CHAPTER 4

INCREASING SPUR GEAR TOOTH STRENGTH
BY PROFILE MODIFICATION

4.1 INTRODUCTION

There is a demand for the gears with higher load carrying capacity and increased fatigue life. Researchers in the gear field have been working on the development of advanced materials, new heat treatment methods, designing the gear with stronger teeth and methods of gear manufacturing to tackle the problem associated with gear failures. In modern gear design and manufacturing, the majority of gear applications are covered by the standard 20° pressure angle involute teeth with trochoidal root fillet generated by the rack, hob, CNC cutting process and etc. This practice has many advantages such as that of interchangeability, insensitivity to change in nominal center distance, commercial availability and easy manufacturing by conventional methods (i.e. hobbing). However the gears made of standard involute profile with less than 17 teeth is liable to undercutting phenomenon in nature. In trochoidal tooth profile, the tooth fillet is generated as the tip of the cutter removes material from the involute profile thus resulting in teeth that have less tooth thickness at the root where the critical section is usually located. This reduces the tooth strength and leads to the crack initiation and propagation while power transmission at the root fillet area. This study is undertaken to eliminate the above draw back by replacing the trochoidal tooth profile with the circular tooth profile to generate the root in the tooth profile called circular root fillet.
4.2 PROFILE MODIFICATION IN SPUR GEAR TOOTH

4.2.1 Geometrical Modeling

Figure 4.1 shows the geometry of spur gear tooth with circular root fillet in which point ‘O’ is the center of the gear, ‘Oy’ is the axis of symmetry of the tooth and ‘B’ is the point where the involute profile starts (from the form circle \( r_f \)).

![Figure 4.1 Spur gear geometry of the circular root fillet](image)

Point ‘A’ is the point of tangency of the circular fillet with the root circle \( r_f \). Point ‘D’ lying on (\( \varepsilon \)2) identical to ‘OA’ represents the center of the circular fillet. Line (\( \varepsilon \)3) is tangent to the root circle at ‘A’ and intersects with line (\( \varepsilon \)1) at ‘C’. The fillet is tangent to the line (\( \varepsilon \)1) at point ‘E’.

Since it is always \( r_s > r_f \), the proposed circular fillet can be implemented without exceptions on all spur gears irrelevant of the number of gear teeth or other manufacturing parameters. A comparison of the
The geometrical shape of a tooth of circular root fillet with that of the standard trochoidal root fillet is presented in Figure 4.2. The geometry of the circular root fillet which coordinates (points A, B, C, D and E) in Figure 4.1 are obtained using the formulas given below:

\[
\begin{align*}
X_B &= r_f \sin \Omega_s \\
Y_B &= r_f \cos \Omega_s \\
Y_C &= \frac{x_c}{\tan \Omega_s} \\
X_A &= r_f \sin (\zeta + \Omega_s) \\
Y_A &= r_f \cos (\zeta + \Omega_s) \\
X_E &= (OC+CE) \sin \Omega_s \\
Y_E &= (OC+CE) \cos \Omega_s \\
X_D &= (r_f + AD) \sin (\zeta + \Omega_s) \\
Y_D &= (r_f + AD) \cos (\zeta + \Omega_s) \\
X_C &= r_f \frac{\tan \Omega_s}{\sin(\zeta + \Omega_s) \tan \Omega_s + \cos(\zeta + \Omega_s)}
\end{align*}
\]
Figure 4.2 Superposition of circular fillet on a standard tooth

The remaining portion of the tooth profile between points ‘B’ and ‘E’ is a straight line. Angle $\frac{\phi}{2}$ that corresponds to the arc $S_s/2$ in Figure 4.1 is given by the Equation (4.1):

$$\frac{\phi}{2} = \frac{S_s/2}{r_s} = \Omega_s$$  (4.1)

Angle $\zeta$ (Figure 4.1) takes values between $0$ and $\zeta_{\text{max}}$ (Equation (4.2)):

$$\zeta_{\text{max}} = \frac{\pi}{Z} - \Omega_s$$  (4.2)

