1]$ will join HU. The remaining students are forced to discontinue education. (See Figure 5)

So the enrolment in AU is $n_A = \beta (1 - x)$, where $x \in (y, 1]$

and hence the profit of AU is given as: $\pi_A = (a_r - E_H) \beta (1 - x)$  \hspace{1cm} (9)

Comparing AU’s pay offs from strategies $(E_L, x \in (y, 1])$ [equation (8)] and $(E_H, x \in (y, 1])$ [equation (9)], we find,

$\pi_A (E_L, x \in (y, 1]) = (a_r - E_L) \beta (1 - x) > \pi_A (E_H, x \in (y, 1]) = (a_r - E_H) \beta (1 - x)$

[Since $E_H > E_L$]

Thus, AU’s strategy $(E_L, x \in (y, 1])$ strictly dominates strategy $(E_H, x \in (y, 1])$. 

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That is to say that if AU chooses a higher cut-off merit index than HU then it does not choose to provide a superior level of educational facilities than HU provides.

**Case (ii): Suppose AU chooses some cut-off merit index \( x = y \).**

As in Case (i), prospective students will determine AU's quality of education depending on the educational expenditure AU chooses to make.

**Sub-case (iia): Suppose that AU offers educational expenditure \( E_L \) along with cut-off \( x = y \).**

Since \( Q_A = y E_L = Q_H \) all rich students with merit \( s_r \in [x, 1] \), will join HU because HU provides the same quality at a lower price \( (a_p) \). The poor students with merit \( s_p \in [y, 1] \) will join HU. The remaining students are forced to discontinue education (See Figure 6).

So the enrolment in AU is \( n_A = 0 \) and hence the profit of AU is also 0.

**Figure 6**

\[
\begin{align*}
&\text{Discontinue} & & \text{HU} \\
&\text{Discontinue} & & \text{HU} \\
0 & & y & & 1 \\
1-\beta & & y & & 1 \\
\end{align*}
\]

**Sub-case (iib): Suppose that AU offers educational expenditure \( E_H \) along with cut-off \( x = y \).**

Since \( Q_A = y E_H, Q_H = y E_L, \) and \( E_H > E_L \), therefore \( Q_A > Q_H \) and all rich students with merit \( s_r \in [x, 1], \) will join AU. The poor students with merit \( s_p \in [y, 1] \) join HU. The remaining students are forced to discontinue education. (See Figure 7)

**Figure 7**

\[
\begin{align*}
&\text{Discontinue} & & \text{HU} \\
&\text{Discontinue} & & \text{HU} \\
0 & & y & & 1 \\
1-\beta & & y & & 1 \\
\end{align*}
\]
So the enrolment in AU is 
\[ n_A = \beta (1 - x), \text{ where } x = y \]
and hence the profit of AU is 
\[ \pi_A = (a_r - E_H) \beta (1 - x) \] (10)

Comparing AU’s pay offs from strategy (EL, \( x = y \)) (which is 0) and (EH, \( x = y \)) [equation (10)], we find 
\[ \pi_A (E_L, x = y) = 0 < \pi_A (E_H, x = y) = (a_r - E_H) \beta (1 - x) \]

Thus, AU’s strategy (EH, \( x = y \)) strictly dominates strategy (EL, \( x = y \)).

That is, in case AU chooses the same cut-off merit index as HU, it chooses a superior level of educational facilities than HU.

**Case (iii): Suppose AU chooses some cut-off merit index \( x < y \).**

As in Cases (i) and (ii), prospective students will determine AU’s quality of education depending on the educational expenditure AU chooses to make.

**Sub-case (iii): Suppose that AU offers educational expenditure E_L along with cut-off \( x \in [0, y] \).**

Since \( Q_A = x \cdot E_L \) and \( Q_H = y \cdot E_L \) and \( x < y \), therefore \( Q_A < Q_H \) and all rich students with merit \( s_r \in [y, 1] \), will join HU. The rich students with merit \( s_r \in [x, y) \) do not have any choice but to join AU. The poor students with merit \( s_p \in [y, 1] \) join HU. The remaining students are forced to discontinue education (See Figure 8).

So the enrolment in AU is 
\[ n_A = \beta (y - x), \text{ where } x \in [0, y) \]
and hence the profit of AU is given as: 
\[ \pi_A = (a_r - E_L) \beta (y - x) \] (11)

![Figure 8](image-url)
Sub-case (iiiib): Suppose that AU offers educational expenditure $E_H$ along with cut-off $x \in [0, y)$.

Since $Q_A = x E_H$ and $Q_H = y E_L$, $E_H > E_L$ and $x < y$, we cannot conclude whether the quality of educational service in AU is perceived to be superior to HU or not.

$Q_A > Q_H$ if and only if $x E_H > y E_L$, i.e., iff $x > E^*y$, and

$Q_A \leq Q_H$ if and only if $x E_H \leq y E_L$, i.e., iff $x \leq E^*y$, where $E^* = E_L / E_H$.

If $x \in (E^*y, y)$, then all rich students with merit $s_r \in [x, 1)$, will join AU. The poor students with merit $s_p \in [y, 1]$ will join HU. The remaining students are forced to discontinue education (See Figure 9).

**Figure 9**

![Figure 9](image)

So, If $x \in (E^*y, y)$, the enrolment in AU is $n_A = \beta (1 - x)$ and hence the profit of AU is given by $\pi_A = (s_r - E_H) \beta (1 - x)$ (12).

**Figure 10**

![Figure 10](image)
If \( x \in [0, E^*y] \), then all rich students with merit \( s_r \in [y, 1] \) will join HU. The rich students with merit \( s_r \in [x, y) \) do not have any choice but to join AU. The poor students with merit \( s_p \in [y, 1] \) will join HU. The remaining students are forced to discontinue education (See Figure 10).

