CHAPTER – I

INTRODUCTION

1.1 IMPORTANCE OF THE FLOW OF THE TWO PHASE FLUIDS

The dynamics of Two phase fluid systems is the study concerned with the motion of liquid or gas containing immiscible, suspended Stokesian solid particles. The modified form of Navier-Stokes equations coupled with Euler’s equations of motion for perfect fluids are used as the equations of motion of fluid phase and particle phase respectively. The presence of inert particles adds an extra force term representing the interaction between particle and fluid phases. The inert particles form a cloud of particles known as pseudo-fluid with negligible viscosity.

For many years, engineers and scientists are concerned with gas-solid particle flows which have got many applications in several industries. The two-phase fluid flows are important in sedimentation pipe flows, fluidized beds, gas purification, transport processes and shock waves.

The problem of gas-particulate flow has gained increased attention due to its various applications in the problem of dispersion and fall-out of
pollutants in air or water. In engineering applications a good knowledge of dusty fluid flows can be obtained from the problem of exhaust nozzle of rockets using solid propellant with added metal powders, conveying of powder material in blood vessels.

The flow of a gas that transports clouds of suspended Stokesian solid particles or liquid droplets through a pipe is of great technical importance. The mathematical analysis of such two-phase flows is more difficult than that of pure gas flows. A simplifying assumption is that the volume occupied by the particles can be neglected. In most of the cases the particles represent one half of the gas particle mixture and the density of the particle is more than a thousand times larger than the gas density. Under such conditions the particle volume fraction is of the order of only $10^{-4}$ and the assumption of a negligible particle volume is then well satisfied. Consequently this assumption is that equilibrium flows of mixtures of particles with a perfect gas can be analyzed like a flow of a perfect gas that has the density and specific heat of the mixture. Published analysis has been concerned mainly with various aspects of deviations from equilibrium that result from the particle velocity and temperature being unable to follow rapid changes in the gas phase. Examples are investigations of steady flow through nozzles (Bailey et al (1961), Soo (1961), Gilbert. et al (1962), Kliegel (1963), Marble (1963), flow behind shock waves (Carrier (1958), Marble F.E (1963a), interaction between particles or propagation of arbitrary pressure waves, (Rudinger (1964), Kriebel (1964)).

At high gas densities (high pressures) or at high particle mass fractions, the particle volume fraction may become sufficiently large so that it should not be neglected. It is therefore of interest to investigate how the results of flow computations are affected on account of the inclusion of the volume fraction of inert particles. We may evaluate the conditions under
which the volume fraction of the particle may be neglected without introducing appreciable errors. As the particles may be considered as incompressible in comparison with the gas, the basic flow equations have the particle volume fraction as an additional value. The discussion is limited to certain aspects of the thermodynamics properties of gas-particle mixtures.

Basic properties of gas-particle mixtures

1. The gas is treated as a perfect gas with constant specific heat.
2. The density and specific heat of the particle material are constant.
3. No mass transfer takes place between the gas and the particles. (Phenomena associated with condensation, evaporation or chemical reactions excluded).
4. Flows are treated as one dimensional continuum flows.
5. The random motion of the particles does not contribute to pressure, i.e. the number of particles is negligible when compared with the number of fluid molecules in the same volume.

In many engineering problems, we come across, more often than not, dusty fluid flows which involve body forces such as buoyant force. Practical application of these flows may be found in heat exchanges utilizing liquid metal or liquid sodium coolants in the area of thermal instability in the study of boiling heat transfer. The use of fluid containing small inert suspended particles may throw more light on several new phenomena which were sofar not known. Examples of dust-laden immiscible fluids separated by an interface occur in the locomotion of microorganisms, where the beating of long slender flagella or fields of hair like cilia propel the organisms, through the surrounding fluid. The micro-ciliary’s transport in the lung the beating of cilia transports mucus up the bronchial tubes. This system is often modeled as two separate layers of fluid-the watery lower serous layer in which the cilia beat and the more viscous upper mucus layer to which foreign particles adhere and are thus removed from lung. Other example includes polymer extrusion
in the petrochemical industry and also flotation processes where the behavior of particles near the bubble free surface is important. There are many industrial applications of two-phase flow. To cite a few,

