Abstract

In Chapter 1 we review the previous works done in the field of parallel solution of boundary value problems. We study, with the help of references, various parallel algorithms for initial value problems and boundary value problems. We also review parallel algorithms for the solution of tridiagonal systems and Navier-Stokes Equations.

In Chapter 2 we develop an efficient parallel algorithm for a class of two-point boundary value problems:

\[ y^{(iv)} = f(x, y), \quad y(0) = A_1, \quad y''(0) = A_2, \quad y(1) = B_1, \quad y''(1) = B_2 \]

with \( \frac{\partial f}{\partial y} \leq 0 \) and \( \frac{\partial f}{\partial y} \) continuous on \([0, 1] \times (+\infty, +\infty)\). We study the convergence of finite difference scheme and show that error estimate is of \( O(h^4) \). Numerical illustration and Theoretical estimates has been calculated by taking example of linear two-point boundary value problem. The Speed-up has been calculated by comparing the time taken by fourth order method with that of second-value method. We also extend our parallel algorithm for the solution of Elliptic PDEs.

In Chapter 3, we develop Parallel Algorithm for the class of initial value problems

\[ y' = f(x, y), \quad y(x_0) = A, \quad x \in [x_0, b] \]

using 3rd order Runge-Kutta method. We extend the analysis to the system of differential equations.
\[ \frac{dy}{dx} = f(x, y), \quad y(x_0) = \eta \]

where \( y = (y_1, y_2, \ldots, y_n)^T \)

Numerical illustration and Theoretical estimates have been calculated by taking several examples both for initial value problems and system of differential equations. Speed-up has been calculated by comparing the results with the time taken using second-order Runge-Kutta method as benchmark.

In Chapter 4, we develop two parallel algorithms for the solution of Quasi-Tridiagonal system (having non-zero elements at top right and bottom left corners on a hypercube multiprocessor with \( p = 2^m \) nodes, \( m \) a positive integer. Such type of equations arises in the solution of TPBVP's having periodic boundary conditions. They also arise in the solution of Navier Stokes Equations. The solution of the Quasi-Tridiagonal system is obtained in four phases using cyclic odd-even reduction process. The Speed-up factors of the parallel algorithm is obtained by comparison with its "best" serial counterpart.