So, if \( x \in [0, E^*y] \), the enrolment in AU is

\[ n_A = \beta (y - x) \]

and hence the profit of AU is given as:

\[ \pi_A = (a_r - E_H) \beta (y - x) \] (13)

Comparing AU's pay off from strategies \((E_L, x \in [0, E^*y])\) [equation 11] and \((E_H, x \in [0, E^*y])\) [equation (13)], we find,

\[ \pi_A (E_L, x \in [0, E^*y]) = (a_r - E_L) \beta (y - x) > \pi_A (E_H, x \in [0, E^*y]) = (a_r - E_H) \beta (y - x) \]

[Since \( E_L < E_H \)]

Thus, AU's strategy \((E_L, x \in [0, E^*y])\) strictly dominates strategy \((E_H, x \in [0, E^*y])\).

That is, if AU chooses a cut-off merit index as low as \( E^*y \) or below, then it also chooses the lower level of educational expenditure. With such low cut-off on merit, it is not possible to provide a better educational quality than HU even with superior educational facilities.

Comparing AU's pay offs from strategies \((E_H, x \in (E^*y, y))\) [equation 12] and \((E_H, x = y)\) [equation (10)], we find,

\[ \pi_A (E_H, x \in (E^*y, y)) = (a_r - E_H) \beta (1 - x) > \pi_A (E_H, x = y) = (a_r - E_H) \beta (1 - y) \]

Thus, AU's strategy \((E_H, x \in (E^*y, y))\) strictly dominates strategy \((E_H, x = y)\). That is, in case AU provides a superior level of educational facilities, it chooses a cut-off merit index below that of HU but above \( E^*y \). Such a cut-off along with \( E_H \) makes the educational quality of AU superior than what is offered by HU.

From the above comparisons of payoffs of the AU we found four sets of strictly dominated strategies for all values of \( y \). These strategies are (1) \((E_H, x \in (y, 1])\), (2) \((E_H, x \in [0, E^*y])\), (3) \((E_H, x = y)\) and (4) \((E_L, x = y)\), for all \( y \in (0, 1) \). The reason for each strategy to be dominated is different.

(1) The strategy \((E_H, x \in (y, 1])\) is dominated by strategy \((E_L, x \in (y, 1])\). This is because if AU is offering any cut-off merit index \( x \in (y, 1] \), its quality of education is perceived to be superior than that of HU, due to choice of a higher cut-off than HU, though it offers the same level of expenditure on educational facilities \( E_L \) as HU. Therefore, the rich students who can afford to pay the price charged by AU will prefer AU to HU and choose to join AU. AU in that case finds no reason to raise its average expenditure on educational facilities to \( E_H \).
The strategy \((E_H, x \in [0, E^*y])\) is dominated by \((E_L, x \in [0, E^*y])\). This is because if AU is offering a cut-off merit index as low as \(x \in [0, E^*y]\), its quality of education is perceived to be inferior to HU, whether AU offers an average expenditure on educational faculties of \(E_H\) or \(E_L\). Therefore, only the rich students with merit \(s_i \in [x, y]\), (those rich students who are rejected from HU) will join AU irrespective of whether it offers \(E_H\) or \(E_L\). In this case also AU find no reason to raise its average expenditure on educational facilities to \(E_H\), as those joining AU have no other choice but to join AU.

The strategy \((E_L, x = y)\) is dominated by strategy \((E_H, x = y)\). This is because if AU offers a cut-off merit index of \(x = y\) along with a low average educational expenditure, its quality of education is perceived to be identical as HU’s. But AU, not being subsidised, cannot charge a price as low as HU. So with strategy \((E_L, x = y)\) AU gets no student and earns no profit. On the other hand if AU offers a cut-off merit index of \(x = y\) along with a high average educational expenditure, its quality of education is perceived to be superior to HU’s. Thus, with strategy \((E_H, x = y)\) rather than \((E_L, x = y)\) the rich students with merit index equal to or greater than \(y\) join AU and AU earns positive profit.

The strategy \((E_H, x = y)\) is dominated by the strategy \((E_H, x \in (E^*y, y))\). This is because if AU offers \(E_H\), AU’s quality of education is perceived to be superior to that of HU, whether AU offers cut-off \(x = y\) or \(x \in (E^*y, y)\). Therefore, choosing \(x = y\) as the cut-off merit index rather than any \(x \in (E^*y, y)\) reduces the size of enrolment and thereby the amount of profit.

The above comparison reveals three non-dominated strategies of AU, namely, 
\[(E_L, x \in [0, y)), \quad (E_H, x \in (E^*y, y)) \quad \text{and} \quad (E_L, x \in (y, 1)).\]

The profit function for each of these strategies is as follows.
\[
\begin{align*}
\pi_A(E_L, x \in [0, y)) &= (a_r - E_L) (y - x) \beta & \text{as per equation (11)} \\
\pi_A(E_H, x \in (E^*y, y)) &= (a_r - E_H) (1 - x) \beta & \text{as per equation (12)} \\
\pi_A(E_L, x \in (y, 1)) &= (a_r - E_L) (1 - x) \beta & \text{as per equation (8)}
\end{align*}
\]

The profit functions given in equations (11), (12) and (8) are decreasing in \(x\).