a) Near Mangalore, from Kudremuch (80 kilo meters above sea level) the iron ore is brought down to Mangalore port by a two-phase flow phenomenon. Finely ground iron ore of mass concentration 4 to 6 percent is mixed with water to make a suspension. It is piped down to Mangalore, the iron ore is separated at the port and shipped to Japan.
b) In cement factories the powdered fine cement is taken to silo by pneumatic conveying which means mixing of fine cement particles with air, the cement dust air is transported through pipelines to the storage bins and the air is devoid of fine particles in a cyclone separator followed by electrostatic precipitators.
c) Applications in fluidized beds:
   1) Coal gasification for power generation.
   2) Reduction of gasoline from petroleum fraction.
   3) Gasoline from natural and synthesis gas.
   4) Manufacture of phthalic anhydride by catalytic oxidation of naphthalene and orthoxyline.
   5) Production of granular polyethylene by polymerization of its gaseous monomer.
   6) Fluidized beds find its niche in expanding semiconductor industry to produce ultra pure silicon and its precursors.
   7) It is used in food pharmaceuticals industries particularly for efficient cultivation of micro-organisms.
   8) Coating metal objects with plastic materials and drying of solids.

The presence of particles in a homogenous fluid makes the dynamical study of flow problems quite complicated. However these problems are usually investigated-under various simplifying assumptions.
The study of flow of Bio-Fluids through various organs of the body demands the knowledge of various disciplines of science and engineering. In recent years this study has become important with a view to explain the mechanism of the flow of blood in the normal arteries and the veins, the flow of urine from the kidneys to the bladder and the flow of various gastronomical juices in the gastrointestinal tract. There is growing awareness that fluid mechanical principles can be applied to construct realistic and simplified mathematical models of physical situations and then find out their solutions in tangible forms which will help us to understand the complex problems of the physiology of the human body. These useful results can be easily utilized by medical scientists and others. In view of this, pulsate two phase flow of blood in arteries is being studied.

1.2 BASIC EQUATIONS OF DUSTY FLUID FLOWS

In order to formulate the fundamental equations of motion of the two-phase fluid flows in a reasonably simple manner and to bring out the essential features certain assumptions are made. They are as follows:

1) The fluid is an incompressible Newtonian fluid.
2) Dust particles are assumed to be spherical in shape, all having the same radius and mass and undeformable.
3) The bulk concentration (i.e. concentration by volume) of the dust is very small so that the net effect of the dust on the fluid particles is equivalent to an extra force $KN_0(q_p - q)$ per unit volume, where $q$ (x,t) is the velocity vector of the fluid, $q_p$ (x,t) is the velocity vector of the dust particles, $N_0$, the number density of the dust particles, i.e., the number of dust particles per unit volume of the mixtures and K, the Stoke’s drag constant which is ‘$6\pi\mu a$’ for spherical particles of radius a, $\mu$ being the coefficient of viscosity of the clean fluid and it is also...
assumed that the Reynolds number of the relative motion of the dust and the fluid is small in comparison with unity.

4) The effect of the dust enters through the two parameters (f), the mass concentration parameter and (τ), the relaxation time of dust particles that is a measure of time for the dust to adjust to changes in the fluid velocity. The former describes how much dust is present and the latter is determined by the size of the individual particles, making the dust fine decreases τ, and making it coarse increases τ, in a manner proportional to the surface area of the particles.

5) The density of the material in the dust particles (ρ₁) is high compared with the fluid density (ρ) so that the mass concentration f may be a significant fraction of unity while the bulk concentration \( \frac{f \rho}{\rho₁} \) is small.

6) The buoyancy force on the particles is neglected since \( \rho/\rho₁ \) is small and so the fluid-phase contributes the entire pressure.

7) For sufficiently small particles, the velocity of sedimentation will be small compared with characteristic velocity and can be neglected.

8) Three dimensionless parameters can be constructed out of the quantities defining the fluid and the dust: the Reynolds number (Re), the mass concentration of dust (f) and relaxation time parameter (τ).

9) The distortion of the flow around the dust particles is neglected.

