Chapter-6: Study on Technical Issues – Illustrations of Reactive Improvement by Relating Key Process Output Variables with Key Process Input Variables with the Help of Regression Analysis and Optimizing the Regression Equations by Applying Mathematical Programming Techniques

6.1 Title: Determining conducive processing conditions for wool fibres

6.1.1 Motivation for this work

Wool fibres are very sensitive to the atmospheric processing condition inside the shop-floor. If the atmospheric processing condition inside the shop-floor is not conducive then wool fibre generates static electrical charges during processing because of its friction with the metallic parts of the machinery. Unless this charge is neutralized by a media which is conductor of electricity, the fibres tend to scatter in all directions resulting in lapping and fibre breakage. This in turn reduces productivity, expressed in terms of Top Yield in the wool combing department. It is to be noted that the atmospheric condition within the shop-floor is again dependent on the ambient condition outside the shop-floor which varies a lot from season to season.

The best conducting material for this purpose is water present in the wool. If the relative humidity percentage (RH%) of the processing area is too low, the moisture content in wool evaporates fast causing accumulation of static charge which subsequently causes fibre breakage, reduction in fibre length (Hauteur length) and decrease of top yield due to generation of more noils or wastes.

6.1.2 Objective of the study

To arrive at the optimum operating conditions, including departmental humidity and temperature, corresponding to which the Actual Hauteur Length, i.e. the Actual Fibre Length and Yield – specifically known as Top Yield – will be maximized and, in turn, the difference between actual Hauteur length and expected Hauteur length will also be maximized. The supplier for a consignment calculates the expected Hauteur length.

6.1.3 Process flow chart at the wool combing department

- Blending of different sub-lots to obtain a homogeneous product
- Removal of natural grease, suint, dust and dirt by washing with synthetic detergent in aqueous bath
6.1.4 Data

Keeping in view the above objective, data are compiled for about one year period from the records of Wool Combing and Engineering department on the following characteristics by maintaining one to one correspondence:-

- Lot number
- Date
- Relative Humidity (RH%) - Minimum
• Relative Humidity (RH%) - Maximum
• Humidity in terms of Grain (grams per cubic feet) - Minimum
• Humidity in terms of Grain (grams per cubic feet) - Maximum
• Dry bulb temperature in 0°F - Minimum
• Dry bulb temperature in 0°F - Maximum
• Expected Hauteur in mm.
• Actual Hauteur in mm.
• Expected Hauteur CV%
• Actual Hauteur CV%

6.1.5 Analysis

Stepwise multiple regression has been used for analyzing the data. A SPSS package (PC version) is made use of for this purpose. The following relationship has been obtained:

\[
\text{Actual Hauteur} = 47.713 + 0.734 \times \text{Maximum Grain} + 0.663 \times \text{Expected Hauteur} - 0.536 \times \text{Expected Hauteur CV%}
\]

With \( R = 0.859 \)

\( R^2 = 0.738 \)

Adjusted \( R^2 = 0.717 \)

Standard error of the estimate = 2.11048

Number of data sets = 41

The corresponding model adequacy checks are provided in Appendix 6.1.1.

It can be concluded that about 72% of the variation in Actual Hauteur (KPOV) can be explained through above relationship with the corresponding KPIVs. The descriptive statistics for the characteristics in the above equation are given in Table 6.1.1.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>N</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Grain</td>
<td>41</td>
<td>7.0</td>
<td>12.8</td>
<td>10.40000</td>
<td>1.40943</td>
</tr>
<tr>
<td>Expected Hauteur</td>
<td>41</td>
<td>64.7</td>
<td>81.9</td>
<td>70.13902</td>
<td>3.20717</td>
</tr>
<tr>
<td>Expected Hauteur CV%</td>
<td>41</td>
<td>33.6</td>
<td>49.7</td>
<td>42.46829</td>
<td>4.12943</td>
</tr>
<tr>
<td>Actual Hauteur</td>
<td>41</td>
<td>72.7</td>
<td>86.8</td>
<td>79.11951</td>
<td>3.96889</td>
</tr>
</tbody>
</table>
Note that the above relationship is valid for the above-mentioned ranges of the independent variables – maximum grain, expected hauteur and expected hauteur CV%.

Figures 6.1.1 and 6.1.2 demonstrate how ‘Actual Hauteur’ and ‘Maximum Grain’ vary over different periods in a year. The effect of seasonality is depicted with the help of these figures.

![Figure 6.1.1. Variation of actual hauteur over different time periods in a year](image1)

![Figure 6.1.2. Variation of maximum grain over different time periods in a year](image2)

Since the objective is to maximize Actual Hauteur, the following formulation can be made subsequent to carrying out regression analysis:
Maximize Actual Hauteur = 47.713 + 0.734 × Maximum Grain + 0.663 × Expected Hauteur – 0.536 × Expected Hauteur CV%
Subject to

\[7.0 \leq \text{Maximum Grain} \leq 12.8\]
\[64.7 \leq \text{Expected Hauteur} \leq 81.9\]
\[33.6 \leq \text{Expected Hauteur CV\%} \leq 49.7\]

6.1.6 Conclusion

In order to maximize Actual Hauteur, the levels shown in Table 6.1.2 of the characteristics are obtained.

**Table 6.1.2.** Recommended levels of characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Recommended Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Grain</td>
<td>12.8 grams per cubic ft.</td>
</tr>
<tr>
<td>Expected Hauteur</td>
<td>81.9 mm</td>
</tr>
<tr>
<td>Expected Hauteur CV%</td>
<td>33.6%</td>
</tr>
</tbody>
</table>

The corresponding Actual Hauteur will be 93.3983 ± 4.1365 with 95% confidence level. Hence, the expected difference between Actual Hauteur and Expected Hauteur is 93.3983 – 81.9000 = 11.4983 mm. The existing difference between Actual Hauteur and Expected Hauteur is about 8.9805 mm. (vide Table 6.1.1.).