Thus, in case AU considers to offer \(E_L\) and \(x\) in the interval \([0, y]\), [equation (11)] it would choose the smallest value of \(x\), \(x = 0\). In case AU considers to offer \(E_H\) and \(x\) in the interval \((E^*y, y)\), [equation (12)] it would choose \(x = E^*y + \epsilon\), where \(\epsilon\) is a small positive number tending to 0. But if \(\epsilon\) is actually equal to 0 then \(Q_A = E_H.E^*y = E_Ly = Q_H\), since \(E^* = E_L / E_H\). In that case, AU, which charges a higher fee than HU, gets no students and its profit is 0;
Therefore, in that case, it is not possible to choose some \( x \) from the interval \((E^*y, y)\) that maximises AU's profit when AU is offering \( E_H \). A maximum of profit does not exist in this case; rather what exists is a supremum of profits at \( x = E^*y + \varepsilon \). Since \( \varepsilon \) is a very small number very close to zero, we can write \( x = E^*y + \varepsilon \) as \( x = E^*y \). Similarly, in case AU considers to offer \( E_L \) and \( x \) in the interval \((y, 1)\) [equation (8)], it would choose \( x = y + \varepsilon \). Due to the same reason as above, a supremum of profit exists at \( x = y + \varepsilon \) and since \( \varepsilon \) is a very small number very close to zero, we can write \( x = y + \varepsilon \) as \( x = y \).

Therefore, given HU's cut-off merit index \( y \), the non-dominated strategies of AU are \((E_L, x_1)\), \((E_H, x_2)\) and \((E_L, x_3)\), where, \( x_1 = 0, x_2 = E^*y \) and \( x_3 = y \).

The profits corresponding to these three non-dominated strategies of the AU, namely, \((E_L, x = y)\) \((E_H, x = E^*y)\) and \((E_L, x = 0)\) are as follows.

\[
\pi^*_A (E_L, x = y) = (a_r - E_L)(1 - y)\beta
\]

\[
\pi^*_A (E_H, x = E^*y) = (a_r - E_H)(1 - E^*y)\beta
\]

\[
\pi^*_A (E_L, x = 0) = (a_r - E_L)(y)\beta
\]

In order to calculate the optimum strategy we proceed systematically as follows.

**Lemma 1:**

\[
\pi^*_A (E_L, y) > \pi^*_A (E_L, 0) \text{ if and only if } y < \frac{1}{2}.
\]

\[
\pi^*_A (E_L, y) = \pi^*_A (E_L, 0) \text{ if and only if } y = \frac{1}{2}.
\]

\[
\pi^*_A (E_L, y) < \pi^*_A (E_L, 0) \text{ if and only if } y > \frac{1}{2}.
\]

For proof see Appendix.

**Lemma 2:**

\[
\pi^*_A (E_L, 0) > \pi^*_A (E_H, E^*y) \text{ if and only if } y > \frac{(A - 1)}{A - 2E^* + AE^*},
\]

\[
\pi^*_A (E_L, 0) = \pi^*_A (E_H, E^*y) \text{ if and only if } y = \frac{(A - 1)}{A - 2E^* + AE^*},
\]

\[
\pi^*_A (E_L, 0) < \pi^*_A (E_H, E^*y) \text{ if and only if } y < \frac{(A - 1)}{A - 2E^* + AE^*},
\]

where \( A = \frac{a_r}{E_H} \).

For proof see Appendix.

**Lemma 3:**

\[
\pi^*_A (E_L, y) > \pi^*_A (E_H, E^*y) \text{ if and only if } y < \frac{1}{A},
\]

\[
\pi^*_A (E_L, y) = \pi^*_A (E_H, E^*y) \text{ if and only if } y = \frac{1}{A},
\]

\[
\pi^*_A (E_L, y) < \pi^*_A (E_H, E^*y) \text{ if and only if } y > \frac{1}{A},
\]

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where \( \Lambda = (a_r / E_H) \).

For proof see Appendix.

**Remark 1:** Comparing \( \pi^*(E_L, y) \), \( \pi^*(E_L, 0) \) and \( \pi^*(E_H, E^*y) \) we found three critical values of \( y \). Firstly, lemma 1 shows that \( \pi^*(E_L, y) > \pi^*(E_L, 0) \) or \( \pi^*(E_L, 0) \) if and only if \( y < \) or \( = 1 / 2 \). Let us denote \( 1 / 2 \) as \( y_1 \). Secondly, lemma 2 shows that \( \pi^*(E_L, 0) > \pi^*(E_H, E^*y) \) if and only if \( y > \) or \( \pi^*(E_H, E^*y) \) if and only if \( y > \) or \( = \) or \( < \frac{(A-1)}{A-2E^*+AE^*} \) where \( A = (a_r / E_H) \). Let us denote \( \frac{(A-1)}{A-2E^*+AE^*} \) as \( y_2 \). Finally, lemma 3 shows that \( \pi^*(E_L, y) > \pi^*(E_H, E^*y) \) if and only if \( y < \) or \( = \) or \( > \frac{(A-1)}{A-2E^*+AE^*} \) and \( A = a_r / E_H \). Let us denote \( \frac{(A-1)}{A-2E^*+AE^*} \) as \( y_3 \).

**Lemma 4:**

\[
y_2 > y_1 > y_3 \quad \text{if and only if} \quad A > 2.
\]
\[
y_2 < y_1 < y_3 \quad \text{if and only if} \quad A < 2 \quad \text{and}
\]
\[
y_2 = y_1 = y_3 \quad \text{if and only if} \quad A = 2, \text{where} \ A = a_r / E_H
\]

For proof see Appendix.

**Proposition III(a):** Given \( a_r > 2E_H \),

If \( y \in [0, \frac{1}{A}] \), AU's optimal strategy is \( (E_L, y) \);

If \( y = \frac{1}{A} \), AU's optimal strategy is either \( (E_L, y) \) or \( (E_H, E^*y) \);

If \( y \in (\frac{1}{A}, \frac{(A-1)}{A-2E^*+AE^*}) \), AU's optimal strategy is \( (E_H, E^*y) \);

If \( y = \frac{(A-1)}{A-2E^*+AE^*} \), AU's optimal strategy is either \( (E_H, E^*y) \) or \( (E_L, 0) \);

If \( y \in (\frac{(A-1)}{A-2E^*+AE^*}, 1] \), AU's optimal strategy is \( (E_L, 0) \).