10) The particle will in general have different temperature than that of the surrounding fluid and therefore there will be temperature defects. Because of these temperature defects, there will be heat transfer between the two phases. The heat transfer from the particle to fluid has the form \( \frac{\rho_p C_p(T_p - T)}{\tau} \) where \( \rho_p = N_0 m \), \( N_0 \) is the number density, \( m \) is the mass of each particle, \( \rho_p \) is the partial density of the dust particles i.e., mass of particle phase per unit volume of mixture of the two
phases, $C_s$ is the specific heat of the solid particle, $C_p$ is the specific heat at constant pressure for the fluid, $\tau_\ell = 1.5 \Pr (C_s / C_p)$ $\tau$ is the thermal equilibrium time, $\Pr = \mu (C_p / k) = \text{Prandtl number}$, $k$ is the thermal conductivity of the fluid, $T$ is the fluid temperature, $T_p$ is the temperature of the dust and $\tau = (2\rho_1 a^2/9\mu)$. Consequently the heat transfer from the fluid to the dust will be negative of the heat transfer from the dust to the fluid.

11) In this thesis, the particle phase is assumed to be relatively dilute so that particle-particle interaction is neglected. Particle-Particle interaction becomes in dense particulate suspensions.

Under these assumptions the basic equations of the dusty fluid are:

**Equation of continuity for the fluid:**
\[ \text{div} \mathbf{q} = 0, \quad (1.1) \]

**Equations of conservation of momentum for the fluid:**
\[ \rho \left[ \frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \text{grad}) \mathbf{q} \right] = -\text{grad} p + \mu \nabla^2 \mathbf{q} + \kappa \mathbf{N} \left( \mathbf{\bar{q}} - \mathbf{q} \right), \quad (1.2) \]

**Equations of conservation of energy for the fluid:**
\[ \rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \frac{\partial}{\partial x} \left( \frac{\partial p}{\partial x} + \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial p}{\partial y} + \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial p}{\partial z} + \frac{\partial w}{\partial z} \right) + \phi_1 + \frac{\rho C_s (T_p - T)}{\tau}, \quad (1.3) \]

Where $\phi_1$ represents the viscous dissipation function and is given by
\[ \phi_1 = 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2 \right]. \]

**Equations of continuity for the particle phase:**
\[ \frac{\partial N_p}{\partial t} + \text{div} (N_p \mathbf{q}_p) = 0, \quad (1.4) \]
Equations of conservation of momentum for the particle phase:

\[
N_o m \left[ \frac{\partial \vec{\rho}_p}{\partial t} + (\vec{q}_p \cdot \nabla) \vec{q}_p \right] = K N_o (\vec{q} - \vec{q}_p), \quad (1.5)
\]

Equations of conservation of energy for the particle phase:

\[
\rho_p C_p \left( \frac{\partial T_p}{\partial t} + u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} + w_p \frac{\partial T_p}{\partial z} \right) = -\rho_p C_p (T_p - T) \frac{\tau}{\tau_f}, \quad (1.6)
\]

The ten independent equations in (1.1 - 1.6) are to be solved for the ten unknowns.

\[ \vec{q} = (u, v, w), \vec{q}_p = (u_p, v_p, w_p), N_o, p, T, T_p. \]

When volume fraction \( \phi \) of particle phase is taken into account the equations of conservation of momentum for the fluid and the dust take the form:

\[
\rho(1 - \phi) \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} = (1 - \phi)(-\nabla^2 \vec{q}) + KN_o (\vec{q} - \vec{q}_p), \quad (1.6a)
\]

\[
N_o m \left[ \frac{\partial \vec{q}_p}{\partial t} + (\vec{q}_p \cdot \nabla) \vec{q}_p \right] = \phi[-\nabla^2 \vec{q} + \nu \nabla^2 \vec{q}] + KN_o (\vec{q} - \vec{q}_p), \quad (1.6b)
\]

### 1.3 BOUNDARY CONDITIONS

The fundamental equations stated in the previous section are to be solved under appropriate boundary conditions to determine the flow fields of the fluid and the dust particles. In general, the boundary conditions are:

**For Fluid Phase**

1. There will be no mass transfer at a solid boundary.
2. The fluid velocity vanishes at a solid boundary.
3. The fluid velocity must approach the free stream value as \( y \) approaches infinity.
4. The temperature of the fluid at the plate is that of the plate.
5. The temperature of the fluid must approach free stream value as \( y \) approaches infinity.
For Particle Phase

1. The dust particles may slip at the boundary and the boundary conditions are to be taken from ambient conditions.
2. The particle phase temperatures must approach their free stream value as y approaches infinity.
3. The exact form of particle-phase boundary condition at a surface is currently unknown and many investigators in the field have used the no-slip condition as for the fluid phase. This is also done in this thesis and the conditions are not written separately for the particle phase so that whatever conditions are valid for the fluid phase are assumed for the particle phase as well.