6.1.7 How to achieve a grain of 12.8 gms/cubic ft?

Since it is the general practice to measure and control the Grain or Relative Humidity % within the shop-floor with a Dry Bulb temperature combined with the difference between Dry Bulb and Wet Bulb temperature, Table 6.1.3 is derived from the hygrometric nomograms.
Table 6.1.3. Relation between atmospheric temperature and grain

<table>
<thead>
<tr>
<th>Dry Bulb Temperature ($^\circ$F)</th>
<th>Wet Bulb Temperature ($^\circ$F)</th>
<th>Grain (grams per cubic feet)</th>
<th>Relative Humidity %</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>86</td>
<td>12.8</td>
<td>92</td>
</tr>
<tr>
<td>89</td>
<td>86</td>
<td>12.8</td>
<td>88</td>
</tr>
<tr>
<td>91</td>
<td>87</td>
<td>12.9</td>
<td>84</td>
</tr>
<tr>
<td>92</td>
<td>87</td>
<td>12.9</td>
<td>81</td>
</tr>
<tr>
<td>93</td>
<td>87</td>
<td>12.6</td>
<td>78</td>
</tr>
<tr>
<td>94</td>
<td>87</td>
<td>12.5</td>
<td>75</td>
</tr>
<tr>
<td>95</td>
<td>88</td>
<td>12.8</td>
<td>75</td>
</tr>
<tr>
<td>96</td>
<td>88</td>
<td>12.8</td>
<td>72</td>
</tr>
</tbody>
</table>

6.1.8 Recommendation

For smooth working of the department, the following levels of dry bulb temperature, wet bulb temperature and relative humidity are to be maintained to obtain a Grain value of 12.8 grams per cubic feet.

Dry Bulb Temperature: 95$^\circ$F to 96$^\circ$F
Wet Bulb Temperature: 88$^\circ$F
Relative Humidity: 72% to 75%

Since the problem of a low top yield is at its greatest during the winter season, owing to a significant fall in both temperature and relative humidity, steam injection is suggested as a remedial measure to achieve the above levels.

It is to be ensured that the overseas supplier provides raw wool with an expected Hauteur length to the tune of 81.9 mm in a consistent manner so that expected Hauteur CV% is obtained in the vicinity of 33.6% or less. The past data in Table 6.1.1. demonstrate that these values are achievable.

The first two combinations of Table 6.1.3. are not suggested due to very high relative humidity (of the order of 88% to 92%) which is again unfavourable for processing of wool fibres.
6.1.9 Bottom line impact

The expected financial gain from this study is estimated based on the seasonal difference i.e. the difference between winter and other non-winter seasons, by the average difference between actual and expected top yield (see Tables 6.1.4 and 6.1.5).

**Table 6.1.4.** Average difference between actual and expected top yield in winter season

<table>
<thead>
<tr>
<th>Months</th>
<th>Actual top yield %</th>
<th>Expected top yield %</th>
<th>Difference in actual and expected top yield %</th>
<th>Average difference in actual and expected top yield %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov’ 00</td>
<td>95.32</td>
<td>93.68</td>
<td>1.64</td>
<td>1.656</td>
</tr>
<tr>
<td>Dec’ 00</td>
<td>95.61</td>
<td>93.95</td>
<td>1.66</td>
<td></td>
</tr>
<tr>
<td>Jan’ 01</td>
<td>95.25</td>
<td>93.58</td>
<td>1.67</td>
<td></td>
</tr>
</tbody>
</table>

**Table 6.1.5.** Average difference between actual and expected top yield in non-winter season

<table>
<thead>
<tr>
<th>Months</th>
<th>Actual top yield %</th>
<th>Expected top yield %</th>
<th>Difference in actual and expected top yield %</th>
<th>Average difference in actual and expected top yield %</th>
</tr>
</thead>
<tbody>
<tr>
<td>May’ 00</td>
<td>95.24</td>
<td>93.42</td>
<td>1.82</td>
<td>2.253</td>
</tr>
<tr>
<td>June’ 00</td>
<td>96.11</td>
<td>93.48</td>
<td>2.63</td>
<td></td>
</tr>
<tr>
<td>July’ 00</td>
<td>95.77</td>
<td>93.46</td>
<td>2.31</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, the difference between two seasons in average difference in actual and expected top yield % = 2.253 –1.656 = 0.597 = 0.6 (approximately).

Average monthly top production = 330,000 kg. So, 0.6% of 330,000 kg. = 1980 kg. The difference in selling price between Wool Top (i.e. good production) and Noil (i.e. waste) is about rupees 250 per kg. Hence, financial gain due to an increase in yield = 1980 × 250 = rupees 4.95 lakhs per month. Considering that there are three months (at times it extends to three and a half months) in the winter season in this region, the conservative estimate of financial gain per annum due to an increase in yield is 3 × rupees 4.95 lakhs = rupees 14.85 lakhs.

To maintain the relative humidity or grain at the prescribed level, steam injection is necessary in the wool combing department during the winter season, which spans about three months. The extent of steam consumption per day will be around 1000 kg for this purpose. The cost of steam generation is about rupees 0.58 per kg. Therefore, the total
expenditure to be incurred per year for steam injection is rupees $1000 \times 0.58 \times 30 \times 3 = 52\,200$ which is rupees 0.522 lakhs.

Hence, the net expected tangible gain from the study = rupees $(14.85 - 0.522)$ lakhs = rupees 14.328 lakhs (approximately).
Appendix 6.1.1. Model adequacy checks for regression analysis

Normal P-P Plot of Standardized Residual

Dependent Variable: Actual Hauteur

![Normal P-P Plot]

Scatterplot

Dependent Variable: ACTUAL HAUTEUR

![Scatterplot]
6.2 Title: Controlling Resistance for Improved Performance of Telecommunication Cables

6.2.1 Motivation for this work

Controlling electrical parameters such as variation in resistance and resistance imbalance is an important part of achieving trouble-free performance of telecommunication systems. Incorrect setting and high variation with regard to these parameters have far-reaching consequences, in the sense that high resistance causes attenuation, low resistance results in higher consumption of copper, and resistance imbalance generates noise. It may be relevant to note here that the resistance imbalance (RI%) between two conductors is given by:

$$RI\% = \left(\frac{|R_1 - R_2|}{R_1 + R_2}\right) \times 100$$

where $R_1$ and $R_2$ are the resistances of two conductors, and resistance is expressed in ohms per km at a temperature of 20°C.

