# Chapter 3

# The Coupled Channels Model for Fusion

Heavy-ion collisions are typically characterised by the presence of many open reaction channels. At energies around the Coulomb barrier the main processes are elastic scattering, the inelastic excitations of low lying surface modes and the transfer of one or a few nucleons and the complete fusion reaction. Generally these processes are treated more or less independently. Elastic scattering data is described using a complex optical potential, where the imaginary part implicitly accounts for the flux lost to other reaction channels. Transfer reactions and directly populated inelastic excitations are then often considered to be perturbations on the elastic scattering in the distorted wave Born approximation. The fusion process is generally described as the penetration of a one-dimensional, real potential barrier, which in turn is usually only loosely identified with the real part of the elastic scattering optical potential. In the past years a much more comprehensive outlook in the form of the coupled channels framework [1] has evolved in which one seeks to describe all the main reaction processes simultaneously. The starting point here is a real potential that provides the bare interaction for the elastic scattering. One then adds coupling interactions to describe the inelastic excitations and transfer reactions. The fusion process is described by imposing the ingoing-wave boundary in all channels. In this way one explicitly accounts for the flux which leaves the elastic channel.

The coupled-channel framework is not only more satisfactory from a theoretical point of view; it is also required by a variety of new data which has been obtained. We saw the clear signature of the failure of the traditional approach of describing fusion in terms of a one-dimensional barrier penetration model. In order to study the influence of inelastic excitations and transfer on the fusion rates it is necessary to perform explicit coupled channels calculations. Another dynamical effect which has been emphasised recently is the energy dependence of the elastic scattering optical potential near the Coulomb barrier, referred to as the threshold anomaly, which is also explained in the framework of the coupled channel approach. A general description of the coupled channels formalism is given in the following sections.

## 3.1 The Coupled Channels Formalism

We consider here fusion reactions at energies below and slightly above the barrier where the coupling effects are the strongest and the number of channels coupled are tractable. At energies high above the barrier so many channels become involved that it is no longer possible to treat them individually. For simplicity we consider only coupling to inelastic channels, ignoring rearrangement processes such as transfer. The total Hamiltonian describing a system of two interacting nuclei can be written as

$$H = -\frac{\hbar^2}{2\mu} \bigtriangledown^2 + h(\xi) + V_0(r) + V_{coup}(r, \xi) \tag{3.1}$$

Here $h(\xi)$ is the internal Hamiltonian for the target nucleus (it is simple to include projectile excitations as well) where $\xi$ stands for the internal dynamical variables. The coupling term $V_{coup}(r, \xi)$ introduces the coupling between the relative motion and the internal degrees of freedom. $V_0(r)$ is the bare potential in the relative distance $r$. The Schrödinger equation for the total wave function $\Psi(r, \xi)$ then has the form

$$\left(-\frac{\hbar^2}{2\mu} \bigtriangledown^2 + h(\xi) + V_0(r) + V_{coup}(r, \xi)\right) \Psi(r, \xi) = E\Psi(r, \xi) \tag{3.2}$$

We introduce here the internal eigenstates $\phi_b(\xi)$ with eigen values $\epsilon_b$ such that

$$h(\xi)\phi_b(\xi) = \epsilon_b\phi_b(\xi) \tag{3.3}$$

and the coupling matrix elements are then defined as

$$V_{bc}(r) = \int d\xi \phi_b^*(\xi) V_{coup}(r,\xi)\phi_c(\xi) \tag{3.4}$$

The coupling matrix elements $V_{bc}$ of the coupling Hamiltonian in the collective model will consist of Coulomb and nuclear components. The total wave function $\Psi(r,\xi)$ can be expanded in terms of the internal eigenstates

$$\Psi(r,\xi) = \sum_b \psi_b(r)\phi_b(\xi) \tag{3.5}$$

From the Schrödinger equation 3.2 we derive coupled channels equations for the channel wave functions $\psi_b(r)$

$$\left(-\frac{\hbar^2}{2\mu}\bigtriangledown^2 + V_0(r) + V_{bb}(r) + \epsilon_b - E\right)\psi_b(r) = \sum_{c(c\neq b)} V_{bc}(r)\psi_c(r) \tag{3.6}$$