**Proof:** If \( a_r > 2E_H \), i.e., \( A > 2 \), from lemma 4, \( y_2 > y_1 > y_3 \), i.e., \( \frac{(A-1)}{A-2E^*+AE^*} > \frac{1}{2} > \frac{1}{A} \).

When \( y \in [0, \frac{1}{A}] \), it follows from lemma 1 - 3 that,

\[
\pi^*(E_L, y) > \pi^*(E_L, 0),
\]
\[
\pi^*(E_L, 0) < \pi^*(E_H, E^*y) \quad \text{and}
\]
\[
\pi^*(E_L, y) > \pi^*(E_H, E^*y).
\]

i.e., \( \pi^*(E_L, y) > \pi^*(E_H, E^*y) > \pi^*(E_L, 0) \).

Therefore, when \( a_r > 2E_H \) and \( y \in [0, \frac{1}{A}] \), AU's optimal strategy is \( (E_L, y) \). (See Figure 11)
When $y = \frac{1}{A}$, it follows from lemmas 1 - 3 that,

\[
\pi^* (E_L, y) > \pi^* (E_L, 0),
\]
\[
\pi^* (E_L, 0) < \pi^* (E_H, E^* y) \text{ and }
\]
\[
\pi^* (E_L, y) = \pi^* (E_H, E^* y).
\]

i.e., $\pi^* (E_L, y) = \pi^* (E_H, E^* y) > \pi^* (E_L, 0)$.

Therefore, when $a_r > 2E_H$ and $y = \frac{1}{A}$, AU's optimal strategy is either $(E_L, y)$ or $(E_H, E^* y)$.

(See Figure 11)

\[
y_1 = \frac{1}{2} \\
y_2 = \frac{(A - 1)}{A - 2E^* + AE^*}
\]

### Figure 11

When $y \in (\frac{1}{A - 2}, \frac{1}{2})$, it follows from lemma 1 - 3 that,

\[
\pi^* (E_L, y) \geq \pi^* (E_L, 0),
\]
\[
\pi^* (E_L, 0) < \pi^* (E_H, E^* y) \text{ and }
\]
\[
\pi^* (E_L, y) < \pi^* (E_H, E^* y).
\]

i.e., $\pi^* (E_H, E^* y) > \pi^* (E_L, y) \geq \pi^* (E_L, 0)$.

Therefore, when $a_r > 2E_H$ and $y \in (\frac{1}{A - 2}, \frac{1}{2})$, AU's optimal strategy is $(E_H, E^* y)$. (See Figure 11)

When $y \in (\frac{1}{A - 2} \frac{(A - 1)}{2}, A - 2E^* + AE^*)$, it follows from lemmas 1 - 3 that,

\[
\pi^* (E_L, y) < \pi^* (E_L, 0),
\]
\[
\pi^* (E_L, 0) < \pi^* (E_H, E^* y) \text{ and }
\]
\[
\pi^* (E_L, y) < \pi^* (E_H, E^* y).
\]

i.e., $\pi^* (E_H, E^* y) > \pi^* (E_L, 0) > \pi^* (E_L, y)$. 

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Therefore, when \( a_r > 2E_H \) and \( y \in \left( \frac{1}{2}, \frac{(A-1)}{A-2E^*+AE^*} \right) \), AU’s optimal strategy is \((E_H, E^*y)\).

(See Figure 11)

When \( y = \frac{(A-1)}{A-2E^*+AE^*} \), it follows from lemmas 1–3 that,

\[
\pi^*_A (E_L, y) < \pi^*_A (E_L, 0),
\]

\[
\pi^*_A (E_L, 0) = \pi^*_A (E_H, E^*y) \quad \text{and}
\]

\[
\pi^*_A (E_L, y) < \pi^*_A (E_H, E^*y).
\]

i.e., \( \pi^*_A (E_H, E^*y) = \pi^*_A (E_L, 0) > \pi^*_A (E_L, y) \)

Therefore, when \( a_r > 2E_H \) and \( y = \frac{(A-1)}{A-2E^*+AE^*} \), AU’s strategy is either \((E_H, E^*y)\) or \((E_L, 0)\).

(See Figure 11)

When \( y \in \left( \frac{(A-1)}{A-2E^*+AE^*}, 1 \right] \), it follows from lemmas 1–3 that,

\[
\pi^*_A (E_L, y) < \pi^*_A (E_L, 0),
\]

\[
\pi^*_A (E_L, 0) > \pi^*_A (E_H, E^*y) \quad \text{and}
\]

\[
\pi^*_A (E_L, y) < \pi^*_A (E_H, E^*y).
\]

i.e., \( \pi^*_A (E_L, 0) > \pi^*_A (E_H, E^*y) > \pi^*_A (E_L, y) \).

Therefore, when \( a_r > 2E_H \) and \( y \in \left( \frac{(A-1)}{A-2E^*+AE^*}, 1 \right] \), AU’s optimal strategy is \((a_r, E_L, 0)\).

(See, Figure 11)

**Remark 2:** When \( A > 2 \) (that is, \( a_r > 2E_H \)) the price that the rich students can afford is relatively high. If the cut-off set by HU is relatively low, i.e., \( y \in [0, 1/A] \), AU chooses \((E_L, y)\).

Since the cut-off set by HU is low, AU can afford to set the cut-off above that of HU and still get the mass of students that generates greater profit than the profit obtainable by choosing \((E_L, 0)\) or \((E_H, E^*y)\). By setting the cut-off just above that of HU, AU increases its perceived quality of educational service only by manipulating the quality of the student body without providing a larger educational expenditure, per student.