In a well-known cable-manufacturing company in India, at the final inspection stage, the amount of rejection due to the above-mentioned electrical failures was around ten percent, of which six per cent was contributed by resistance alone. This amounted to high losses of quality and hence it was of interest to study how to reduce this failure.

In any continuous manufacturing process, the overall variability with respect to any quality characteristic is contributed to by the variabilities at each stage of the process. Discussion with the technicians concerned led to attention being focused on the tandem and twinning stages, to overcome the high levels of electrical failure of the telecommunication cable.

6.2.2 Objective

The objective of the study was to identify the stage of the process, which was most responsible for variation in resistance and resistance imbalance, and to evolve a workable range for the process parameters of this contributory stage, so that the extent of nonconformance at the final inspection stage due to the above-mentioned failure would be reduced.

6.2.3 The process

The flow charts for the overall process, the tandem process and the twinning process are given below.
6.2.3.1 The overall process

- PICKLING
- ROD BREAKDOWN
- TANDEM
- TWINNING
- DRUM TWISTER
- JELLY FILLING
- SHEATHING
- ARMOURING
- JACKETING
- FINAL CABLE
6.2.3.2 The tandem process

1. Copper wire of 2.5 mm diameter
2. Wire drawing by series of dies to required diameter
3. Annealing
4. Preheating
5. Insulation
6. Cooling by water
7. Winding onto tandem bobbin
8. Bobbin containing insulated conductor wire
6.2.3.3 The twinning process

TANDEM BOBBIN CONTAINING 31 KM OF INSULATED CONDUCTOR WIRE

TWINNING AT SET LAY OR TWIST

BOBBIN CONTAINING 10.15 KM TWINNED CONDUCTOR WIRE
6.2.4 Data and measurement

At the outset data on resistance were collected at the twinning stage. There were five twinning machines. Each twinning machine has two sides – the left side and the right side. Corresponding to each side of a machine both input resistance and output resistance were recorded maintaining one to one correspondence. Nine observations were recorded separately for input resistance and output resistance corresponding to each side of a twinning machine. Therefore, $9 \times 2 \times 2 \times 5 = 180$ measurements were taken for resistance. In order to measure resistance, one metre sample of wire was collected from each bobbin subsequent to removing the initial 4 to 5 metres conductor wire. For each one metre sample, resistance was measured at room temperature by Wheatstone Bridge principle. Thereafter, it was expressed in Ohms per Kilometre at $20^0$ C by adopting the following formula:

$$R_{20} = \frac{R \text{ at room temperature}}{1 + 0.00393(\text{room temperature} - 20)}$$

Data were also collected on resistance at the tandem stage and the corresponding process parameters i.e. line speed, annealing voltage, preheating voltage, insulation temperature or head-2 temperature. At 32 random time points such data were collected maintaining one to one correspondence between resistance and the corresponding parameters.

6.2.5 Analysis

6.2.5.1 Analysis of covariance

The technique – analysis of covariance (ANCOVA) was used for identifying the contributory stage of the process for resistance related problems. ANCOVA was carried out with the observations on resistance after twinning (Output Resistance) considering the corresponding observations on resistance after tandem (Input Resistance) as concomitant variable. The following model of nested ANCOVA was considered for analysis:

$$Y_{ijk} = \mu + \alpha_i + \gamma_{(j)(i)} + \beta(X_{ijk} - \bar{X}_{(i)00}) + e_{k(i)} \quad i=1(1)9, \ j=1(1)2, \ k=1(1)9$$

Where $Y_{ijk}$ = Output Resistance corresponding to $k$th observation for $j$th side of the $i$th Twinning machine

$\mu$ = Overall Mean

$\alpha_i$ = The effect due to the $i$th machine

$\gamma_{(j)(i)}$ = The effect due to the $j$th side of the $i$th machine
\[ \beta = \text{Regression Coefficient of } Y \text{ on } X \]

\[ X_{ijk} = \text{Input Resistance corresponding to } k\text{th observation for the } j\text{th side of the } i\text{th Twinning machine} \]

\[ \bar{X}_{000} = \frac{\sum_{i=1}^{5} \sum_{j=1}^{2} \sum_{k=1}^{9} X_{ijk}}{5 \times 2 \times 9} \]

\[ e_{ki(j)} = \text{Random error component with NID } (0, \sigma^2) \]

The above model is a fixed effect nested ANCOVA model with the following assumptions [Huitema (37)]

- Significance of regression coefficient
- Statistical independence of covariate and treatment
- Homogeneity of within group regression
- Normality of error component
- Homogeneity of conditional variance of Output Resistance

The computational details and tests of the above assumptions for ANCOVA are given in Appendix 6.2.1. The following table furnishes the results of the nested ANCOVA.

**Table 6.2.1. Results of nested ANCOVA**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Square</th>
<th>Adjusted for Regression</th>
<th>F0</th>
<th>Tab F (5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>XX</td>
<td>YX</td>
<td>YY</td>
<td></td>
</tr>
<tr>
<td>Machine</td>
<td>4</td>
<td>3.0057</td>
<td>1.8792</td>
<td>1.6569</td>
<td></td>
</tr>
<tr>
<td>Side Within Machine</td>
<td>5</td>
<td>2.1122</td>
<td>0.7898</td>
<td>2.2817</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>80</td>
<td>24.3633</td>
<td>18.3919</td>
<td>36.9026</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>89</td>
<td>29.4812</td>
<td>21.0609</td>
<td>40.8412</td>
<td></td>
</tr>
<tr>
<td>Adjusted Side Within Machine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine + Error</td>
<td>84</td>
<td>27.3689</td>
<td>20.2711</td>
<td>38.5595</td>
<td></td>
</tr>
<tr>
<td>Adjusted Machine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
To investigate the effect of the concomitant variable, the F value obtained was 47.65. The corresponding tabulated F is 3.97 with $\alpha = 5\%$ with DF 1, 79. It revealed the significant effect of the input resistance i.e. the resistance after the tandem stage on the output resistance i.e. the resistance after the twinning stage.