This system of coupled Schrödinger equations is to be solved subject to the boundary condition that the solution in the incident channel is composed of an ingoing and an outgoing wave, while all other channels contain only outgoing waves. The coefficients of the outgoing waves then determine the various reaction cross sections. In practical applications imaginary potentials are introduced into the coupled channels equations to account for the bulk of the flux which is lost from the direct channels to the compound reaction. Such a model is useful for calculating direct reactions. The fusion cross section can also be calculated from the difference between the total flux lost from the entrance channel and the flux which appears in the direct reaction channels [2, 3].

However, this approach is not suited to study the channel coupling effects on barrier penetration since imaginary potentials suppress the coupling which acts as the nuclei inter-penetrate. The artificial limitations resulting from using strongly

absorbing imaginary potentials in the coupled equations can be removed by imposing an ingoing boundary condition (IWBC) to account for the flux which leads to fusion. The form of the IWBC as suggested by Rawitscher [4] is

$$\psi_b(r) \propto \frac{1}{\sqrt{k_b(r)}} \exp\left[-i\int_{R_b}^{r} k_b(r')dr'\right] \tag{3.7}$$

where $k_b$ is the asymptotic wave number in the channel $b$ and $R_b$ is the starting point of the integration, taken to be inside the Coulomb barrier. The coupled equations solved under ingoing-wave boundary conditions provide a more realistic framework for describing fusion reactions. In the limit of the coupling strength going to zero, this reduces to the standard one-dimensional barrier penetration model.

The coupled system of equation (eqn. 3.6) with the boundary conditions can be solved either by direct numerical integration methods or through iterative methods. Within the coupled channels formalism one determines the total reaction cross section and the total cross section in the excited channels. This difference is equal to the ingoing flux at $R_b$ and is equated with the fusion cross section. The IWBC is applied at some point inside the barrier to obtain the S-matrices. The fusion cross section is then obtained based on the unitarity relation as

$$\sigma = \frac{\pi}{k^2}\sum_{\ell}(2\ell+1)\left(1-\sum_{i}|S_i(\ell)|^2\right) \tag{3.8}$$

where the index $i$ designates different scattering channels, which have been explicitly dealt with in the coupled channels calculations.

## 3.2 The Simplified Coupled Channels Formalism

The framework of the coupled channels calculations outlined above is numerically too complex and gives little insight in the actual consequences. One can introduce certain approximations which will not only simplify the numerical calculations but will also give clear understanding of the effect of channel coupling in terms of the

distribution of potential barriers. The approach of the simplified coupled channels calculations of Dasso *et al.*, [5] is described here.

One can significantly reduce the dimension of the coupled channels calculations by ignoring the changing of the centrifugal barrier due to the finite multi-polarity of the nuclear intrinsic excitation. This is called the no-Coriolis approximation, rotating frame approximation or the iso-centrifugal approximation. This approximation is reasonable in the case of heavy ion fusion reactions as the fusion cross sections are not affected by this approximation. Another simplification comes from ignoring the finite excitation energies of the internal degrees of freedom, *i.e.* $\epsilon_b = 0$. It is also reasonable to presume that the nuclear structure of the colliding nuclei is not disturbed until fusion occurs inside the barrier. In this adiabatic limit, where the intrinsic structure of the nuclei are not disturbed by the relative motion, the relative and intrinsic part in the coupling interaction can be separated to give

$$\langle b|V_{coup}(r,\xi)|\ c\rangle = F(r)\ \langle b\ |G(\xi)\ |c\rangle = F(r)G_{bc} \tag{3.9}$$

