CHAPTER 3
GAME THEORITIC APPROACH FOR SPECTRUM SHARING

3.1 INTRODUCTION

Cognitive Radio Networks (CRN) is a wireless network consisting of several categories of users. A primary user is the license holder of a spectrum band, but, on the other hand, the secondary users intend to use idle license band (spectrum hole). These secondary users employ their cognitive capabilities to use the spectrum holes without harming the primary users.

One of the main challenges in operating these wireless networks is in the modus operandi of an efficient spectrum allocation that takes into account economic efficiency and spatial reusability simultaneously. Auction theory has been widely used for spectrum allocation in CRNs for realizing the fairness and improving efficiency. Unfortunately, the existing designs approach neither considers spectrum reuse nor the truthfulness when applied to double spectrum auctions. Thus a suitable framework is needed to achieve truthfulness and other economic properties while significantly improving spectrum utilization.

3.2 GAME THEORY

Game theory (Jane Wei Huang et al. 2009) is the study of optimization in situations of strategic interaction among individuals. In game
theory, these strategic interactions are called games and the participated entities are called players. In a typical game, each player has to apply a tactic to play from a set of possible strategies and an individual’s payoff is determined by a combination of strategies concerning all players. Games can be either static or dynamic but a static game is played once, while it is played several times in dynamic game.

The cornerstone of the game theory is the Nash Equilibrium – the combination of each player’s strategies such that each player has the highest payoff given every other player is playing their equilibrium strategy. Essentially, in a Nash Equilibrium each player is responding to the others’ strategies in the best possible way. A Nash Equilibrium is the outcome that will occur when all players are playing their best responses.

3.3 GAME THEORY AND STRATEGIC BEHAVIOR

The formal structure of the game theory models forces each player to consider the actions of the others when picking a strategy. In a game model, a player is supposed to act by maximizing their utility. Utility within such game approach has been measured by the payoffs, typically at the end of the game.

Some game-theoretic models require a utility function. A utility function maps the utility of a bundle of goods and services to a real number. Some examples of utility functions are listed below:

- \( u(x_1, x_2) = x_1 x_2 \)
- \( u(x_1, x_2) = ax_1 + bx_2 \)
- \( u(x_1, x_2) = \max \{ax_1, bx_2\} \)
The first example above is a multiplicative function, in which to have 16 units of utility requires 4 of each bundle, or 8 of bundle $x_1$ and 2 of bundle $x_2$, or 1 of bundle $x_1$ and 16 of bundle $x_2$. The other examples are additive and maximum functions.

### 3.4 NASH EQUILIBRIUM

Nash equilibrium (Mehdi Ghamari et al. 2012) is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. A profile of strategies is said to be a pure strategy Nash Equilibrium if $s_i^*$ a best response strategy against $s_{i-1}^* - \forall i = 1, ..., n$.

### 3.5 NASH-STABLE

A Coalition Structure (CS) (Xiaojuan Liao et al. 2012) $\Pi$ is Nash stable if $\forall \psi_i \in \Psi$ such that

$$
\psi_i \in \Omega_k, \Omega_k \in \Pi, (\Omega_k, \Pi) \geq_i ((\Omega_i \cup \psi_i), \Pi'), \forall \Omega_k \in \Pi, \text{ where }
$$

$$
\Pi' = (\Pi \setminus \Omega_k, \Omega_l \cup \Omega_k \setminus \psi_i, \Omega_l \cup i)
$$

(3.1)

According to the definition, there is no SU who wants to switch from one coalition to another or leave the current coalition to become non-cooperative, when CS is Nash-stable.

The distributed coalition formation algorithm will converge to a final CS $\Pi_r$ that is Nash-stable.

If the converged CS $\Pi_r$ is not a Nash-stable, there must be a $\psi_i \in \Psi$, $\Omega_k \in \Pi_r$, $\Omega_l \in \Pi_r$, where $\psi_i \in \Omega_k$, s.t. $(\Omega_i \cup \psi_i, \Pi') \geq_i (\Omega_k, \Pi_r)$.
According to the switch rule, SU $\psi_i$ still has the incentive to perform switch operation. Therefore, it contradicts with the fact that $\Pi_r$ is a CS converged based on the switch rule.