1.3.1 FREE CONVECTION HEAT TRANSFER

A free convection flow is caused by the action of body forces on the fluid, that is, forces which are proportional to the mass or density of the fluid. The flow is generally produced in the following way. Let us consider a plate fixed in quiescent fluid subject to a body force. When the plate is in isothermal equilibrium with the fluid, with hydrostatic pressure balances the body forces. If a temperature gradient is imposed on the fluid normal to the body force by heating (or cooling) the plate there results a deficit (or excess) of body force because of diminished (or increased) density, with the fluid particles near the plate subjected to greater deficit (or excess) than those away from the plate. This imbalance of the forces accelerates the fluid particles near the plate more rapidly than those farther away and free convection sets in. Generally free convection takes place in the field of gravity where the aforementioned deficit or excess of the force is the Archimedean or buoyancy force. In the case of the dust particles the buoyancy force is neglected [see section 1.2 assumptions. (6)]
1.4 TWO-DIMENSIONAL BOUNDARY LAYER EQUATIONS

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1.7)
\]

The equations governing the two-dimensional flow of a dusty fluid are

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{KN_o}{\rho} (u_p - u), \quad (1.8)
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{KN_o}{\rho} (v_p - v), \quad (1.9)
\]

\[
\rho C_p \left( \frac{\partial T_p}{\partial t} + u \frac{\partial T_p}{\partial x} + v \frac{\partial T_p}{\partial y} \right) = \frac{\partial}{\partial x} \left( k \frac{\partial T_p}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T_p}{\partial y} \right) + \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + \phi + \frac{\rho_p C_p (T_p - T)}{\tau_f}, \quad (1.10)
\]

\[
\frac{\partial N_a}{\partial t} + \frac{\partial}{\partial x} (N_a u_p) + \frac{\partial}{\partial y} (N_a v_p) = 0, \quad (1.11)
\]

\[
\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = \frac{K}{m} (u - u_p), \quad (1.12)
\]

\[
\frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = \frac{K}{m} (v - v_p), \quad (1.13)
\]

\[
\rho_p C_p \left( \frac{\partial T_p}{\partial t} + u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} \right) = - \frac{\rho_p C_u (T_p - T)}{\tau_f}, \quad (1.14)
\]

The usual boundary layer approximations are made. Also from the Stokes' drag, we have \((KN_o/\rho) \sim o(1)\), i.e., \(6\pi a N_0\) must be of order \((1/\delta^2)\), because as usual \(v\) must be of order \(\delta^2\) and as \(\alpha\) is very small, \(N_0\) must be very large which we always assume for dusty fluid flows. From (1.9), in order that the usual boundary layer approximation may be assumed for \(\frac{\partial p}{\partial y}\)

\[
\left( \frac{\partial p}{\partial y} \right) \sim o(\delta) \text{ we must have } (v_p - v) \sim o(\delta) \text{ and hence } v_p \sim o(\delta) \text{ and } \rho_p \text{ is of the order of } \rho.
\]
Since the fluid is incompressible all dissipation terms can be neglected. Also \( k \) and \( \mu \) can be assumed to be constants and then for steady flow the boundary layer equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1.15)
\]

\[
u \frac{\partial u}{\partial y} + v \frac{\partial v}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial y^2} + KN_0 \frac{(u_p - u)}{\rho}, \quad (1.16)
\]

\[
\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \rho_p C_s \frac{(T_p - T)}{\tau_T}, \quad (1.17)
\]

\[
\frac{\partial}{\partial x} (N_0 u_p) + \frac{\partial}{\partial y} (N_0 v_p) = 0, \quad (1.18)
\]

\[
u \frac{\partial u_p}{\partial x} + v \frac{\partial v_p}{\partial y} = \frac{K}{m} (u - u_p), \quad (1.19)
\]

\[
\rho_p C_s \left( u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} \right) = -\rho_p C_s \frac{(T_p - T)}{\tau_T}, \quad (1.20)
\]

For uniformly distributed dust particles i.e., when \( N_0 = \) constant throughout the flow field, there are only six unknown viz., \( u, v, u_p, v_p, T, T_p \) which can be found by solving the six equations (1.15)-(1.20).