A further approximation is the constant coupling approximation where the coupling strength F is treated effectively as being spatially constant for all channels and replaced by a representative value $F_o$ at the barrier position. Rearranging the eqn 3.6 as

$$\left(-\frac{\hbar^2}{2\mu}\bigtriangledown^2 + V_0(r) - E\right)\psi_b(r) = \sum_c \langle b|h(\xi) + V_{coup}(r,\xi)|c\rangle\ \psi_c(r) \tag{3.10}$$

and in the adiabatic and constant coupling limit the right side of the above equation becomes

$$\langle b|h(\xi) + F(r)G(\xi)|c\rangle = \epsilon_b\delta_{bc} + F_oG_{bc} = M_{bc} \tag{3.11}$$

where $M_{bc}$ is the coupling matrix. Eqn. 3.10 can then be decoupled to give

$$\left[\bigtriangledown^2 + \frac{2\mu}{\hbar^2}\left(E - V_0(r) - \lambda_b\right)\right]\sum_b\ U_{cb}\psi_b(r) = 0 \tag{3.12}$$

where $\lambda_b$ are the eigen values of $h + V_{coup}$, describing the interacting intrinsic system. $U_{cb}$ denotes the unitary transformation which diagonalises the matrix $M_{bc}$ to give a

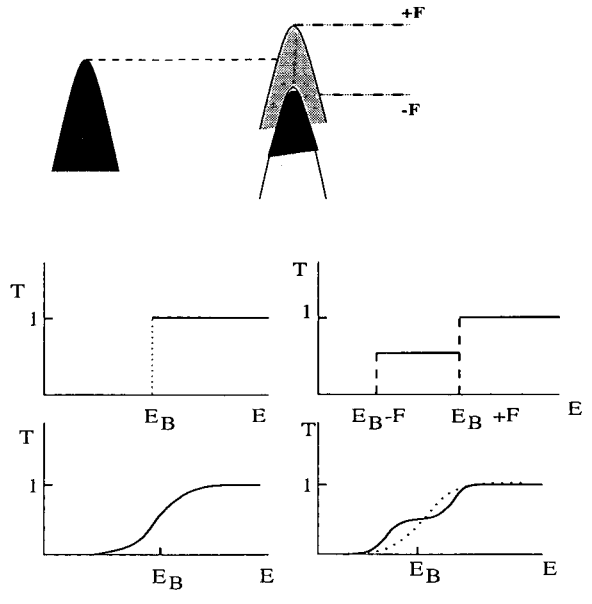


Figure 3.1: Schematic of the splitting of the single barrier into two as a result of coupling. The classical as well as the quantum mechanical picture is showm.

set of eigen values $\lambda_b$.

$$\sum_{jk} U_{bj} M_{jk} U_{kc}^{-1} = \lambda_b \, \delta_{bc} \tag{3.13}$$

Eqn. 3.12 indicates that the effect of coupling is to replace the barrier $V_B$ by a set of barriers $[V_B + \lambda_b]$ which confronts the incoming flux. Within the spectrum of barriers generated by the coupling interactions are those which are higher than the original barrier as well as those which are lower than the uncoupled barrier. This effect is shown pictorially in Fig. 3.1 which shows that the barrier encountered by the incident flux in the no-coupling limit is bifurcated by the coupling F. The total transmission for the coupling and no-coupling cases is also shown in the figure. The effect of coupling is to increase the transmission below the uncoupled barrier where as the transmission above the barrier is decreased.

The total transmission function is given by a weighted average of the overlap factor $\langle U_{b0} \rangle^2$ of the initial state with the eigen vectors of the matrix M corresponding

to the eigen values $\lambda_b$.

$$T_\ell(E) = \sum_b |U_{b0}|^2 \, T_\ell \, [E, V(r) + \lambda_b] \tag{3.14}$$

where $U_{b0}$ is the overlap probability and 0 denotes the entrance channel.