### 3.6 COALITION FORMATION

Consider a wireless network consisting of $M$ PUs or sellers $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_M\}$, who hold a certain amount of licensed spectrum band. As a matter of fact, the PUs may not use the whole licensed spectrum all the time and are willing to lease the idle channels to some of the $N$ SUs or buyers denoted as $\Psi = \{1, 2, \ldots, N\}$.

Assume that the idle time on PU’s channel is relatively long compared to the demand from the SU’s. The spectrum auction takes place periodically, with a period $T$, and during that the channel states remain unchanged. Assume that the channel conditions are homogenous to SU’s so that the requirement of one SU can be fulfilled by any channel. Further denote the seller $\lambda_i$’s bid as $s_i$, $\lambda_i \in \Lambda$, which represents the minimum payment on which it is willing to sell its channel, and buyer $j$’s bid as $b_j$, $j \in \Psi$, which represents the declared price on which it is willing to pay for a channel.

Assume that SUs are distributed over a large area and SUs locate far enough with each other can share spectrum without interference. Hence, SUs may form a coalition as a single buyer to participate in the auction. If the coalition (group of SUs) wins the auction, all SUs in the group can share the spectrum without interference. Once the buyers form a group, denote it as coalition $\Omega_k = \{U_i \in S \Psi; S \subset \Psi\}$. As any buyer belongs to a certain coalition, the coalition structure (CS) of buyers is defined as $\Pi = \{\Omega_1, \ldots, \Omega_n\}$

\[
\sum_{k=1}^{n} |\Omega_k| = N
\]  

(3.2)
Model the SUs’ interference on each other’s as a conflict graph $G = \{V,E\}$, where $V = \Psi$ and edge $(i, j) \in E$ if $\Psi_i, \Psi_j$ interfere with each other. The coalition with a single SU is also allowed.

The following Figure 3.1 shows 3 coalitions in a single partition.

![Figure 3.1 Coalition Structure](image)

The partition of SUs must satisfy:

$$\Psi = \bigcup_{k=1}^{K} \Omega_k; \Omega_m \cap \Omega_n = \emptyset; \forall \ m \neq n \in \{1,2, \ldots, K\} \quad (3.3)$$

There is a trustworthy third party agent (TPA) in any given area for facilitating the auction process. The TPA acts as an auctioneer, and its function can be divided into three parts:

- Initially, all buyers and sellers submit their bids $b_j$ or $s_t$ to the TPA and the TPA keeps these bids confidential to SUs and PUs throughout the whole process.
• During the coalition formation process, the TPA communicates with SUs to assist them to join or leave coalitions and reserve the detail information to avoid possible manipulation of SUs.

• When the coalition formation process converges, the TPA decides who finally wins the auction and then allocates the spectrum.

3.7 DOUBLE AUCTION

Incorporating the coalition formation process, the virtual auctions to achieve a high economic efficiency (Ning Zhang et al. 2012), which means more profits, would be achieved by PUs. Thus, the TPAs role can be listed as follows:

• Initialization At the beginning, all sellers and buyers submit their asks $s_i, i = [1, \ldots, M]$ and bids $b_j, j = [1, \ldots, N]$ to the TPA. The TPA keeps these ask and bids confidential throughout the whole spectrum auction process.

• Virtual auction process The TPA initializes the partition of SUs as $\Pi_0 = \{\{1\}, \{2\} \ldots \{N\}\}$ and performs the virtual double auctions for larger profit. Note that the coalition’s bid is lower than the sum of SUs’ bids inside the coalition in order to ensure the truthfulness. Finally, this process terminates when the TPA cannot obtain a larger profit by changing the partition of SUs. Since the virtual auctions are performed multiple times, while no one is regarded as a real one to decide the winning SUs, the process is called as the virtual auction process.
• **Spectrum auction** According to the final partition of SUs, the TPA performs the auction and decides the winning SUs, and finally charges them.