### 6.2.5.2 Regression analysis

In order to control the resistance after the tandem operation, a relationship was established between the process parameters and the corresponding resistance by the stepwise regression method. SPSS package was used for this purpose. Stepwise regression, with a quadratic relationship between resistance and the process parameters concerned, generated a regression equation involving three process parameters with multiple $R^2$ as 0.7864, as follows:

$$\hat{R} = -24.151349 + 0.132372 \text{ AV} - 0.170549 \text{ PV} + 0.935788 H_2 - 0.002026 H_2^2$$

where $\hat{R}$ = estimated resistance after tandem operation is the KPOV  
$AV$ = annealing voltage is the KPIV  
$PV$ = preheating voltage is the KPIV  
$H_2$ = head-2 temperature is the KPIV

with multiple $R^2 = 0.78$  
adjusted $R^2 = 0.75$

and standard error of estimate = 0.23 (ohm)

Table 6.2.2. Results of stepwise regression

<table>
<thead>
<tr>
<th>Step No.</th>
<th>Variable Entered</th>
<th>Multiple $R^2$</th>
<th>Adjusted $R^2$</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$H_2^2$</td>
<td>0.33277</td>
<td>0.31053</td>
<td>0.39515</td>
</tr>
<tr>
<td>2</td>
<td>$AV$</td>
<td>0.55810</td>
<td>0.52762</td>
<td>0.32707</td>
</tr>
<tr>
<td>3</td>
<td>$H_2$</td>
<td>0.70825</td>
<td>0.67699</td>
<td>0.27046</td>
</tr>
<tr>
<td>4</td>
<td>$PV$</td>
<td>0.78637</td>
<td>0.75472</td>
<td>0.23469</td>
</tr>
</tbody>
</table>

Table 6.2.3. ANOVA for regression

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>$F_0$</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>4</td>
<td>5.52060</td>
<td>1.38015</td>
<td>24.846</td>
<td>Significant at 1% level</td>
</tr>
<tr>
<td>Residual</td>
<td>27</td>
<td>1.49980</td>
<td>0.05555</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>7.02040</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The adequacy checks for the regression model (normal probability plot of residuals and predicted resistance versus residual plot) were found to be satisfactory.

### 6.2.5.3 Mathematical programming

After identifying the relationship between \( \hat{R} \) and the process parameters at the tandem stage, workable ranges were determined for the latter to satisfy the specification of resistance after the tandem operation \((86 \pm 1\text{-ohm})\). The following steps were used to determine the workable ranges.

**Step 1**

Let \( \tilde{X} \bullet = (AV \bullet, PV \bullet, H_2 \bullet) \) be the solution of the following problem

\[
\text{Min} \left[ \hat{R}(\tilde{X}) - T \right]^2
\]

subject to

\[
\hat{R}(\tilde{X}) \leq U - 3\hat{\sigma} \\
\hat{R}(\tilde{X}) \geq L + 3\hat{\sigma}
\]

\( \tilde{X} \) is the area of exploration

where \( \hat{R}(\tilde{X}) \) is the regression equation

- \( T \) is the target value for resistance
- \( U \) is the USL for resistance
- \( L \) is the LSL for resistance
- \( \hat{\sigma} \) is the standard error of the estimate

\[
\tilde{X} = (AV, PV, H_2)
\]

**Step 2**

Varying \( \tilde{X} \) around \( \tilde{X} \bullet \) with a suitable skip and checking corresponding \( \hat{R}(\tilde{X}) \) with USL and LSL, the workable range for \( \tilde{X} \) was obtained. With the help of the IMSL package [IMSL (38)], \( \tilde{X} \bullet \) was obtained as \( AV \bullet = 25, PV \bullet = 7, H_2 \bullet = 230 \); corresponding \( \hat{R}(\tilde{X} \bullet) = 85.997 \).

The workable range was obtained as follows:

- \( AV = (24 - 28) \)
- \( PV = (7 - 8.5) \)
- \( H_2 = (228 - 234) \)
6.2.6 Conclusion

- From nested ANCOVA it was quite evident that the twinning stage could be ruled out as a source of variation in resistance and resistance imbalance, as there were no significant differences between twinning machines and between the sides within the machines.

- The significance of the concomitant variable in the nested ANCOVA revealed that the variation in resistance at the tandem stage was an important area to investigate.

- From the relationship between the process parameters and resistance at the tandem stage, it was quite clear that annealing voltage, preheating voltage and head-2 temperature made a significant contribution to resistance, in the area being explored. The above finding was ratified by technological consideration. The other parameter considered for study, i.e. line speed, had no significant effect on resistance in this area.

6.2.7 Recommendation for process control

Variation in resistance at the tandem stage was identified as the major contributory factor to the problem of nonconformance of resistance and resistance imbalance in the final cable. Variation could be reduced by maintaining the process parameters at the tandem stage as follows:

- annealing voltage: (24 – 28) volts
- preheating voltage: (7 – 8.5) volts
- head-2 temperature: (228 – 234)° centigrade

6.2.8 Improvements achieved

The above recommendation has been implemented. A significant improvement has been noticed in the percentage of nonconformance of resistance at the final stage. Initially it was six per cent and after implementation it has been reduced to two per cent.