The fusion cross sections is calculated using a parabolic approximation [6] for the barrier. This yields the total fusion cross section as [7]

$$\sigma_{fus}(E) = \frac{R_B^2 \hbar\omega}{2E} \sum_b |U_{b0}|^2 ln\left[1 + \exp\left\{\frac{2\pi}{\hbar\omega}(E - V_B - \lambda_b)\right\}\right] \tag{3.15}$$

Here $\hbar\omega$ is the curvature of the parabolic barrier. For energies much below the barrier,

$$\sigma_{fus}(E) = \left\{\sum_b |U_{b0}|^2 \exp(-\lambda_b \frac{2\pi}{\hbar\omega})\right\}\left[\frac{R_B^2 \hbar\omega}{2E} \exp\{\frac{2\pi}{\hbar\omega}(E - V_B)\}\right] \tag{3.16}$$

Here the quantity in the square bracket can be defined as the cross section without any coupling. The quantity in the braces can be treated as the enhancement factor due to coupling. Also it is important to note that the lowest values of (*i.e.*, the most negative) $\lambda_b$ plays a role in the sub-barrier energies, and their contribution becomes more dominant, by lowering the barrier.

### 3.2.1 The Simplified Coupled Channels Code - CCMOD

A brief description of the code CCMOD [8] which has been extensively used to analyse sub-barrier fusion data is given in this section. CCMOD is a modified version of the code CCDEF [5] which was developed by Dasso *et al.* using the above mentioned formalism. In both the codes the various channels (such as inelastic excitations and transfer channel etc.) are treated as independent modes which couple to the ground state. The code CCMOD has however certain differences from CCDEF. A brief description of the code is given below.

#### The Potentials

The Coulomb and nuclear potentials are calculated considering the general case of a deformed colliding nuclei where the potentials are dependent on the orientation of

the two nuclei. The Coulomb potential between two deformed nuclei with deformation parameters $\beta_2$ and $\beta_4$ is

$$V_c(r,\theta_1,\theta_2) = \frac{Z_p Z_t e^2}{r}\left[1 + \sum_{\lambda=2,4:i=1,2} \frac{3}{2\lambda+1}\left(\frac{R_i^0}{r}\right)^{\lambda} \beta_{\lambda i} Y_{\lambda 0}(\theta_i)\right] \tag{3.17}$$

where $\theta_i$ is the angle the symmetry axis of the nuclei makes with the collision axis. The orientation dependent nuclear radii is given by

$$R_i(\theta_i) = R_i^0\left[1 + \beta_{2i} Y_{20}(\theta_i) + \beta_{4i} Y_{40}(\theta_i)\right] \tag{3.18}$$

where

$$R_i^0 = 1.233 A_i^{\frac{1}{3}} - 0.98 A_i^{-\frac{1}{3}} \tag{3.19}$$

The nuclear potential used in this code is a Woods-Saxon parameterisation of the Akyüz Winther potential [9] which has the form

$$V_n(r,\theta_1,\theta_2) = -V_0\left[1 + \exp\left\{\frac{r - R(\theta_1,\theta_2)}{a}\right\}\right]^{-1} \tag{3.20}$$

with

$$R(\theta_1,\theta_2) = R_1(\theta_1) + R_2(\theta_2) + 0.29 \tag{3.21}$$

The influence of the deformed nuclear shape is taken into account within the sudden approximation. In this limit, the characteristic time of the interaction is short compared to the period of the rotational motion of the deformed system.

### The Coupling Strength

1. For inelastic excitations to collective states, the coupling strength $F$ is computed from the deformation parameters $\beta_{\lambda i}$ as [5],

$$F_{\lambda_i}(r) = \frac{\beta_{\lambda_i}}{\sqrt{4\pi}}\left[-R\left(\frac{dV_n(r)}{dr}\right)_{R_{ib}} + \frac{3 Z_p Z_t e^2}{(2\lambda+1)}\frac{R_i^{\lambda}}{R_{ib}^{\lambda+1}}\right] \tag{3.22}$$

   Here, $R_i$ is the radius of the nucleus (Eqn. 3.19) which is excited and $R_{ib}$ is the position of the barrier and $V_n$ the nuclear potential. The first term gives

excitations due to the nuclear part and the second terms gives the contribution of Coulomb excitation. The factor $1/\sqrt{4\pi}$ results from averaging over all directions of relative coordinates. The deformation parameter is taken either from the experimental data or calculated from the electromagnetic reduced transition probabilities $B_i(E\lambda)\uparrow$ values given by

$$\beta_{\lambda i} = \frac{4\pi}{3ZR_i^{\lambda}} \left[\frac{B(E_\lambda)\uparrow}{e^2}\right]^{1/2} \tag{3.23}$$

where $\lambda$ is the multi polarity of the transition.