A cognitive network consists of $m$ multiple primary users and $n$ multiple secondary users (Figure 3.2). The primary user’s locations are arbitrarily with a minimum distance $R_0$ between any two primary transmitters. The cognitive transmitters are distributed randomly and uniformly with density $\lambda$.

![Network model](image)

**Figure 3.2 Network model**

### 3.7.1 Possible Operations for Coalition Formation

Assume that the TPA changes the partition $\Pi$ by moving one SU at a time, thus the auctioneer’s possible operations (Figure 3.3) can be summarized as follows:
• **Joining operation** A buyer in a coalition of its own joins an existing coalition, thus a larger coalition is formed, i.e. \((jj, \Omega_2) \rightarrow (\Omega_2, j))\).

• **Switching operation** A buyer leaves the current coalition (with more than two buyers) and switches to another one, i.e. \((\Omega_1; \Omega_2) \rightarrow (\Omega_1/j; \Omega_2,j))\).

• **Leaving operation** A buyer leaves the current coalition, and forms a singleton coalition of its own, i.e. \((\Omega_1) \rightarrow (\Omega_1/j, \{j\}))\).

![Figure 3.3 Possible Operations](image-url)
3.7.2 Economic Properties

Four parameters are used to describe economic properties: bid, true valuation, clearing price, and bidder utility. For a seller $m$, $B^s_m$ is its bid i.e., the minimum payment required to sell a channel; $V^s_m$ is its true valuation of the channel; $P^s_m$ is the actual payment received if it wins the auction; utility modeled as below

$$U^s_m = P^s_m - V^s_m \text{, if it wins the auction, and 0 otherwise.}$$

(3.4)

For a buyer $n$, $B^b_n$ is its bid i.e., the maximum price it is willing to pay for a channel; $V^b_n$ is it’s true valuation of a channel; $P^b_n$ is the price it pays if it wins the auction, and its utility is

$$U^b_n = V^b_n - P^b_n \text{ if it wins, and 0 otherwise.}$$

(3.5)

There are three economic properties as below

- **Truthfulness**: A double auction is truthful if no matter how other players bid, no seller $m$ or buyer $n$ can improve its own utility by bidding untruthfully ($B^s_m \neq V^s_m$ or $B^b_n \neq V^b_n$). In untruthful auctions, selfish bidders can manipulate their bids to game. In truthful auctions, the dominate strategy for bidders is to bid truthfully, thereby eliminating the fear of market manipulation.

- **Individual Rationality**: A double auction is individual rational if no winning seller is paid less than its bid and no winning buyer pays more than its bid:

$$P^s_m \geq B^s_m \text{, } P^b_n \leq B^b_n \text{ } \forall \text{ seller m and buyer n}$$
- **Ex-post Budget Balance:** A double auction is ex-post budget balanced if the auctioneer’s profit $\Phi \geq 0$. The profit is defined as the difference between the revenue collected from buyers and the expense paid to sellers:

$$\Phi = \sum_{n=1}^{N} P_{n}^{b} - \sum_{m=1}^{M} P_{m}^{s} \geq 0 \quad (3.6)$$

The previous property ensures that the auctioneer has incentives to setup the auction.

### 3.7.3 McAfee Double Auction

In this model, multiple POs need simultaneous sale of sell their spectrum, thus the double auction mechanism is adopted for spectrum leasing problem. Most existing double auctions use McAfee’s design (Tan Xuezhi et al. 2009), which achieves key economic properties such as truthfulness, individual rationality, Ex-post budget balance, etc. At the cost of the least profitable trade to achieve truthfulness, the design matches buyers to sellers one by one to make the auction profitable. The brief procedure of the McAfee double auction is summarized as follows:

i) Sort asks in non-decreasing (for sellers) and bids in non-increasing (for buyers’ coalitions) orders:

$$s_{1} \leq s_{2} \ldots \leq s_{M}$$

$$\Theta_{1} \geq \Theta_{2} \geq \ldots \geq \Theta_{N}$$

Where $\Theta_{i}$ is the bid of buyer’s coalition $\Omega_{i}$.