As high variation in resistance increases resistance imbalance, it is expected that, by exercising proper control over resistance, the levels of failure to meet specification due to resistance imbalance will also be reduced.
Appendix 6.2.1. Computational details and tests for ANCOVA assumptions

\[
A_{xx} = \sum_{i=1}^{5} \frac{x_{i00}^2}{2 \times 9} - \frac{x_{000}^2}{5 \times 2 \times 9} = 3.00567
\]

\[
A_{yy} = \sum_{i=1}^{5} \frac{y_{i00}^2}{2 \times 9} - \frac{y_{000}^2}{5 \times 2 \times 9} = 1.65693
\]

\[
A_{xy} = \sum_{i=1}^{5} \frac{x_{i00} y_{i00}}{2 \times 9} - \frac{x_{000} y_{000}}{5 \times 2 \times 9} = 1.87918
\]

\[
B_{xx} = \sum_{i=1}^{5} \sum_{j=1}^{2} \frac{x_{ij0}^2}{9} - \frac{1}{2 \times 9} \sum_{i=1}^{5} x_{i00}^2 = 2.11206
\]

\[
B_{yy} = \sum_{i=1}^{5} \sum_{j=1}^{2} \frac{y_{ij0}^2}{9} - \frac{1}{2 \times 9} \sum_{i=1}^{5} y_{i00}^2 = 2.28167
\]

\[
B_{xy} = \sum_{i=1}^{5} \sum_{j=1}^{2} \frac{x_{ij0} y_{ij0}}{9} - \frac{1}{2 \times 9} \sum_{i=1}^{5} x_{i00} y_{i00} = 0.78978
\]

\[
S_{xx} = \sum_{j=1}^{2} \sum_{k=1}^{9} x_{jk}^2 - \frac{1}{5 \times 2 \times 9} \sum_{i=1}^{5} x_{i00}^2 = 29.48116
\]

\[
S_{yy} = \sum_{j=1}^{2} \sum_{k=1}^{9} y_{jk}^2 - \frac{1}{5 \times 2 \times 9} \sum_{i=1}^{5} y_{i00}^2 = 40.84129
\]

\[
S_{xy} = \sum_{j=1}^{2} \sum_{k=1}^{9} x_{jk} y_{jk} - \frac{1}{5 \times 2 \times 9} \sum_{i=1}^{5} x_{i00} y_{i00} = 21.06085
\]

\[
E_{xx} = S_{xx} - A_{xx} - B_{xx} = 24.36327
\]

\[
E_{yy} = S_{yy} - A_{yy} - B_{yy} = 36.9026
\]

\[
E_{xy} = S_{xy} - A_{xy} - B_{xy} = 18.39189
\]

\[
SSE = E_{yy} - \frac{E_{xy}^2}{E_{xx}}
\]

\[
SSE' = (E_{xy} + B_{yy}) - \frac{(E_{xy} + B_{yy})^2}{(E_{xx} + B_{xx})}
\]
\[ \text{SSE}' = \left( E_{yy} + A_{yy} \right) - \left( \frac{E_{xy} + A_{xy}}{E_{xx} + A_{xx}} \right)^2 \]

To test \( H_0 : \gamma_{j(i)} = 0 \forall j \) for fixed \( i \)

Against \( H_1 : \gamma_{j(i)} \neq 0 \) for at least one \( j \) for fixed \( i \)

The following statistic was used

\[ F = \frac{(\text{SSE}' - \text{SSE})/(2-1)}{\text{SSE}/(90-10-1)} = 1.56 \]

To test \( H_{02} : \alpha_i = 0 \forall i \)

Against \( H_{12} : \alpha_i \neq 0 \) for at least one \( i \)

The following \( F \) statistic was used

\[ F = \frac{(\text{SSE}' - \text{SSE})/4}{\text{SSE}/(90-10-1)} = 0.45 \]
## Computational procedure for the nested ANCOVA table

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Square</th>
<th>Adjusted for Regression</th>
<th>$F_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>XX</td>
<td>XY</td>
<td>YY</td>
</tr>
<tr>
<td>Machine</td>
<td>4</td>
<td>$A_{xx}$</td>
<td>$A_{xy}$</td>
<td>$A_{yy}$</td>
</tr>
<tr>
<td>Side</td>
<td>5</td>
<td>$B_{xx}$</td>
<td>$B_{xy}$</td>
<td>$B_{yy}$</td>
</tr>
<tr>
<td>Within Machine</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>80</td>
<td>$E_{xx}$</td>
<td>$E_{xy}$</td>
<td>$E_{yy}$</td>
</tr>
<tr>
<td>Total</td>
<td>89</td>
<td>$S_{xx}$</td>
<td>$S_{xy}$</td>
<td>$S_{yy}$</td>
</tr>
<tr>
<td>Side</td>
<td>85</td>
<td>$E_{xx} + B_{xx}$</td>
<td>$E_{xy} + B_{xy}$</td>
<td>$E_{yy} + B_{yy}$</td>
</tr>
<tr>
<td>Within Machine</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted Machine</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine + Error</td>
<td>84</td>
<td>$E_{xx} + A_{xx}$</td>
<td>$E_{xy} + A_{xy}$</td>
<td>$E_{yy} + A_{yy}$</td>
</tr>
<tr>
<td>Adjusted Machine</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SSE' - SSE = 84 - 79

MSE' = MSE

SSE'' - SSE = 83 - 79

MSE'' = MSE
II. Tests for assumptions of ANCOVA model:

(i) To test significance of $\beta$ the following $F$ statistic was used

$$F = \frac{E_{xy}^2 / E_{xx}}{SSE/79}$$

(ii) Since from technical point of view it is quite obvious that twinning machine cannot influence the resistance before twinning operation, independence of covariate (input resistance) and treatments (twinning machines and sides within machine) can be assumed. So no statistical test is provided for this assumption.

(iii) To test homogeneity of within group regression the test procedure was adopted like this. Suppose the within group residual sum of square = SSE. The sum of squares for residuals for each machine side combination was obtained as $C_{yy} = \frac{C_{xy}^2}{C_{xx}}$.

