Chapter 2

Penetration depth in a mixed wave superconductor

We have calculated the penetration depth in a mixed wave superconductor. At low temperatures the ground state has $d_{2\Sigma}$ symmetry which at elevated temperatures becomes $x-y$ of the type of $s$ wave. We have used the experimental measurements of the surface resistance of $Bi_2Sr_2CaCu_2O_{8+\delta}$ crystals of $T_c \sim 52K$ at microwave frequencies to extract the London penetration depth as a function of temperature. It is found that the experimental data is consistent with the interpretation of $d_{2\Sigma}$ wave at low temperatures becoming $s$ wave at high temperatures. The $d_{2\Sigma} + is$ wave is estimated to start changing over to $s$ at a temperature of $\sim 40K$. The theoretically calculated values of the penetration depth as a function of temperature are in reasonable agreement with those extracted from the experimental measurements.
In this chapter, we calculate the penetration depth for a system of mixed wave symmetry, \( d_{x^2 - y^2} \) ground state. The calculation is performed for a system which has \( d_{x^2 - y^2} \) at low temperatures and \( s \) wave symmetry at high temperatures. The experimentally deduced values from the surface resistivity of \( Bi_2Sr_2CaCu_2O_{8+\delta} \) crystals agree with the mixed wave interpretation.

The diamagnetic current is given by

\[
j_D = -\left(\frac{e^2}{mc}\right) \sum_{k,q,\sigma} c_{k+q,\sigma}^\dagger c_{k,\sigma} e^{-iq\cdot x} A(x)
\]

so that the problem of determining the London penetration depth [1-3] is reduced to that of the conversion of summation into integration by the use of appropriate density of states [3] and multiplying the gap \( A \) by the factor \( / \), where

\[
\begin{align*}
\alpha &= 1 \\
\beta &= \cos 2\phi \\
\gamma &= \sin 2\phi
\end{align*}
\]

The density of states can be described in terms of the complete elliptic integral so that the London penetration depth [4] is found to be,

\[
\frac{\lambda_{x^2 - y^2}(0)}{\lambda_{x^2 - y^2}(T)} = 1 - \frac{4}{\pi k_B T} \int_0^{\Delta_{x^2 - y^2}} dE f_E \left( 1 - f_E \right) E/\Delta_{x^2 - y^2} K(E/\Delta_{x^2 - y^2})
\]

\[
+ \int_{\Delta_{x^2 - y^2}}^{\infty} dE f_E \left( 1 - f_E \right) K(\Delta_{x^2 - y^2}/E)
\]

which relates the temperature dependence of the penetration depth with that of the gap \( A_{x^2 - y^2} \). The temperature dependence of the gap is determined by [3],

\[
\gamma^{-1} = 2\pi k_B T \left< |f|^2 \right> \sum_n \frac{|f|^2}{\sqrt{\omega_n^2 + \Delta^2 |f|^2}}
\]
where $f$ is given by (2) and $\gamma$ is the dimensionless coupling constant, $\omega_n$ is the Matsubara frequency and the sum is cutoff at $\omega_n = \epsilon_c$. Due to the crystal field in metals the Fermi surface becomes anisotropic so that at low temperatures the ground state is of $d^2_{x^2-y^2}$ symmetry. However, in such a case upon warming from low temperatures, the ground state may change from $d^2_{x^2-y^2}$ to $s$. The London penetration depth for $d_{xy}$ may be written from (3) by replacing $\Delta_{x^2-y^2}$ by $\Delta_{xy}$. However this symmetry occurs only near the surface of the sample[5]. In the Landau model, the higher temperature phase is more symmetric than the lower temperature phase. Therefore, we predict that at higher temperatures, the superconductor will become the $s$-wave type. Therefore, the $d^2_{x^2-y^2}$ state should change to $s$ state. This is possible only when the order parameter is complex, $d^2_{x^2-y^2} + i$. This result is in conformity with the calculations of the minimization of free energy[1]. For the complex order parameter, the penetration depth is given by

\[
\left| \frac{\lambda_{x^2-y^2,s}}{\lambda_{x^2-y^2,s}(T)} \right|^2 = 1 - \frac{4}{\pi k_B T} \int_{\Delta_{x^2-y^2}} dE f_E(1-f_E) \frac{E}{\Delta_s} K \left( \frac{(E^2 - \Delta_{x^2-y^2} f^2)^{1/2}}{\Delta_s} \right) \]

\[
+ \int_{\Delta_{x^2-y^2}} dE f_E(1-f_E) \frac{E}{(E^2 - \Delta_{x^2-y^2} f^2)^{1/2}} \times K \left( \frac{\Delta_s}{(E^2 - \Delta_{x^2-y^2} f^2)^{1/2}} \right)
\]