ii) Find $k = \text{argmax}_{k} s_{k} \leq \Theta_{k}; 1 \leq k \leq \min (M,K)$, the index of the least profitable trade. The first $k-1$ sellers and $k-1$ coalitions are winners;
iii) Charge all the winning coalitions equally to the bid of the $k^{th}$ ranked coalition $\Theta_k$. Pay all the winning sellers equally with the bid of the $k^{th}$ ranked seller $s_k$.

Consider the scenario where the number of buyers is larger than that of the channels owned by sellers, which is $N > M$, for sake of one seller’s spectrum being shared by multiple buyers. Considering the auction result with the current partition as $\Pi = \{\Omega_1, \ldots, \Omega_K\}$, these coalitions are classified into two types, where type I refers to winning coalitions and type II refers to losing coalitions.

### 3.7.4 Critical Coalition

The coalition $\Omega_K$ which satisfies $\Theta_k \geq s_k; \Theta_{k+1} < s_{k+1}$ is called the critical coalition. It is obvious that $\Omega_K$ belongs to type II coalition according to the auction mechanism described above. However, the McAfee mechanism here is for buyers that each has a corresponding single user, but not for coalitions of users.

### 3.7.5 Bid Definition of Coalition

Truthfulness means all buyers submitting the bids reflecting their usage on spectrum bands. This must be considered in the bid definition of coalitions. Therefore, the coalition $\Omega_l$’s bid is defined as the number of buyers multiplied by the minimum bid across the coalition, expressed as

$$\Theta_l = |\Omega_l| \cdot \min/ b_j / \forall j \in \Omega_l$$

(3.7)

The utility of SU which belongs to winning coalition $\Omega_l$ can be derived as

$$u_j = b_j - (1/|\Omega_l|)P_l; \forall j \in \Omega_l$$

(3.8)
Spectrum utilization can be reviewed as the best spectrum usage, meaning that the spectrum should be leased to SUs who need them most. In the spectrum auction, SUs define their bids based on actual usage, such as the transmission rate, channel capacity, and the data pattern. The optimization goal is defined as the social welfare maximization, which equals to the summation of winning SUs’ bids. Thus, the optimal partition of buyers’ needs to be found $\Pi^* = \{\Omega_1, \ldots, \Omega_k^*\}$ maximizing the following:

$$S - \text{OPT}: S^* = \sum_{\Omega \in \{\Omega_1, \ldots, \Omega_{k-1}\}} b_j$$ (3.9)

where coalition $\Omega_{l,k}$ is the critical coalition.

$$\Theta_l \geq \Theta_m, \forall l < m \in [1, \ldots, K]$$ (3.10)

Equation (3.10) means that the coalitions are sorted in the descending order according to their bids.

$$\Omega_l \cap \Omega_m = \emptyset, \bigcup_{l=1}^{K^*} \Omega_l = \Psi$$ (3.11)

Equation (3.11) ensures that $\Pi^*$ is a partition of SUs and each SU belongs to a certain coalition.

$$\Theta_l \geq s_l, \forall 1 \leq l \leq k$$ (3.12)

Equation (3.12) is the requirement of the McAfee double auction mechanism, so that the variable $k$ can be derived under the current partition.

$$(m, n) \notin E, \forall m, n \in \Omega_l, l \in [1, \ldots, K^*]$$ (3.13)

Equation (3.13) is the interference constraint among SUs. The direct solution to this question is to list all possible partitions which satisfy the above constraints, and find the optimal one by comparing the achieved social
welfare of SUs. However, the number of the possible partitions is a Bell number, thus it is an NP-hard problem for an exhaustive search. Rather than solving this problem by an exhaustive search for all possible partitions, the coalition formation process is incorporated into the auction mechanism to find a suboptimal solution.