When \( A > 2 \) (that is, \( a_r > 2E_H \)) and the cut-off set by HU is relatively high, i.e., \( y \in ((A-1)/(A-2E^*+AE^*), 1] \), AU chooses \((E_L, 0)\). For this range of \( y \), the mass of rich students who cannot get into HU is large. AU thus generates more profit than any other strategy by catering to this residual mass of rich students.
Given $A > 2$ (that is, $a_r > 2E_H$) when, $y \in ([1/A], [(A-1)/(A-2E^*+AE^*)])$, $A_U$ chooses $(E_H, E^*y)$. Since the affordability of the rich students is very high, $A_U$ will be able to raise profit by offering $E_H$ and at the same time attracting all rich students with quality above $E^*y$. Note that $A_U$'s cut-off in this case is $E^*y$, which is lower than its cut off $y$ when it offers $E_L$ and gets the rich students who could also join $H_U$. In this case $A_U$ can set its cut-off below that of $H_U$ and still attract relatively better quality rich students by providing a higher educational expenditure $E_H$, per student. All the rich students having quality $s_r \in [E^*y, 1]$ gets enrolled in $A_U$.

When $y = \frac{1}{A}$, $A_U$ is indifferent between $(E_L, y)$ and $(E_H, E^*y)$. When $y = \frac{A-1}{A-2E^*+AE^*}$, $A_U$ is indifferent between $(E_H, E^*y)$ and $(E_L, 0)$.

**Proposition III (b):** Given $a_r < 2E_H$

If $y \in [0, \frac{1}{2})$, $A_U$'s optimal strategy is $(E_L, y)$.

If $y = \frac{1}{2}$, $A_U$'s optimal strategy is either $(E_L, y)$ or $(E_L, 0)$.

If $y \in (\frac{1}{2}, 1]$, $A_U$'s optimal strategy is $(E_L, 0)$.

**Proof:** If $a_r < 2E_H$, i.e., $A < 2$, it follows from lemmas 4 that $y_1 < y_3$, $y_2 < y_3$ and $y_2 < y_1$. So, $y_2 < y_1 < y_3$, i.e., $\frac{1}{2} > \frac{A-1}{A-2E^*+AE^*}$.

When $y \in [0, \frac{A-1}{A-2E^*+AE^*}]$, it follows from lemma 1 - 3 that,

\[\pi^*_A (E_L, y) > \pi^*_A (E_L, 0),\]

\[\pi^*_A (E_L, 0) \leq \pi^*_A (E_H, E^*y)\]

\[\pi^*_A (E_L, y) > \pi^*_A (E_H, E^*y),\]

i.e., $\pi^*_A (E_L, y) > \pi^*_A (E_H, E^*y) \geq \pi^*_A (E_L, 0)$.

Therefore, when $A < 2$ (that is, $a_r < 2E_H$) and $y \in [0, \frac{A-1}{A-2E^*+AE^*}]$, $A_U$'s optimal strategy is $(E_L, y)$. (See Figure 12)
When \( y \in \left( \frac{1}{2}, \frac{1}{A} \right) \), it follows from lemmas 1 – 3 that,

\[
\pi^*_A (E_L, y) > \pi^*_A (E_L, 0),
\]

\[
\pi^*_A (E_L, 0) > \pi^*_A (E_H, E^*y) \text{ and }
\]

\[
\pi^*_A (E_L, y) > \pi^*_A (E_H, E^*y).
\]

i.e., \( \pi^*_A (E_L, y) > \pi^*_A (E_L, 0) > \pi^*_A (E_H, E^*y) \).

Therefore, when \( a_r < 2E_H \) and \( y \in \left( \frac{1}{2}, \frac{1}{A} \right) \), AU's optimal strategy is \((E_L, y)\).

(See Figure 12)

When \( y = \frac{1}{2} \), it follows from lemma 1 – 3 that,

\[
\pi^*_A (E_L, y) = \pi^*_A (E_L, 0),
\]

\[
\pi^*_A (E_L, 0) > \pi^*_A (E_H, E^*y) \text{ and }
\]

\[
\pi^*_A (E_L, y) > \pi^*_A (E_H, E^*y),
\]

i.e., \( \pi^*_A (E_L, y) = \pi^*_A (E_L, 0) > \pi^*_A (E_H, E^*y) \).

Therefore, when \( \lambda < 2 \) (that is, \( a_r < 2E_H \)) and \( y = \frac{1}{2} \), AU’s optimal strategy is either \((E_L, y)\) or \((E_L, 0)\). (See Figure 12)

When \( y \in \left( \frac{1}{2}, \frac{1}{A} \right) \), it follows from lemma 1 – 3 that,

\[
\pi^*_A (E_L, y) < \pi^*_A (E_L, 0),
\]

\[
\pi^*_A (E_L, 0) > \pi^*_A (E_H, E^*y) \text{ and }
\]

\[
\pi^*_A (E_L, y) \geq \pi^*_A (E_H, E^*y).
\]

i.e., \( \pi^*_A (E_L, 0) > \pi^*_A (E_L, y) \geq \pi^*_A (E_H, E^*y) \)
Therefore, when \( a_t < 2E_H \) and \( y \in \left( \frac{1}{2}, 1 \right] \), AU's optimal strategy is \((E_L, 0)\). (See Figure 12)

When \( y \in \left( \frac{1}{A}, 1 \right] \), it follows from lemma 1–3 that,

\[
\begin{align*}
\pi^*(E_L, y) &< \pi^*(E_L, 0) \text{ and,} \\
\pi^*(E_L, 0) &> \pi^*(E_H, E^*y) \\
\pi^*(E_L, y) &< \pi^*(E_H, E^*y), \\
\text{i.e., } \pi^*(E_L, 0) &> \pi^*(E_H, E^*y) > \pi^*(E_L, y)
\end{align*}
\]

Therefore, when \( a_t < 2E_H \) and \( y \in \left( \frac{1}{A}, 1 \right] \), AU's optimal strategy is \((E_L, 0)\). (See Figure 12)

Remark 3: When \( A < 2 \) (that is, \( a_t < 2E_H \)) the price that the rich students can afford is not very high. Providing a higher educational expenditure, per student, by choosing \((E_H, E^*y)\) becomes a dominated strategy for all values of \( y \in [0, 1] \).