(5)

where

\[
\Delta_{x^2-y^2,s} = (\Delta_{x^2-y^2} + \Delta_s)^{1/2}
\]

\[
f_E = (e^{E/k_BT} + 1)^{-1}
\]

(6)

The surface microwave resistivity is proportional to $\mu_0 \omega \lambda_{ab}(0)$ where the permeability $\mu_0$ and the microwave frequency $\omega$ are known so that the penetration depth can be determined. The experimental measurements have been performed by Jacobs et al[6] using a single crystal.
of $Bi_2Sr_2CaCu_2O_{8+\delta}$ from which the penetration depth has been deduced as a function of temperature. We have taken a few experimental points from their work and shown in Fig.1. The experimental values are linear up to a temperature of $T < 40K$.

Figure 1: Temperature dependence of the London penetration depth $\lambda_{ab}(T)$ showing linear slopes corresponding to $d_{x^2-y^2}$ which agrees with the experimentally measured values. The curves marked as $d_{x^2-y^2}$(th), $s$-wave (th 100K) and $s$-wave (th 82K) are all theoretically calculated. The dots are deduced from some of the experimental measurements of the microwave surface resistivity of $Bi_2Sr_2CaCu_2O_{8+\delta}$ single crystals. The theoretically calculated penetration depth for the $s$ wave is in reasonable agreement with the experimentally measured values for $T > 60K$. 

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It is found that the theoretical expression (3) for $d_{2\Sigma}$ state is also linear\cite{4},

$$\frac{\lambda x^2 - y^2(T)}{\lambda x^2 - y^2(0)} \simeq 1 + 0.69k_B T/\Delta x^2 - y^2(0)$$

(7)

Therefore, it is identified that the low temperature behaviour is caused by a $d_{2\Sigma}$ type state. The experimentally measured slope for $d_{2\Sigma}$ is $\frac{\delta \lambda}{dT} \Delta x^2 - y^2(0) \sim 10 A/K$. The theoretical value of the slope $\frac{\delta \lambda}{dT} \frac{\lambda(T)}{\lambda(0)} \sim 15.3 K^{-1}$ is larger than the experimental value. The theoretically calculated value of $\lambda x^2 - y^2(T)/\lambda x^2 - y^2(0)$ for $T_c = 82K$ and $\lambda(0) = 2600 \AA$, is also shown in Fig.1. This value depends on temperature a bit more strongly than the experimentally deduced values. Using $\Delta x = 1.77k_B T_c(1-T/T_c)^{1/2}$ we find $\lambda_x(T)/\lambda_x(0)$ for two different values of $T_c = 82K$ and 100K which are also shown in Fig.1. These calculated values of $\lambda_x(T)/\lambda_x(0)$ are less than the experimentally measured values. But for $T > 60K$ only $s$ wave agrees with the experimental data. Thus we see that for $T < 40K$ the order parameter has $d_{2\Sigma}$ symmetry becoming $s$ wave for $60 < T < 82K$. The occurrence of multiple symmetries in the gap is permitted\cite{7} by the group theory. According to the expression (5) both the $s$ wave and the $d_{2\Sigma}$ wave type gaps occur at all temperatures with the average value given by the first of the expressions in (6) for a given value of the ratio $\Delta d_{2\Sigma}/\Delta$, so that the calculated $\chi(T)$ for the mixed wave occurs in between the two curves, $\lambda d_{2\Sigma}(T)$ and $\lambda_s(T)$. It may be noted that in the case of $^3He$ the $|0>$ and $|\pm 1>$ states are phase separated. Therefore it is possible to construct the theory in such a way that at low temperatures only the $d_{2\Sigma}$ phase occurs and at elevated temperatures only the $s$ phase occurs with a phase boundary separating the two phases. However evidence for such a phase boundary has not yet been reported in high $T_c$ superconductors.
References