3.8 COALITION FORMATION ALGORITHM

The coalition formation algorithm developed as in Table 3.1.

Table 3.1 Coalition Formation Algorithm

<table>
<thead>
<tr>
<th>Initial State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: All buyers and sellers submit their bids $b_i, s_j, \psi_i \in \Psi, \lambda_j \in \Lambda$ to the TPA.</td>
</tr>
<tr>
<td>2: Buyers bid non-cooperatively, the partition is $\Omega_i = { \psi_i }, i \in { 1, \ldots, N }$, $\sum_{i=1}^{N}</td>
</tr>
<tr>
<td>3: $\forall \psi_i, h(i) = \emptyset$</td>
</tr>
</tbody>
</table>

**Coalition Formation Process**

| 3: Repeat |
| 4: for $j = 1 : N$ do |
| 5: the auctioneer informs all SUs the latest cardinality of all coalition $|\Omega_i|\forall \Omega_i \in \Pi$ |
| 6: for $k = 1 : |\Pi|$ do |
| 7: if $\psi_i \notin \Omega_k$ |
| 8: buyer $\psi_j$ compute the coalition bid of $\Omega_k' = \{ \Omega_k \cup j \}$, and derive the value of preference function $g_j$ according to (2) (4) |
| 9: if $g_i(\Omega_k', \Pi') \geq g_i(\Omega_k, \Pi) \& \Omega_k' \notin h(j)$ |
| 10: buyer $\psi_j$ jump to coalition $\Omega_k'$ |
Table 3.1 (Continued)

11: \[ h(j) = h(j) \cup \Omega_k \textbf{break} \]
12: \[ \text{else continue} \]
13: \[ \text{end for} \]
14: \[ \text{end for} \]
15: \[ \text{end for} \]
16: \[ \text{end for} \]
17: \[ \text{Until } \text{CS converges to Nash-stable partition} \]

**Allocation Process**

18: After the coalition structure reach Nash stable, the auctioneer announces

the final winners according to McAfee double auction scheme and

allocates the channels.

19: \[ \text{end} \]

### 3.8.1 Algorithm Flow Chart

Coalition Structure Generation
- EDP algorithm

Solve optimization problem
- Finite convergence time
- Nash stable

Resource allocation
- divide the value of solution
- keep key properties intact
**STEPS**

- **Initialisation** All sellers and buyers submit their asks $s_{i,i} \in [1, \ldots, M]$ and bids $b_{j,j} \in [1, \ldots, N]$ to the auctioneer. The auctioneer (Figure 3.4) initializes partition.

![Figure 3.4 Auctioning](image1)

- **Conflict graph construction** Model the SUs’ interference on each other’s as a conflict graph $G = (V, E)$, where $V = \Psi$ and edge $(i, j) \in E$ if $\psi_i$ and $\psi_j$ interfere with each other. The coalition with a single SU is also allowed.

![Figure 3.5 (Continued)](image2)
Buyers  Tradable sellers
B1 {S1, S5}
B2 {S4, S5}
B3 {S5}
B4 {S5}
B5 {S2, S5}
B6 {S3, S5}
B7 {S2, S3}

Figure 3.5 Conflict graph construction

- **Cooperative Coalition Formation** The auctioneer performs the virtual double auctions for larger profit using game theoretical concepts.

- **Initiate switch request TPA (Third Party Auctioneer) Auction**
  - One SU initiates a switch request to certain coalition with no interference with them (destination coalition) and sends request to TPA.
  - TPA calculates the payment of members of destination coalition and payment of requesting SU and sends information back to the SU and destination coalition.