Given \( A < 2 \) and the cut-off set by HU is low, i.e., \( y \in [0, 1/2] \), AU chooses \((E_L, y)\). By setting the cut-off just above that of HU, AU enhances its quality of education as perceived by the prospective students without providing a higher educational expenditure, per student. Since the cut-off set by HU is low, AU gets the required mass of students even when it sets the cut-off above that of HU.

When the cut-off set by HU is high, i.e., \( y \in (1/2, 1) \), AU chooses \((E_L, 0)\) and provides an educational quality inferior than HU. AU admits all rich students who are rejected in HU. As explained earlier, the size of this residual mass of student is large enough to generate greater profit than any other strategy does. Since these students do not have a choice other than AU, AU does not need to attract students with a higher expenditure on facilities.

When \( y = 1/2 \), AU is indifferent between \((E_L, y)\) and \((E_L, 0)\).

**Proposition III (c):** Given \( a_t = 2E_H \)

If \( y \in [0, 1/2] \), AU's optimal strategy is \((E_L, y)\).

If \( y = 1/2 \), AU’s optimal strategy is either \((E_L, y)\) or \((E_L, 0)\) or \((E_H, E^*y)\).

If \( y \in (1/2, 1) \), AU’s optimal strategy is \((a_t, E_L, 0)\).

**Proof:** If \( a_t = 2E_H \), i.e., \( A = 2 \), it follows from lemma 4, \( y_2 = y_1 = y_3 \), i.e., \( \frac{1}{A} = \frac{1}{2} = \frac{(A - 1)}{A - 2E^* + AE^*} \)
When \(y \in [0, 1/2)\), it follows from lemma 1–3 that,

\[
\begin{align*}
\pi^*_A (E_L, y) &> \pi^*_A (E_L, 0), \\
\pi^*_A (E_L, 0) &< \pi^*_A (E_H, E^*y) \text{ and} \\
\pi^*_A (E_L, y) &> \pi^*_A (E_H, E^*y).
\end{align*}
\]

i.e., \(\pi^*_A (E_L, y) > \pi^*_A (E_H, E^*y) > \pi^*_A (E_L, 0)\).

Therefore, when \(a_r = 2E_H \) and \(y \in [0, 1/2)\), AU’s optimal strategy is \((E_L, y)\). (See Figure 13)

When \(y = 1/2\), it follows from lemma 1–3 that,

\[
\begin{align*}
\pi^*_A (E_L, y) &= \pi^*_A (E_L, 0), \\
\pi^*_A (E_L, 0) &= \pi^*_A (E_H, E^*y) \text{ and} \\
\pi^*_A (E_L, y) &= \pi^*_A (E_H, E^*y).
\end{align*}
\]

i.e., \(\pi^*_A (E_L, y) = \pi^*_A (E_L, 0) = \pi^*_A (E_H, E^*y)\).

Therefore, when \(a_r = 2E_H \) and \(y = 1/2\), AU’s optimal strategy is either \((E_L, y)\), or \((E_H, E^*y)\), or \((E_L, 0)\). (See Figure 13)

\[
\begin{array}{ccc}
0 & 1/2 & 1 \\
\hline
\text{AU’s choice is } (E_L, y) & \text{AU’s choice is } (E_L, 0) \\
\text{AU is indifferent between } & & \\
(E_L, y), (E_L, 0) \text{ and } (E_H, E^*y)
\end{array}
\]

\textbf{Figure 13}

When \(y \in (1/2, 1]\), it follows from lemma 1–3 that,

\[
\begin{align*}
\pi^*_A (E_L, y) &< \pi^*_A (E_L, 0), \\
\pi^*_A (E_L, 0) &> \pi^*_A (E_H, E^*y) \text{ and} \\
\pi^*_A (E_L, y) &< \pi^*_A (E_H, E^*y) \\
\text{i.e., } \pi^*_A (E_L, y) &< \pi^*_A (E_H, E^*y) < \pi^*_A (E_L, 0)
\end{align*}
\]

Therefore, when \(a_r = 2E_H \) and \(y \in (1/2, 1]\), AU’s optimal strategy is \((E_L, 0)\). (See Figure 13) \(\square\)

\textbf{Remark 4:} When \(\Lambda = 2\) (that is, \(a_r = 2E_H\)) and the cut-off set by HU is low, i.e., \(y \in [0, 1/2]\), AU chooses \((E_L, y)\). This allows AU to increase the quality of educational service without providing a larger educational expenditure, per student. But when the cut-off set by HU is high, that is, \(y \in (1/2, 1]\), AU does not get the required mass of students by setting its cut-off
above that of HU and hence admits all rich students who does not get admission in HU by choosing $(E_i, 0)$. When $y = 1/2$, AU is indifferent between $(E_i, y)$, $(E_{ii}, E^*y)$ and $(E_i, 0)$. 

From our discussion so far, we find that there exist three classes of Nash equilibria in this game.