### 1.5 REVIEW OF THE LITERATURE ON DUSTY FLUID FLOWS

In this review, we confine our attention to (1) Fundamental aspects of dusty fluid flows, (2) Parallel flows in dusty fluids, (3) Channel flows of dusty fluids, (4) Dusty fluid flows through porous medium and (5) two phase fluid flows with heat transfer.

**1. Fundamental aspects of dusty fluid flows:**

Several authors have made an extensive study of the Fundamental aspects of the flows of dusty fluids. As far back as in 1945, Leo Briggs conducted experiments in laboratory and observed that the gas handling capacity of a cyclonic dust collected increases considerably with dust
concentration. The experimental studies conducted by Sproull (1961) and Kazakevich and Krapivin (1958) showed that the concentration of the dust is the main factor and that the particle size and bulk density of the dust are relatively unimportant and that the aerodynamic resistance of the dusty gas is less than that of the clean gas. Soo (1961, 1964, 1965a, 1965b, 1966, 1967, 1973, 1975a, 1975b, 1977) had done a good amount of work concerning the problems of dusty fluid flows. Saffman (1962) formulated a model of dusty fluid under the simplifying assumptions given in Sec. 1.2 of this chapter. He considered the effect of dust particles on the stability of laminar flow by investigating the effect of particles on the critical Reynolds number for transition from laminar to turbulent flow. Marble (1962), Pai (1970), Jain (1975) and Goddard (1977) made a thorough study of the dynamics of gas-particle flow systems. They first introduced the new dimensionless parameters and indicated their general effect on the solution. Secondly the dynamic and thermodynamic relations which govern the gas-particulate motion were developed by them. Finally, the analytical solutions of several typical problems were summarized as an illustration of the effects of the new dimensionless parameters. Nayfeh (1966) discussed oscillating two-phase flow through a rigid pipe. Some problems concerning the structure of shock waves, sound attenuation and many flow-field problems were discussed by Marble (1970). Wallies (1969) studied the one-dimensional two-phase flow. Davidson (1969) considered the motion of mixtures of a fluid and solid lumps or particles which commonly occur in various fields of engineering viz., hydraulic, mechanical and chemical. Torobin and Gauvin (1959, 1960), Crooke (1972, 1975), Hinch (1977) and Purcell (1978) studied some fundamental aspects of dusty fluid flows. An extreme principle for infinite slow viscous flows containing particles was studied by William (1976). Donald (1979) discussed the stability of Stokes' layer of a dusty gas. Flow of a dusty fluid past a wavy moving wall was elaborated by Nag (1980). Datta and Mishra (1980) discussed two dimensional stagnation point flow of a dusty


(2) Parallel flows in dusty fluids:

Michael and Miller (1966), using the formulation of Saffman gave analytical solution for two problems of a dusty gas occupying the semi infinite space above the rigid plane. They considered the motion induced in the dusty
gas in the cases when the plane moves parallel to itself (a) in simple harmonic motion and (b) impulsively from rest with uniform velocity. Healy et al (1973) studied the response of a viscous incompressible non diffusive fluid containing a dilute suspension of small spherical particles and occupying a semi infinite region bounded by an infinite plane, when the latter is slowly moved parallel to itself in a prescribed manner normal to the direction of stratification. He showed that, in particular, when the system density decays exponentially with distance from the plate, the problem is equivalent to the axial flow of the same system in a long tube. Peddieson Jr (1976) obtained analytical and numerical solutions for five unsteady parallel flows of particulate suspensions which exhibit boundary layer characteristics. Barron (1977) discussed the steady plane flow of a viscous dusty fluid with parallel velocity fields with the dust particle distribution function to be a variable. Nag et al (1979) investigated the unsteady couette flow of a dusty gas between two parallel plates by considering one of the plates to be at rest and the other moving at different velocities. They applied the Laplace Transform technique. Mitra (1981a) discussed the flow of a dusty gas induced by two infinitely extended parallel plates when the lower plate is at rest and the upper plate begins to oscillate harmonically in its own plane. The couette flow of a dusty fluid with one of a horizontal moving boundaries being stopped suddenly was studied by Ramanaprasad and Pattabi Ramacharyulu (1981). Mitra (1982) considered the unsteady flow of dusty gas between two parallel plates, one being at rest and the other oscillating. Datta and Mishra (1983) studied the unsteady couette flow and heat transfer of a dusty fluid filling the gap between two infinite parallel plates kept at arbitrary temperatures. The problem of a dusty fluid flow past an infinite flat plate which starts moving parallel to itself in an arbitrary time dependent velocity when the pressure is uniform was studied by Mitra (1983a). Hazem Attia (2002) discussed unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties. Saito (2002) made a numerical analysis of dusty-gas flows. Ghosh