- **Preference relation** A preference relation $\geq_i$ is defined as a complete, reflexive and transitive binary relation over all possible coalitions that $\psi_i$ can form. For any SU $\psi_i \in \Psi$, consider two coalitions $\Omega_1, \Omega_2$ and their corresponding CS $\Pi_1, \Pi_2$. $(\Omega_1, \Pi_1) \geq_i (\Omega_2, \Pi_2)$ indicates that $SU_i$ prefers to be in
coalition $\Omega_1$ with CS $\Pi_1$ over being $\Omega_2$ with CS $\Pi_2$. In our work, the preference relation for any SU $i$ is defined as:

$$(\Omega_1, \Pi_1) \succeq_i (\Omega_2, \Pi_2) \iff g_i(\Omega_1, \Pi_1) \geq g_i(\Omega_2, \Pi_2) \quad (3.14)$$

where $\Omega_1 \in \Pi_1, \Omega_2 \in \Pi_2$ and $\Pi_1, \Pi_2$ are two possible CSs and $g_i$ is a function to show the preference defined as follows:

$$g_i(\Omega, \Pi) = r_i(\Omega, \Pi), \text{if } (r_i(\Omega, \Pi) \geq r_j(\Omega \setminus \{\psi_i\}, \Pi) \forall j \in \Omega \setminus \{\psi_i\}) \& \Omega \notin h(i) \text{ or } (|\Omega| = 1) \quad (3.15)$$

Where $h(i)$ is the history records of SU $i$ containing the coalition’s whose size is larger than one and that has been visited.

- **Inform TPA and Update**- The TPA depending on the interference condition takes the decision on the coalition structure on to which partition it should belong to.

- **Convergence**- As the SUs make decision sequentially, $\Pi_i, n_i$ is used to denote the CS in $i^{th}$ decision process and $n_i$ records the total switch number. Two notations are used to record the sequence because in some decision process, no switch will be made. The switch sequence is recorded as:

$$\Pi^s = \{\Pi_{1,1}, \Pi_{2,2}, \Pi_{3,2}, \ldots, \Pi_{T,nT}\} \quad (3.16)$$

For any switch operation, there will be two kinds of consequences C1. The new CS has not been visited; C2. While this CS was. If every switch operation leads to C1, due to fact of the number of all possible CSs being bounded by a Bell
Number, the number of switch operation is finite. Therefore, the
CS will converge. If C2 occurs, due to restrictions on the switch
rule for SUs to form the coalitions existed in the history unless
they chooses to become non-cooperative, there must exist some
non-cooperative SUs in the revisited CS. Without loss of
generality, assume there is no SUs want to become non-
cooperative. If cooperative SUs or the non-cooperative SUs
choose to switch to an existing coalition, then, according to the
switch rule, they must form a new CS. The algorithm terminates
when all the non-cooperative SU and cooperative SUs choose to
remain unchanged. Therefore, there will be no circulations.
Every time C2 occurs, the next CS is either a new one or
terminates. Therefore, the algorithm will terminate in all cases.

- **Pricing** To maintain truthfulness, TRUST pays each winning
seller m by the $k^{th}$ seller’s $B^k_s$ bid. TRUST charges each winning
buyer group l by the $k^{th}$ buyer group’s bid $\pi_k$. This group price
is evenly shared among all the buyers in the group l:

$$P^n_b = \pi_k / n_1, n_1 \in G.$$  \hspace{1cm} (3.17)

No charges or payments are made to losing buyers and sellers.
The auctioneer’s profit is:

$$\Phi = (k - 1) \cdot (\pi_k - B^k_s)$$ \hspace{1cm} (3.18)

- **Channel Allocation** After the coalition structure reach Nash
stable, the auctioneer announces the finally winners according
to McAfee double auction scheme and allocates the channels.
3.9 COALITION STRUCTURE GENERATION (CSG)

The goal of the coalition generation problem is to find the combination of disjoint coalitions that maximize the global value by aggregation. The existing dynamic programming (DP) algorithm is able to find an optimal solution to the CSG problem. It explores the complete solution space with complexity $O(3^n)$. Since it performs redundant calculations its memory requirements are large. The proposed Extended Dynamic Programming (EDP) algorithm reduces this drawback, introducing conditions that avoid the generation and evaluation of a large number of splittings. EDP explores a fraction of the splittings explored by the DP algorithm.