1. AU chooses $(a, E_i, y + \varepsilon)$. The rich students with quality $s_r \in [y + \varepsilon, 1]$ are in a position to choose between AU education and HU education and takes admission in AU. This class of equilibria is generated under the following parametric conditions:

- When $a_r > 2E_H$ and $y \in [0, \frac{1-\varepsilon}{A}]$
- or when $a_r < 2E_H$ and $y \in [0, \frac{1-\varepsilon}{2}]
- or when $a_r = 2E_H$ and $y \in [0, \frac{1-\varepsilon}{2}]

Under this class of Nash equilibria, all rich students with merit index $s_r \in [y + \varepsilon, 1]$, will apply to and join AU. The rich students with merit index $s_r \in [y, y + \varepsilon)$ and the poor students with merit index $s_p \in [y, 1]$ do not have any choice but to join HU. The remaining students are forced to discontinue education (see Figure 5).

Therefore under the above parametric conditions the enrolment in AU is $n_A = \beta \left(1 - y - \varepsilon\right)$ and hence the profit of AU is given as: $\pi^*_A = (a_r - E_i) \beta \left(1 - y - \varepsilon\right)$.

2. AU chooses $(a, E_{ii}, E^*y + \varepsilon)$. The rich students with merit index $s_r \in [y, 1]$ are in a position to choose between AU and HU and takes admission in AU. This class of equilibria is generated under the following parametric conditions:

- When $a_r > 2E_H$ and $y \in \left[\frac{1-\varepsilon}{A}, \frac{(A-1)(1-\varepsilon)}{A-2E^*+AE^*}\right]$
- Or when $a_r = 2E_H$ and $y = \frac{1-\varepsilon}{2}$.

Under this class of Nash equilibria, all rich students with merit index $s_r \in [E^*y + \varepsilon, 1]$, will apply to and join AU. The poor students with merit index $s_p \in [y, 1]$ join HU. The remaining students are forced to discontinue education (see Figure 9).

Therefore, under this parametric condition the enrolment in AU is $n_A = \beta \left(1 - E^*y - \varepsilon\right)$ and hence the profit of AU is given as: $\pi^*_A = (a_r - E_H) \beta \left(1 - E^*y - \varepsilon\right)$.

3. AU chooses $(a, E_i, 0)$. The rich students with merit index $s_r \in [y, 1]$ are in a position to choose between AU and HU and takes admission in HU. This class of equilibria is generated under the following parametric conditions:
When, \( a_r > 2E_H \) and \( y \in \left[ \frac{(A-1)(1-\epsilon)}{A-2E_f+AE^*}, 1 \right] \).

Or when, \( a_r < 2E_H \) and \( y \in \left[ \frac{1-\epsilon}{2}, 1 \right] \).

Or when, \( a_r = 2E_H \) and \( y \in \left[ \frac{1-\epsilon}{2}, 1 \right] \).

Under this class of Nash equilibria, all rich students with merit index \( s_r \in [y, 1] \), will apply to and join HU. The rich students with merit index \( s_r \in [0, y) \) do not have any choice but to join AU, and the poor students with merit index \( s_p \in [y, 1] \) do not have any choice but to join HU. The remaining students are forced to discontinue education (see Figure 7).

Therefore, under the above parametric conditions the enrolment in AU is \( n_A = \beta y \) and hence the profit of AU is given as: \( \pi^*_A = (a_r - E_d) \beta y \)

Section 7.3 Conclusion

We have analysed in this chapter the effects of trade liberalisation in education services via the mode of Commercial Presence on an importing country. The specific issues that were addressed are (1) quality of education an Affiliate University is likely to offer to its prospective students; and (2) alterations in the access to education as result of trade in education. We found that a Affiliate University offers a quality of education (measured by the amount of educational resources incurred per student) that is relatively superior than the domestic university of the importing country if and only if two conditions are satisfied. These conditions are - (1) it can charge a price beyond some threshold value for a superior educational quality and (2) the domestic university maintains a cut-off merit index that is neither too high nor too low. (See the second class of Nash equilibria in the previous page).

It is apparent from the analysis in this chapter that impact of trade in education on the access to education for students in the importing country depends on the distribution of wealth in the country. In absence of the Affiliate University, access to education was limited to students with \( s \in [y, 1] \) irrespective of their economic background. When the AU is allowed to operate, it offers education to only rich students. Under different parametric conditions AU offers either \((a_r, E_d, y+\epsilon)\) or \((a_r, E_H, E^*y+\epsilon)\) or \((a_r, E_L, 0)\). If AU offers \((a_r, E_d, y+\epsilon)\) access to education remains unchanged with only reallocation of some rich students from the domestic to the Affiliate University. In case AU offers \((a_r, E_d, 0)\), access to education for poor students remain unchanged whereas rich students with \( s \in [0, y) \) get access to AU education. In case AU offers \((a_r, E_H, E^*y+\epsilon)\) access to education for poor students remain unchanged whereas rich students with \( s \in [E^*y+\epsilon, y) \) get access to education. In this case, not only education becomes accessible for larger number of rich students, those rich students who can now access education
have access to higher amount of educational resources on an average. Therefore, operation of the Affiliate University makes distribution of access to education inequitable relative to the situation that prevailed in absence of the Affiliate University.

Finally, gain from trade is unambiguous for the exporting country. Profit earned by the Affiliate University, if transferred to its home campus can be used to improve its educational quality either by raising educational resources offered to its students or by offering a lower price to some or all students and thereby expanding the pool of applicants and hence admitting students more selectively.

Economic theory so far have considered either students’ quality or teaching quality as educational quality offered by an institution. Resources used on students, though is referred to another dimension of educational quality in literature on education, it had not received attention from theorisation in the field of economics. The above model incorporates both students’ quality and resources used on students as determinants of educational quality of an institution. Also, the existing literature determines volume of production or size of the school as dependent, at least partially, on the fixed establishment costs of the educational institute. This model departs from this aspect of the literature by showing that the size of the school can be limited even in the absence of a fixed cost when consumers perceive education as determined by quality of other consumers. Nevertheless, the model is not devoid of restrictive assumptions. Relaxing these restrictive assumptions should open up avenues for further theorisation in the area. For example, that the domestic university has a fixed cut-off merit index for admission of students and it does not alter in response to the choices of the Affiliate University is a restrictive assumption. Possibilities could be explored by introducing a strategically deciding domestic university. Furthermore, students in the importing country are assumed to consider only two options, namely, studying in either the Home University or in the Affiliate University. Realistically students are also expected to explore the option of moving to a foreign country for education. These are some of the important ways this model can be used for further research in the area.