(3) **Channel flows of dusty fluids:**

Reddy (1972), obtained an exact solution in terms of an infinite series of the flow of a dusty viscous liquid through a rectangular channel under the influence of an exponential pressure gradient. The dusty gas flow between two infinite flat plates oscillating in their own planes was discussed by Vimala (1972). Mukherjee et al (1973) studied the flow of a dusty viscous liquid between two parallel flat plates. Dube and Sharma (1975a) discussed in their note on the flow of a dusty viscous liquid in a channel bounded by two parallel flat plates under the influence of a constant pressure gradient. Sharma (1975) and Mathur et al (1976) discussed similar problems of unsteady flow of a dusty viscous liquid in a channel bounded by two parallel plates. Gupta and Gupta (1976) reported the flow of a dusty gas under the influence of a time varying pressure gradient through a rectangular channel. Explicit expressions were obtained for the velocities of the gas and the particles by using operational methods. Sharma and Dube (1976) studied the unsteady flow of a Rivlin-Erickson fluid with a uniform distribution of dust particles through a square channel. Kishore and Pandey (1977a) discussed the flow of a dusty viscous liquid in a parallel plate channel under the influence of a pressure gradient varying linearly with time. Radhakrishnamacharya (1978) studied the pulsate flow of a dusty fluid through a two-dimensional constricted channel. Ramanaprasad and Pattabi Ramacharyulu (1980) presented an analytic

(4) Dusty fluid flows in porous medium:


(5) Dusty fluid flows with heat transfer:

Oscillating free convection over two dimensional bodies in a dusty fluid was discussed by Ramamurthy (1987). The same author (1990) elaborated free convection effects on the Stokes problem for an infinite vertical plate in a dusty fluid. Effects of variable viscosity and viscous dissipation of the flow of a third grade fluid in a pipe was studied by M Massoudi (1995).


1.6 PROBLEMS INVESTIGATED

In chapter – II, the couette dusty flow between two horizontal parallel porous flat plates with transverse sinusoidal injection of the dusty fluid at the stationary plate and its corresponding removal by constant suction through the plate in uniform motion are analyzed. Due to this type of injection velocity the dusty flow becomes three dimensional. Perturbation method is used to obtain the expressions for the velocity, temperature fields of both the fluid and dust. Graphs of these variables are drawn and results are given based on the graphs. It is found that the velocity profiles of both the fluid and dust in the main flow direction decrease with the increase of mass concentration of the dust particles. The velocity profiles of both fluid and dust in cross flow direction increase with an increase in the mass concentration of the dust particles up to the middle of the channel and thereafter an opposite trend is noticed. The skin friction components $T_x$ and $T_z$ in both the main flow and transverse flow directions increase with an increase in the mass concentration of the dust particles (or) injection parameter. The heat transfer coefficient decreases with the increase of the injection parameter but increases with the increase in the mass concentration of the dust particles.

The objective of chapter III is to discuss the effect of periodic permeability on the free convective flow of a dusty viscous incompressible fluid through a highly porous medium. The porous medium is bounded by an infinite vertical porous plate. Assuming the free stream velocity to be uniform, analytical solutions are obtained for the dusty flow field, the
temperature field, the skin friction and the rate of heat transfer, using perturbation method. In above cases the permeability of the porous medium is assumed as per the equation (3.1). The problem becomes three dimensional due to such a type of permeability variation. Graphs are drawn for the velocity profiles of the fluid, dust, temperature profiles of the fluid and dust when all the parameter are varied and conclusions are made based on these graphs. It is found that the velocity profiles of both the fluid and dust decrease with either an increase in the mass concentration of the dust particles (or) an increase in the Grashof number (or) an increase in the permeability of the porous medium (or) an increase in the Reynolds number. The skin friction component $T_x$ increases with an increase in the mass concentration of the dust particles (or) an increase in Prandtl number while it decreases with an increase in Grashof number (or) increase in the permeability of the porous medium (or) increase in the Reynolds number. The Nusselt number $Nu$ (in the case of water $Pr=7.0$) increases with an increase in mass concentration of the dust particles (or) increase in Reynolds number, while it decreases with an increase in the permeability of the porous medium. The Nusselt number $Nu$ for (in the case of air $Pr=0.7$) decreases with an increase in the mass concentration of the dust particles (or) increase in the permeability of the porous medium (or) increase in Reynolds number.