Where $C_{yy} = \sum_k Y_{ijk}^2 - \left( \frac{\sum_k Y_{ijk}}{9} \right)^2$ \quad $C_{xx} = \sum_k X_{ijk}^2 - \left( \sum_k X_{ijk} \right)^2 / 9$

$$C_{xy} = \sum_k X_{ijk} Y_{ijk} - \left( \sum_k X_{ijk} \sum_k Y_{ijk} \right) / 9$$

Now summing over the residuals of 10 machine side combinations pooled individual residual sum of squares ($SS_{resi}$) was obtained. To test the homogeneity of regression the following $F$ statistic was used

$$F = \frac{\left( SSE - SS_{resi} \right) / 9}{SS_{resi} / 70} = 2.18$$

(iv) To test homogeneity of conditional variance for each group, the test procedure was adopted like this.

$$S_{y/x}^2 = \frac{(1 - r_w^2) C_{yy}}{n_k - C - 1}$$

where $r_w = \frac{E_{xy}}{\sqrt{E_{xx} E_{yy}}}$

$n_k = 9$, $C = \text{number of covariates} = 1$. The test statistic $F = \frac{\text{Max} S_{y/x}^2}{\text{Min} S_{y/x}^2}$ follows Bonferroni’s $F$ statistic $F_{C', 7, 7}$ where $C' = 45$. The computed $F = 2.536$ and the tabulated $F$ at $\alpha = 0.1$ is 14.505. For a test at level $\alpha$ experimentwise, the critical value is $F_B(\alpha/2, C', n_{\text{largest}} - C, n_{\text{smallest}} - C)$ where $\alpha/2$ is one half the desired experimentwise $\alpha$ level and $C'$ is $[10(10-1)]/2$, which is the “number of comparisons,” and 10 is the number of groups. If the obtained statistic $F_B$ obtained is equal to or exceeds the critical value of $F_B$, the
hypothesis of equal conditional variances $H_0$: $\sigma^2_{y_{1/x}} = \sigma^2_{y_{2/x}} = \sigma^2_{y_{3/x}} = \ldots \sigma^2_{y_{10/x}}$ is rejected [Huitema (37)].
6.3 Title: Reduction of Carbon Percentage of Hot Metal Produced by a Mini Blast Furnace

6.3.1 Motivation for this work

The study pertains to a ductile iron (DI) plant in India. The plant manufactures DI pipes of various sizes ranging from 80 to 1000 mm. The hot metal is prepared in a Mini Blast Furnace (MBF) for this DI pipe manufacture. One of the vital chemical constituents for this hot-metal preparation is the carbon percentage in hot metal [C%]. The specification for [C%] is supposed to be within 3.6% to 4.0% for obtaining a smooth flow of production. The problem is of producing hot metal, containing [C%] higher than the upper specification limit. The adverse effect of high [C%] is twofold.

First, it results in brittleness in the ultimate product – the DI pipe. Second, it causes environmental pollution in the form of emission of shiny and tiny carbon particles to the atmosphere. It may be worthwhile to note here that these silverlike shiny carbon particles are formed due to a sudden temperature drop which takes place when the carbon particle along with the MBF fume, having a very high temperature of the order of 1300-1400°C, suddenly come into contact with the atmosphere.

The first adverse effect of a high [C%] (i.e., generation of brittleness in DI pipes) is accorded priority by the plant personnel because it is directly related to its core business area of manufacturing quality DI pipes. Keeping that in mind, the practice adopted by the plant is to add scrap to the induction furnace or Mini Hill Furnace (MHF), the next stage of hot-metal preparation after MBF. Needless to say, the very purpose of this scrap addition is to reduce [C%] so that brittleness is avoided. As a matter of fact, even though the scrap addition is done with the intention of avoiding brittleness, it is undesirable because it achieves quality by compromising cost of production and productivity. The cost of production and productivity are compromised as follows. The addition of metal scrap reduces the hot-metal temperature at the induction furnace or MHF. To compensate for such a loss in temperature, extra heating is required for additional time. Because of this extra heating, the cost of production increases, and because of the extra time required, the productivity gets hampered. Because the MBF plant was newly commissioned at the time of carrying out this study, adequate control did not exist in the process due to the ad hoc operational approach. This study is an exploratory one to arrive at the process control features.

The study is undertaken with a view to finding feasible and beneficial ways and means of reducing [C%] in the hot metal produced by MBF.
6.3.2 The process flowchart for ductile iron pipe manufacture

- Mini Blast Furnace
- Induction Furnace
- Channel Furnace

- Mg Converter → Pipe Casting
- Annealing Furnace
- Zn Coating on Outer Surface → Sample Cutting for Mechanical Test

- Mg Converter
- Pipe Casting
- Annealing Furnace
- Zn Coating on Outer Surface

- Mg Converter → Mg Converter
- Pipe Casting
- Sample Cutting for Mechanical Test

- Socket Barrel Grinding → Hydro Testing
- Cutting & Rectification
- Weighing & Visual Inspection
- Cement Lining in Inner Surface

- Bitumen Coating on Outer Surface
- Cement Lining Rectification
- Stenciling
- Stenciling
- Stock Yard
- Customer
6.3.3 Background information

The magnitude of the problem of high [C%] in the hot metal produced by MBF can be conceived of from the existing distribution of [C%] provided in Table 6.3.1. It may be worthwhile to recall here that the specification of [C%] is 3.6% to 4.0%. It can be observed from Table 6.3.1 that about 89% of the tapping subsequent to MBF operation contain carbon above 4%.