2. Coupling to transfer channels is included by directly specifying the values of the coupling strength F at the position of the unperturbed barrier. The spatial variation of the form factor is taken to be that of the one particle transfer form factor [10] given by

$$F(r) = \frac{F_o}{\sqrt{4\pi}} \exp\left[-\frac{(r - R_p - R_t)}{1.2fm}\right] \tag{3.24}$$

**The Eigen Values and Eigen Functions**

To solve a set of "n" uncoupled equations, one has to diagonalise the matrix $M_{bc}$ (Eqn. 3.11) to get the eigen values $\lambda_b$ and the weight factors $U_{b0}$ (Eqn. 3.13). The matrix $M_{bc}$ has the form

$$M = \begin{pmatrix} 0 & F_1(R_B) & F_2(R_B) & \cdots & F_n(R_B) \\ F_1(R_B) & -Q_1 & 0 & \cdots & F_n(R_B) \\ \vdots & \vdots & \vdots & \ddots & \cdots \\ F_n(R_B) & 0 & 0 & \cdots & -Q_n \end{pmatrix} \tag{3.25}$$

where $F_i$'s are calculated using Eq. (3.22) or the value at the barrier position is directly specified. In the CCMOD code the above matrix is diagonalised to get the eigen values and eigen vectors. In the CCDEF code, two channels are coupled at a time and the resulting matrix

$$M = \begin{pmatrix} 0 & F_i \\ F_i & -Q_i \end{pmatrix} \tag{3.26}$$

is diagonalised to yield the eigen values [1],

$$\lambda_{i\pm} = \frac{1}{2}\left(-Q_i \pm \sqrt{Q_i^2 + 4F_i^2}\right) \tag{3.27}$$

and eigen vectors [1],

$$U_{i\pm}^2 = \frac{4F_i^2}{4F_i^2 + \left[Q_i \mp \sqrt{4F_i^2 + Q_i^2}\right]^2} \tag{3.28}$$

The eigen values of the "n" channels coupled to the entrance channel are then obtained by taking all the possible linear combinations of the eigen values obtained from Eqn. 3.27. The corresponding weight factors are calculated by taking a product of the weight factors corresponding to the eigen values which appear in the linear combination. Thus, while the coupling of 'n' channels to the entrance channel should result in 'n+1' different eigen states and eigen values, this method will generate $2^n$ eigen states and eigen values. However, only 'n+1' states out of the $2^n$ will have significant effects.

### Finite Range Effects

The constant coupling approximation used in the CCDEF code, when used for the cases of strong coupling results in the over prediction of the low energy fusion data. Dasso and Landowne modified the above model and extended this model to strong coupling scheme [11]. Here the matrix has to be diagonalised in the barrier region to obtain the eigen vectors and eigen values as a function of $r$. The transmission coefficient $T_\ell(E)$ is then given by,

$$T_\ell(E) = \sum_b |U_{b0}(R)|^2 T_\ell[E, V_\ell(r) + \lambda_b(r)] \tag{3.29}$$

Here the weighting factors are fixed at a reasonable distance $R$, which is usually the position of the unperturbed barrier or the average position of the eigen barriers. In the CCDEF code, for the strong coupling case, the quantity $V_\ell(r) + \lambda_b(r)$ is

evaluated by expressing it in terms of a second order expansion in the vicinity of the unperturbed barrier $V_{b\ell}$, *i.e.*,

$$V_\ell(r)+\lambda_b(r) = V_{b\ell}(R_B)+\lambda_b(R_B)+\lambda_b'(R_B)(r-R_B)+\frac{1}{2}\left[V_\ell^{"}(R_B)+\lambda_b^{"}(R_B)\right](r-R_B)^2 \tag{3.30}$$