An exact solution of oscillatory Ekman boundary layer dusty flow through a porous medium bounded by two horizontal flat plates is obtained by comparing harmonics & non harmonics terms in chapter IV. One of the plates is at rest and the other oscillates in its own plane with velocity about a non-zero mean velocity $U_0$ Origin is chosen on the lower plate and $z$ axis is taken perpendicular to the plates. The entire system rotates about an axis normal to the plates. The effects of Coriolis force on the permeability of the porous medium of the flow fields are studied. Graphs are drawn for amplitude $|A_0|$ and phase angle $\theta_0$ due to $u_0$ & $v_0$ for steady flow and conclusions are made
based on these graphs. It is found that the primary and secondary velocity components \( u_0 \) and \( v_0 \) decrease with an increase of the rotation parameter \( R \) but increase with the increase of the permeability parameter \( k_0 \). The amplitude of the resultant velocity and phase angle increases with an increase in the permeability parameter (or) an increase in rotation parameter \( k_0 \). The shear stress increases with increase in rotation parameter. However increase in the permeability leads to an increase in the shear stress for small rotation but a reverse effect is noted for large rotation.

The purpose of chapter V section 1 is to study the effect of non-constant unsteady 2-dimensional free convective flow during the motion of a viscous incompressible dusty fluid through a highly porous medium. The porous medium is bounded by a vertical plane surface of constant temperature. The surface absorbs the dusty fluid with constant velocity and the free stream velocity of the fluid vibrates about a mean constant value. Analytical expressions for the velocity of the fluid and dust are given. The effects of Grashof number and permeability parameter upon the velocity field are also shown in a graphic representation. The velocity profiles of both fluid and dust decrease with an increase in the mass concentration of the dust particles (or) an increase in the frequency parameter (or) an increase in the permeability parameter (or) an increase in the Grashof number. The temperature profiles of both the fluid and dust increase with an increase in the relaxation time of the dust particles. Both the profiles decrease steadily from “one” in the lower plate to “zero” at the other plate.

The unsteady two-dimensional free convective flow of dusty fluid through a porous medium bounded by an infinite vertical plate is considered when the temperature of the plate is oscillating with the time about a constant non-zero mean value is discussed in chapter V section. The x-axis is taken along the plate in the direction of the flow and y axis is taken normal to it.
analytical solution for the velocity field for both the fluid and dust are derived and the effects of $k_0$ (the permeability parameter), $\omega$ (the frequency parameter), $f$ (the mass concentration parameter) on the velocity field (both the fluid and dust) are discussed. It is found that the velocity profiles of both fluid and dust increase with an increase in mass concentration of dust particles.

Steady laminar free convection flow of an electrically conducting dusty fluid along a porous hot vertical plate in the presence of heat source / sink is investigated in chapter VI. The x axis is taken along the plate in upward direction and y axis is taken normal to it. A transverse constant magnetic field is applied in direction of y axis. The governing equations of motion are solved by a regular perturbation technique. Graphs are drawn for each variable and conclusions are made based on these graphs. The skin-friction coefficient and Nusselt number are derived and tabulated. The velocity and temperature distributions of both the fluid and dust are shown graphically taking two cases namely, case I: when $Gr > 0$ (i.e. flow on a cooled plate) and Case II: $Gr < 0$ (i.e. flow on a heated plate). The velocity profiles of both the fluid and dust increase due to an increase in the mass concentration of the dust particles when $Gr > 0$ (flow on a cooled plate) and reverse effect is seen when $Gr < 0$ (flow on a heated plate). The temperature decreases when $Gr > 0$ (flow on a cooled plate) and opposite trend is maintained when $Gr < 0$ (flow on a heated plate). Skin friction coefficient decreases with an increase in mass concentration parameter and Nusselt number increases due to an increase in mass concentration of the dust particles when $Gr > 0$ (flow on a cooled plate), but both behave in opposite manner when $Gr < 0$ (flow on a heated plate).