Table 6.3.1. Frequency distribution of [C%] at MBF

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>Frequency</th>
<th>Cumulative Frequency (greater than type)</th>
<th>% Cumulative Relative Frequency (greater than type)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.555 – 3.645</td>
<td>2</td>
<td>270</td>
<td>100.00</td>
</tr>
<tr>
<td>3.645 – 3.735</td>
<td>0</td>
<td>268</td>
<td>99.26</td>
</tr>
<tr>
<td>3.735 – 3.825</td>
<td>6</td>
<td>268</td>
<td>99.26</td>
</tr>
<tr>
<td>3.825 – 3.915</td>
<td>10</td>
<td>262</td>
<td>97.04</td>
</tr>
<tr>
<td>3.915 – 4.005</td>
<td>11</td>
<td>252</td>
<td>93.34</td>
</tr>
<tr>
<td>4.005 – 4.095</td>
<td>35</td>
<td>241</td>
<td>89.26</td>
</tr>
<tr>
<td>4.095 – 4.185</td>
<td>66</td>
<td>206</td>
<td>76.30</td>
</tr>
<tr>
<td>4.185 – 4.275</td>
<td>103</td>
<td>140</td>
<td>51.85</td>
</tr>
<tr>
<td>4.275 – 4.365</td>
<td>37</td>
<td>37</td>
<td>13.70</td>
</tr>
<tr>
<td>Total</td>
<td>270</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.3.4 Data

At the outset, 270 data sets are collected on the chemical composition after MBF operation. These observations are the outcome of spectrographic analysis of the MBF samples. Chemical compositions of C%, Si%, Mn%, S%, and P% of MBF metal are thus found.

Subsequently, 38 data sets are collected on the MBF shop floor on Si% of a charge mix and the corresponding process parameters [i.e., hot blast temperature (HBT), hot blast volume (HBV), and iron to carbon ratio (Fe/C)].

In addition, 30 data sets are collected on S% of a charge mix and the relevant process parameters [i.e., sulfur loading (amount of S in the charge mix from coke), slag basicity (CaO/SiO$_2$ ratio in slag), slag volume, slag temperature, MgO% in slag].

The process parameters are noted down corresponding to a charge mix from the control panel and then the MBF metal sample is sent to the laboratory for spectrographic analysis of Si% or S%.

6.3.5 Analysis and result

Because the objective of the study is to obtain feasible and beneficial ways and means of reducing [C%], the relationship between [C%] and other chemical constituents (viz. [Si%], [S%], [Mn%], [P%]) was examined at the existing techno-economic setup.

The technique adopted for the purpose was stepwise regression. The SPSS package was used for this purpose. The best relation arrived at was between [C%] versus [Si%] and [S%]. The number of observations considered for the purpose was 270. The relationship used was

\[
[C\%] = 4.453 - 0.054[S\%]^2 - 32.656[S\%]^2
\]

with adjusted $R^2 = 0.65$ and standard error (S.E.) = 0.076. Relation (1) holds under the following conditions:

- Normal operation of the MBF
- Variation of [P%] between 0.17% and 0.83%
- Variation of [Mn%] between 0.02% and 0.08%
- Variation of [Si%] between 0.99% and 4.40%
- Variation of [S%] between 0.01% and 0.13%

It is obvious from Eq. (1) that the relation between [C%] versus [Si%] and [C%] versus [S%] is inverse in nature. This implies that if [Si%] and [S%] increase within the ranges of 0.99 – 4.40% and 0.01 – 0.13%, respectively, [C%] decreases and vice versa.
Because the objective is to control $[C\%]$ on the lower side (in particular within the range 3.6 – 4.0%), it is clear from relation (1) that this is achievable by adjusting both $[Si\%]$ and $[S\%]$ on the higher side. This task of finding the optimum composition of $[Si\%]$ and $[S\%]$ corresponding to a target $C\%$ of 3.8% is performed by the nonlinear programming method with the help of an IMSL package [IMSL (38)]. The formulation of the problem is

\[
\begin{align*}
\text{Minimize} & \quad (4.453 - 0.054[Si\%]^2 - 32.656[S\%]^2 - 3.8)^2 \\
\text{Subject to} & \quad 4.453 - 0.054[Si\%]^2 - 32.656[S\%]^2 = 3.8
\end{align*}
\]

The optimum composition of $[Si\%]$ and $[S\%]$ is $[Si\%] = 2.951\%$ and $[S\%] = 0.076\%$. It is to be noted that these optimum compositions remain within the existing operating zones of $[Si\%]$ and $[S\%]$. It may be recalled that the existing operating zone of $[Si\%]$ is 0.99 – 4.40% and that of $[S\%]$ is 0.01 – 0.13%.

Having found the optimum composition of $[Si\%]$ and $[S\%]$ for reducing $[C\%]$, the next logical step is to establish a relation among $[Si\%]$, $[S\%]$, and the corresponding process parameters. This task is also accomplished by resorting to stepwise regression with the help of an SPSS package. The relationship is

\[
[Si\%] = 26.540 - 20.147(Fe/C) + 8.153(Fe/C)^2 - 0.011(Fe/C)HBT
\]  
(2)

with adjusted $R^2 = 0.67$, standard error = 0.189, and number of observations = 38. The above relationship holds good for the following conditions:

- Normal operation of the MBF
- Variation of HBT within 740 – 760\(^0\)C
- Variation of Fe/C within 1.37 – 1.76
- Variation of HBV within 19,430 – 30,070

\[
[S\%] = 0.062 + 0.040(\text{sulfur loading}) - 0.189(\text{slag basicity})
\]  
(3)

with adjusted $R^2 = 0.72$, standard error = 0.011, and number of observations = 30. Relation (3) holds for the following conditions:

- Normal operation of the MBF
- Variation of sulfur loading within 3.67 kg/THM (tons of hot metal) to 4.42 kg/THM
- Variation of slag basicity within 0.76 to 1.07
- Variation of slag volume within 225.3 kg/THM to 274.1 kg/THM
- Variation of theoretical slag temperature within 1518\(^0\)C to 1571\(^0\)C

The optimum levels of the process parameters Fe/C and HBT corresponding to Eq. (2) for meeting the target $[Si\%]$ of 2.951 are determined by solving the following nonlinear programming (NLP) with the help of an IMSL package [IMSL (38)].

\[
\begin{align*}
\text{Minimize} & \quad [26.540 - 20.147(Fe/C) + 8.153(Fe/C)^2 - 0.011(Fe/C)HBT - 2.951]^2 \\
\text{Subject to} & \quad 26.540 - 20.147(Fe/C) + 8.153(Fe/C)^2 - 0.011(Fe/C)HBT = 2.951
\end{align*}
\]
The optimum level of Fe/C and HBT is Fe/C = 1.40 and HBT = 747.5. It is to be noted that these optimum levels remain within the existing operating zones of Fe/C and HBT. It may be recalled that the existing operating zone of Fe/C is 1.37 – 1.76 and that of HBT is 740 – 760°C.