The position of the eigen barriers $R_{B\lambda}$ are given by the condition that $V_\ell(r)+\lambda_b(r)$ is maximum for $R_{Bb} = r$. In CCMOD, the coupling matrix is diagonalised to obtain a set of $\lambda_b(r)$ as function of $r$. The barrier position for each channel is then obtained by finding the value of $r$ for which $V_\ell(r)+\lambda_b(r)$ is maximum. Once the barrier positions are known, the barrier heights $V_\ell(R_{Bb})+\lambda_b(R_{B\beta})$ and the curvature $\hbar\omega$ are calculated. The weight factors are calculated at the position of the unperturbed barrier. The transmission coefficient $T_\ell$ (Eqn. 3.29) are calculated and the corresponding cross sections are determined using Eqn. 3.15.

## 3.3 Exact Coupled Channels Calculations

In the case of exact calculations, the coupled Schrödinger equations are rigorously solved by means of numerical integration. In a coupled channels calculations for direct reaction, the absorption due to inelastic excitation is explicitly treated. The imaginary part of the complex potential is used to describe the absorption into all other channels not included in the coupled elastic-inelastic channel space. This model predicts cross section for the elastic, inelastic and reaction cross sections. For extending this model to fusion, one has to isolate the fusion component from the transfer part. This is usually accomplished by introducing an ingoing-wave boundary condition (IWBC) [12]. In this approach of solving the Schrödinger equation, one specifies the value of the logarithmic derivative of the radial wave function at a boundary radius $R_{bc}$, chosen to lie inside the outer maximum of the interaction potential. In the vicinity of the boundary one assumes that there are only ingoing waves. The boundary condition at $R_{bc}$ replaces the usual boundary condition that the wave function is regular at the origin. An imaginary potential is not needed

to describe the absorption into channels other than inelastic one, when using the ingoing wave boundary conditions. In its place, it is considered that the flux crossing the boundary $R_{bc}$ leads to fusion.

Simplified calculations have the limitation that they take the frozen approximation where the excitation energies of the states are taken to be zero. In addition to this, the method of finding the eigen values and eigen vectors by diagonalising the matrix at the position of the uncoupled barrier is too simplified. Apart from this, the coupling interaction is often taken to the first order of $\beta$R. Also, the vibrational levels are truncated at the one-phonon level.

It has been demonstrated that non-linear couplings [13] significantly affect the shape of fusion barrier distributions and thus the linear coupling approximation is inadequate in quantitative comparison with the recent high quality data of fusion cross sections [14]. It was shown that the discrepancies are very pronounced for heavy symmetric systems although even for very asymmetric systems the effect is non-negligible. In view of this we have performed complete coupled channels calculations in the analysis of our high precision fusion data. The coupled channels code CCFULL [15] has been used for these calculations. This program includes the nuclear couplings to full order and thus it does not introduce the expansion of the coupling potential. The Coulomb couplings are however included to first order only. The no-Coriolis approximation or isocentrifugal approximation, is used in the program where one can replace the angular momentum of the relative motion in each channel by the total angular momentum $J$ [16, 17]. The program takes full account of the finite excitation energies of intrinsic motions. It includes Coulomb excitations and uses the incoming wave boundary condition inside the Coulomb barrier. A brief account of the code CCFULL is given in the following section.

### 3.3.1 The Exact Coupled Channels Code - CCFULL

The coupled-channels equations are written as

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{J(J+1)\hbar^2}{2\mu r^2}+V_N^{(0)}(r)+\frac{Z_pZ_te^2}{r}+\epsilon_n-E\right]\psi_n(r)+\sum_m V_{nm}(r)\psi_m(r)=0, \tag{3.31}$$

where $r$ is the radial component of the coordinate of the relative motion and $\mu$ is the reduced mass respectively. Here E is the bombarding energy in the center of mass frame and $\epsilon_n$ is the excitation energy of the $n$-th channel. $V_{nm}$ are the matrix elements of the coupling Hamiltonian, which in the collective model consists of Coulomb and nuclear components. $V_N^{(0)}$ is the nuclear potential in the entrance channel. In the program, the Woods-Saxon parameterisation

$$V_N^{(0)}(r)=-\frac{V_0}{1+\exp((r-R_0)/a)};\quad R_0=r_0\left(A_p^{1/3}+A_t^{1/3}\right), \tag{3.32}$$

is adopted for the nuclear potential $V_N^{(0)}$.