Table 3.2 Coalition Structure Generation Algorithm

```
PSEUDO CODE
1. for m=2 → n do
2.   for C ← coalitions Of size(m) do
3.     max_value ← value[C]
4.     (lower_bound, higher_bound) ← EDP bounds(n,m)
5.     C_1 ← get First Split(C,lower_bound)
6.     while ( size Of(C_1) ≤ higher_bound ) do
7.       C_2 ← C - C_1
8.       if (max_value < value[C_1] + value[C_2]) then
9.         max_value = value[C_1] + value[C_2]
10.      end if
11.     C_1 ← get Next Split(C_1,C)
12.    end while
13.   value[C] ← max_value
14. end for
15. end for
```
Description

User \((a_x)\) A single user, where \(x\) indicates the user identifier.

Users \((A)\) The set of all available users. \(A = \{a_1, a_2, \ldots, a_n\}\).

Coalition \((C)\) \(C \subseteq A\). \(C\) is a subset of \(A\) that contains the users participating in a coalition. Its size is denoted as the number of users forming the coalition.

Split The operation performing a binary partition of a coalition.

Splitting The result of the split operation. A splitting is a 2-tuple represented by \((C_1, C_2)\), where \(|C_1| \cdot |C_2| > 0\), \(C_1 \cap C_2 = \emptyset\).

Coalition Structure (CS) A collection of disjoint coalitions whose union yields the entire set of agents. \(CS \subseteq 2^A\) where for any \(C_i, C_j \in CS\).

i. The outer loop, lines 1-14, where the coalition size is selected.

ii. The intermediate loop, lines 2-13, where the coalitions of a fixed size are generated.

iii. The inner loop, lines 5-11, where every coalition is split and evaluated.

iv. The first splitting in the inner loop is generated by the get First Split function in line 5 and the rest by the get Next Split function in line 11. Lines 7-9 assess the value of the best splitting, which is stored in memory in line 13.
3.10 RESULTS AND ANALYSIS

The time convergence of DP and EDP algorithm to an optimal coalition structure against the number of secondary users is presented in Figure 3.6. It can been observed that the proposed EDP algorithm evaluates only a subset of splitting and converges at a much lower time than the existing DP.

![Convergence Time vs No. of SUs](image)

**Figure 3.6 Convergence time**

The single threaded implementation of the EDP algorithm, it is analyzes the operations of generating and evaluating splitting inside the inner loop, which consume about 99% of the execution time. All memory accesses correspond to reads from the vector of coalition values performed in the inner loop of the algorithm, and a few writes on the intermediate loop. The total number of data read operations done by the DP algorithm is around $2 \times 3^N$. As
EDP evaluates only a subset of the splitting, number of data read operations is considerably lesser than the former. The data-reuse degree of the algorithm is high.

The bad performance behavior of the memory access pattern arises for vectors that do not fit into the processor's cache. The simplest and most efficient approach is always to parallelize the outer loop of a program. DP and EDP, though, exhibit loop-carried dependencies on the outer loop: the optimal values for coalitions of size m must be generated before using them for generating the optimal values for coalitions of size m + 1. The intermediate loop generates all the coalitions of a given size, and for each coalition it analyzes all the splitting of certain sizes.

3.11 SUMMARY

The game theoretic approach of spectrum sharing need to deal with the theory of the Nash Equilibrium in which the combination of each player’s strategies leads a state, where each player has the highest payoff given every other player, playing their equilibrium strategy. Each player considers the actions of the others when picking a strategy. Considering a typical cognitive radio system, coalition formation among secondary user is important because if the coalition wins the auction, all SUs in the group can share the spectrum seamlessly. In double auctioning, McAfee’s design concept is very popular as it aims to achieve many economic properties. When coalition structure achieves Nash equilibrium, the auctioneer declares the final winners as per McAfee double auction method, while allocating the channels among SUs. The existing dynamic programming (DP) method can find an optimal solution to CSG problem but with complexity of O(3^n). The proposed EDP algorithm reduces complexity and hence the time of execution by applying conditions that avoid the generation and evaluation of large number of splitting.