The optimum levels of the process parameters sulfur loading and slag basicity corresponding to Eq. (3) for meeting the target [S%] of 0.076 are determined by solving the following linear programming problem (LPP) [Taha (94)]:

\[
\text{Minimize } (0.062 + 0.040(\text{sulfur loading}) - 0.189(\text{slag basicity}) - 0.076) \\
\text{Subject to } 3.67 \leq \text{sulfur loading} \leq 4.42 \\
\quad 0.76 \leq \text{slag basicity} \leq 1.07 \\
0.062 + 0.040(\text{sulfur loading}) - 0.189(\text{slag basicity}) = 0.076
\]

The optimum level of sulfur loading and slag basicity is sulfur loading = 3.92 and slag basicity = 0.76.

The adequacy checks for the regression model (normal probability plot of residuals and predicted versus residual plot) were found to be satisfactory [Mukhopadhyay, A. R. (66)]. The adequacy checks – normal probability plots of error and predicted response versus error plots – were checked as suggested by [Draper and Smith (26)].

6.3.6 Conclusion

- Because the desirable range of [C%] is 3.6 – 4.0%, the target for [C%] is 3.8%. To meet the target [C%] of 3.8%, [Si%] should be maintained at 2.951% and [S%] should be maintained at 0.076%.

- To obtain [Si%] at 2.951%, the Fe/C ratio should be maintained at 1.40 and HBT should be maintained at 747.5°C.

- To obtain [S%] at 0.076%, sulfur loading should be maintained at 3.92 and slag basicity should be maintained at 0.76.

6.3.7 Improvements achieved after implementation

Subsequent to implementing the recommendations, the distribution of [C%] of MBF hot metal is found. Comparison with the distribution of [C%] after scrap addition at the MHF, the practice adhered to earlier to reduce [C%] in the hot metal produced by the MBF reveals clearly that with respect to both the mean and standard deviation, the recommendations resulted in statistically significant lower values.
Table 6.3.2. Comparison of distribution of [C%] at the MBF after implementation with that of [C%] at the MHF before implementation following the practice of scrap addition

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>[C%] at MHF Before Implementation</th>
<th>[C%] at MBF after Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Cumulative Frequency (less than type)</td>
</tr>
<tr>
<td>3.585-3.645</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>3.645-3.705</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>3.705-3.765</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>3.765-3.825</td>
<td>41</td>
<td>66</td>
</tr>
<tr>
<td>3.825-3.885</td>
<td>66</td>
<td>132</td>
</tr>
<tr>
<td>3.885-3.945</td>
<td>20</td>
<td>152</td>
</tr>
<tr>
<td>3.945-4.005</td>
<td>1</td>
<td>153</td>
</tr>
<tr>
<td>4.005-4.065</td>
<td>1</td>
<td>154</td>
</tr>
<tr>
<td>Minimum</td>
<td></td>
<td>3.59</td>
</tr>
<tr>
<td>Maximum</td>
<td></td>
<td>4.06</td>
</tr>
<tr>
<td>( \bar{X} )</td>
<td></td>
<td>3.8234</td>
</tr>
<tr>
<td>( \sigma_{n-1} )</td>
<td></td>
<td>0.0714</td>
</tr>
</tbody>
</table>

Consequently, the earlier practice of scrap addition at the MHF was discarded. The departmental procedure and work instruction with regard to hot-metal procurement from the MBF to the MHF were modified accordingly.

### 6.3.8 Bottom-line impact

It is to be noted that before implementation, 89% of the tapping subsequent to the MBF operation contained [C%] above 4%. Iron scrap used to be added to the MHF with a view to reducing [C%] by 10% of a day’s production of about 290 metric tons (MT). The cost of scrap is about Rs 6000/MT. Hence, 10% scrap addition per day implies an extra Rs 174,000 (Rs 6000 × 29) per day.

Again, for 10% addition of scrap, loss in temperature used to take place at 65°C per MT at the MHF. It is to be noted that 100°C per MT loss in temperature implies an additional consumption of 45 units of electricity. Because the cost of 1 unit of electricity was about Rs 4 during the period of conducting this study, the cost of additional electricity consumption for this scrap addition is (Rs 4 × 65 × 45/100) per MT = Rs 117/MT (i.e., Rs 117 × 290 per day = Rs 33,930 per day).

Hence, total cost incurred for reduction of [C%] is Rs 207,930 per day (approximately) (Rs 174,000 + Rs 33,930 per day). The implementation of the above-mentioned recommendations resulted in a cost savings of Rs 207,930 per day (approximately).

It may be worthwhile to mention here that the concerned technicians were thinking of installing steam injection methodology as an alternative to the practice of scrap addition.
at the MHF for reducing [C%]. However, to facilitate steam injection, the HBT has to be raised from 750 ± 10°C to 950 ± 10°C. In order to do this, one would have to change the heating system from the metallic blast preheater (MBP) to stoves. This change itself would cost around Rs 4 million. The total additional cost for steam injection will be around Rs 120 million, including the cost of conversion from the MBP to stoves.

6.3.9 Cross-linkage with chapters 7 and 8

This chapter (6) described how one used to relate key process output variables with corresponding key process input variables in manufacturing processes by applying regression analysis and subsequently how one used to optimize the regression equations by applying the relevant mathematical programming techniques without consciously following DMAIC or other steps for six sigma project management before the advent of six sigma.

Chapter 8 describes how improvement in manufacturing processes with similar methodology can be achieved by consciously following DMAIC steps after the advent of six sigma. This systematic and structured approach of problem solving in a real life situation helps the project team members immensely to think in the same direction. However, the very next chapter (chapter 7) discusses how one can improve the manufacturing processes and simultaneously holds the gain by applying appropriate process management or process control techniques in various situations. Here the emphasis is primarily on holding the gain.