The coupled-channels equations are solved by imposing the boundary conditions that there are only incoming waves at $r=r_{min}$, and there are only outgoing waves at infinity for all channels except the entrance channel ($n$=0), which has an incoming wave with amplitude unity. The program CCFULL adopts the minimum position of the Coulomb pocket inside the barrier for $r_{\min}$. Practically, the numerical solution is matched to a linear combination of incoming and outgoing and Coulomb wave functions at finite distance $r_{\max}$ beyond which both the nuclear potential and the Coulomb coupling are sufficiently small. The boundary conditions are thus expressed as

$$\psi_n(r) \rightarrow T_n\exp\left(-i\int_{r_{\min}}^{r}k_n(r')dr'\right) \qquad r\le r_{min}, \tag{3.33}$$

$$\rightarrow H_J^{(-)}(k_nr)\delta_{n,0}+R_nH_J^{(+)}(k_nr) \qquad r>r_{\max}, \tag{3.34}$$

where $k_n(r)$ is the local wave number for the $n$-th channel and $k_n=k_n(r=\infty)$. $H_J^{(-)}$ and $H_J^{(+)}$ in eq. (3.34) is the incoming and the outgoing Coulomb functions, respectively.

In order to ensure that there are only incoming waves at $r \to r_{min}$, the program CCFULL solves the coupled-channels equations outer wards from $r_{min}$, first by setting [18]

$$\psi_n(r_{min}) = 1, \qquad \psi_m(r_{min}) = 0 \quad (m \neq n), \tag{3.35}$$

$$\frac{d}{dr}\psi_n(r_{min}) = -ik_n(r_{min}), \qquad \frac{d}{dr}\psi_m(r_{min}) = 0 \quad (m \neq n). \tag{3.36}$$

The solution of the coupled equations with the proper boundary conditions (3.33) and (3.34) is given by a linear combination of $\chi_{nm}$ as,

$$\psi_m(r) = \sum_n T_n \chi_{nm}(r). \tag{3.37}$$

where $\chi_{nm}$ is the wave function of the m$^{th}$ channel given by a superposition of the incoming and outgoing Coulomb waves as

$$\chi_{nm}(r) = C_{nm} H_J^{(-)}(k_m r) + D_{nm} H_J^{(+)}(k_m r) \qquad r \to r_{\max}. \tag{3.38}$$

Equation 3.37 satisfies the boundary condition (3.33) at $r = r_{min}$. At $r = r_{\max}$, it leads to

$$\psi_m(r_{\max}) = \sum_n T_n \chi_{nm}(r_{\max}) = \sum_n T_n \left( C_{nm} H_J^{(-)}(k_m r_{\max}) + D_{nm} H_J^{(+)}(k_m r_{\max}) \right). \tag{3.39}$$

By comparing eqs. (3.34) and (3.39), one finds

$$\sum_n T_n C_{nm} = \delta_{m,0}. \tag{3.40}$$

The transmission coefficients are then finally obtained by

$$T_n = \left( C^{-1} \right)_{n0}. \tag{3.41}$$

In case of inclusive process, where the intrinsic degree of freedom emerges in any final state, a summation over all possible intrinsic states, yields the inclusive penetrability as

$$P_J(E) = \sum_n \frac{k_n(r_{\min})}{k_0} |T_n|^2. \tag{3.42}$$

The fusion cross section and the mean angular momentum of compound nucleus are then calculated by

$$\sigma_{fus}(E) = \sum_J \sigma_J(E) = \frac{\pi}{k_0^2}\sum_J (2J+1)P_J(E), \tag{3.43}$$

$$\begin{aligned} < l > &= \sum_J J\sigma_J(E) / \sum_J \sigma_J(E), \\ &= \left(\frac{\pi}{k_0^2}\sum_J J(2J+1)P_J(E)\right) \Big/ \left(\frac{\pi}{k_0^2}\sum_J (2J+1)P_J(E)\right) \end{aligned} \tag{3.44}$$