**APPENDIX**

*Proof of Lemma 1:*

Comparing (14) and (16) we find that,

\[
\pi^*_{A, E_{L_0}, y} > or = or < \pi^*_{A, E_{L_0}}
\]

\[
\text{or} \quad (a_r - E_{L_0}) (1 - y) \beta > or = or < (a_r - E_{L_0}) \beta y
\]

\[
\text{or} \quad (1 - y) > or = or < y
\]
Proof of Lemma 2:

Comparing (16) and (15) we find that,

\[ \pi^*_A(E_I, 0) > \begin{cases} \pi^*_A(E_H, E^*y) \\ (a_r - E_I) \beta y \end{cases} \]

\[ \psi \begin{cases} (a_r - E_I) (1 - E^*y) \beta \\ \left( \frac{a_r}{E_H} - 1 \right) (1 - E^*y) \end{cases} \]

\[ \psi \begin{cases} (A - E^*) y \end{cases} \]

\[ \psi \begin{cases} (A - 1) (1 - E^*y) \end{cases} \quad \text{[A = (a_r / E_I)]} \]

\[ \psi \begin{cases} (A - 1) E^*y + (A - E^*) y \end{cases} \]

\[ \psi \begin{cases} (A - 2E^* + AE^*) y \end{cases} \]

\[ \psi \begin{cases} y \end{cases} \]

Note that \( [A - 2E^* + AE^*] = [A - E^* + AE^* - E^*] = [A - E^* + E^*(A - 1)] > 0 \), since \( A > 1 > E^* \)

Proof of Lemma 3:

Comparing (14) and (15) we find that,

\[ \pi^*_A(E_I, y) > \begin{cases} \pi^*_A(E_H, E^*y) \\ (a_r - E_I) \beta (1 - y) \end{cases} \]

\[ \psi \begin{cases} (a_r - E_I) \beta (1 - E^*y) \end{cases} \]

\[ \psi \begin{cases} \left( \frac{a_r}{E_H} - 1 \right) (1 - E^*y) \end{cases} \]

\[ \psi \begin{cases} (A - E^*) (1 - y) \end{cases} \]

\[ \psi \begin{cases} (A - 1) (1 - E^*y) \end{cases} \quad \text{[A = (a_r / E_I)]} \]

\[ \psi \begin{cases} (A - 1) E^*y - (A - E^*) y \end{cases} \]

\[ \psi \begin{cases} (E^*-1) Ay \end{cases} \]

\[ \psi \begin{cases} (1 - E^*) Ay \end{cases} \]

\[ \psi \begin{cases} y \end{cases} \]

\[ \psi \begin{cases} 1 / A \text{ as} (1 - E^*) > 0 \end{cases} \]

Proof of Lemma 4:

As denoted in remark 1, \( y_1 = 1 / 2 \),

\[ y_2 = \frac{(A - 1)}{A - 2E^* + AE^*} \quad \text{and} \quad y_3 = 1 / A \]

\[ y_1 \begin{cases} > \text{or} < y_2 \end{cases} \]

\[ \psi \begin{cases} 1 / 2 \end{cases} \begin{cases} > \text{or} < \end{cases} \frac{(A - 1)}{A - 2E^* + AE^*} \]

\[ \psi \begin{cases} (A - 2E^* + AE^*) \end{cases} \]

\[ \psi \begin{cases} > \text{or} < 2A - 2 \end{cases} \]

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\( y_1 > \text{or} = \text{or} < y_2 \quad \Leftrightarrow \quad A < \text{or} = \text{or} > 2 \quad \text{as} \quad (1 - E^*) > 0 \quad (17) \)

\( y_1 > \text{or} = \text{or} < y_3 \quad \Leftrightarrow \quad 1 / 2 > \text{or} = \text{or} < 1 / A \quad \Leftrightarrow A > \text{or} = \text{or} < 2 \quad (18) \)

\( y_2 > \text{or} = \text{or} < y_3 \quad \Leftrightarrow \quad \frac{(A - 1)}{A - 2E^* + AE^*} > \text{or} = \text{or} < 1 / A \)

\( \Leftrightarrow A (A - 1) > \text{or} = \text{or} < (A - 2E^* + AE^*) \)

\( \Leftrightarrow A^2 - 2A + 2E^* - AE^* > \text{or} = \text{or} < 0 \)

\( \Leftrightarrow A (A - 2) - E^* (A - 2) > \text{or} = \text{or} < 0 \)

\( \Leftrightarrow (A - 2) > \text{or} = \text{or} < 0 \)

[By (3) \( a_r > E_L \Rightarrow (a_r / E_H) > (E_L / E_H) \). So, \( A > E^* \).]

\( \Leftrightarrow A > \text{or} = \text{or} < 2 \quad (19) \)

Comparing inequations (17), (18) and (19) we get

\( y_2 > y_1 \) and \( y_1 > y_3 \) and \( y_2 > y_3 \) \quad \Leftrightarrow \quad y_2 > y_1 > y_3 \quad \text{if and only if} \quad A > 2. \)

\( y_2 < y_1 \) and \( y_1 < y_3 \) and \( y_2 < y_3 \) \quad \Leftrightarrow \quad y_2 < y_1 < y_3 \quad \text{if and only if} \quad A < 2 \)

and

\( y_2 = y_1 = y_3 \) if and only if \( A = 2 \), where \( A = a_r / E_H \).