### Coupling Matrix Elements

### Rotational coupling

For the case of rotational coupling in the target nucleus, the nuclear coupling Hamiltonian can be generated by changing the target radius in the nuclear potential (3.32) to a dynamical operator

$$R_0 \rightarrow R_0 + \hat{O} = R_0 + \beta_2 R_t Y_{20} + \beta_4 R_t Y_{40}, \tag{3.45}$$

where $R_T$ is parameterised as $r_{\text{coup}} A_T^{1/3}$, and $\beta_2$ and $\beta_4$ are the quadrapole and hexadecapole deformation parameters of the deformed target nucleus, respectively. The nuclear coupling Hamiltonian is thus given by

$$V_N(r, \hat{O}) = -\frac{V_0}{1 + \exp((r - R_0 - \hat{O})/a)}. \tag{3.46}$$

One needs the matrix elements of this coupling Hamiltonian between the $|n> = |I0>$ and $|m> = |I'0>$ states of the ground state rotational band of the target. This is done by using the matrix algebra [19]. Here one looks for the eigen values and eigen vectors of the operator $\hat{O}$ which satisfies

$$\hat{O}|\alpha> = \lambda_\alpha |\alpha> . \tag{3.47}$$

In the program CCFULL, this is done by diagonalising the matrix $\hat{O}$.

For the Coulomb interaction of the deformed target, the program CCFULL includes up to the second order with respect to $\beta_2$ and to the first order of $\beta_4$.

### Vibrational coupling

The need for all order nuclear couplings for the case where the vibration can be approximated by the harmonic oscillator was shown in ref [13]. In realistic case, however, phonon spectra are often truncated at some level, and thus the intrinsic motion deviates from the harmonic limit even when the excitation energies are equally spaced.

For vibrational coupling, the operator $\hat{O}$ in the nuclear coupling Hamiltonian is given by

$$\hat{O} = \frac{\beta_\lambda}{\sqrt{4\pi}} R_t (a^\dagger_{\lambda 0} + a_{\lambda 0}), \tag{3.48}$$

where $\lambda$ is the multi polarity of the vibrational mode and $a^\dagger_{\lambda 0}(a_{\lambda 0})$ is the creation (annihilation) operator of the phonon. The matrix element of this operator between the $n$-phonon state $|n>$ and the $m$-phonon state $|m>$ is given by

$$\hat{O}_{nm} = \frac{\beta_\lambda}{\sqrt{4\pi}} R_t (\sqrt{m}\delta_{n,m-1} + \sqrt{n}\delta_{n,m+1}). \tag{3.49}$$

The program CCFULL uses the linear coupling approximation for the Coulomb coupling of the vibrational degree of freedom. The Coulomb coupling matrix elements are thus read

$$V^{(C)}_{nm}(r) = \frac{\beta_\lambda}{\sqrt{4\pi}} \frac{3}{2\lambda+1} Z_p Z_t e^2 \frac{R_T^\lambda}{r^{\lambda+1}} (\sqrt{m}\delta_{n,m-1} + \sqrt{n}\delta_{n,m+1}). \tag{3.50}$$

The total coupling matrix element is given by the sum of $V^{(N)}_{nm}$ and $V^{(C)}_{nm}$.

### Transfer coupling

The program CCFULL includes a pair-transfer coupling between the ground states. It uses the macroscopic coupling form factor given by [20]

$$F_{trans}(r) = F_t \frac{dV^{(0)}_N}{dr}, \tag{3.51}$$

where $F_t$ is the coupling strength.

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