### 4.3 DESIGNING THE MODIFIED SPUR GEAR PROFILE

In actual practice, the trochoidal root fillet is formed in the gears during the manufacturing process depending on the tip radius of the hob. It has been proved that the bending stress decreases gradually in gears as the number of teeth increases and the total contact ratio increases (Spitas et al. 2005). To overcome the above problem, a novel method namely, circular root fillet instead of standard trochoidal root fillet is introduced in gears having less than 17 teeth to decrease the bending stress at the root and also to avoid the gear tooth failure due to undercutting. According to Gitin Maitra (1998), if a gear is undercut for one reason or another, it may become sometimes necessary to know the magnitude of the undercutting radius. Under such circumstances he proposed a formula (Equation (4.3)) to find out the minimum number of teeth to avoid undercutting which is as follows:

$$Z_{\text{min}} = \frac{2}{\sin^{\frac{1}{2}} \alpha}$$  (4.3)
This expression is valid for standard gear tooth with the addendum of the rack being equal to the module ‘mn’. However the undercut–free minimum number of teeth is given by Equation (4.4):

\[ Z_{\text{min}} = \frac{2h_{ca}}{mn \sin \gamma} \]  

(4.4)

where, \( h_{ca} \) is = the addendum of the rack cutter without tip fillet rounding

Table 4.1 gives the specifications of the 13 teeth spur gear used in this investigation. These design specifications have been arrived from the KISSsoft (2008) a calculation programs for machine design according to DIN 3990 method ‘B’ standards for the given centre distance. The law of gearing (Townsend Dennis 1991) requires that the mating gear should have the same normal pressure angle ‘\( \alpha \)’ and the same module ‘mn’ in order to be able to mesh properly. The above points were considered in this analysis. The virtual model of the spur gear with 13 teeth having circular as well as trochoidal root fillet are modeled in Pro-E wildfire 3.0 software and analyzed in ANSYS software version 11.0 to evaluate the performance.

**Table 4.1 Specifications of spur gear**

<table>
<thead>
<tr>
<th>Gear tooth type</th>
<th>Standard involute full depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear number of teeth (( Z_1 ))</td>
<td>13</td>
</tr>
<tr>
<td>Normal module (mn)</td>
<td>2 mm</td>
</tr>
<tr>
<td>Face width (b)</td>
<td>12.85 mm</td>
</tr>
<tr>
<td>Normal pressure angle (( \alpha ))</td>
<td>20°</td>
</tr>
<tr>
<td>Helix angle (( \beta ))</td>
<td>Spur gear</td>
</tr>
<tr>
<td>Tooth root fillet</td>
<td>Trochoidal and circular (proposed)</td>
</tr>
<tr>
<td>Centre distance (( a_1 ))</td>
<td>26 mm</td>
</tr>
<tr>
<td>Pitch circle diameter (d)</td>
<td>26 mm</td>
</tr>
<tr>
<td>Addendum modi co efficient (( X_1 ))</td>
<td>0.05 mm</td>
</tr>
<tr>
<td>Total contact ratio ((\epsilon_g))</td>
<td>1.442</td>
</tr>
<tr>
<td>Gear ratio (u)</td>
<td>1.0</td>
</tr>
<tr>
<td>Addendum (( h_a ))</td>
<td>2.1 mm</td>
</tr>
</tbody>
</table>
4.4 FORCE ANALYSES

The load transmitting capability of the gear tooth is analyzed and checked for designing a gear system. The effective circumferential force on the tooth at the pitch circle of the gear while in meshing is estimated. Two kinds of stresses are induced in gear pair during power transmission from one shaft to another shaft. They are: i) bending stress—induced on gear teeth due to the tangential force developed by the power and ii) Surface contact stress or compressive stress. The load is assumed as uniformly distributed along the face width of the tooth.

4.4.1 Force Components

When the mating gears are engaged, the line of contact starts from the bottom of the tooth to the tip of the tooth along the tooth profile for the pinion and from the tip of the tooth to the bottom of the tooth for the gear. While the force is acting at the tip of the tooth, the long distance of action from the root causes maximum stress at the bottom of the tooth. Hence, the tangential force was applied at the tip of the tooth along the face width during bending stress analysis. Referring now to Figure 4.3, the normal force ‘$F_n$’ acts along the pressure line. The normal force ‘$F_n$’ is resolved into two components, namely,  i) Tangential force ($F_t$) and ii) Radial force ($F_r$). This normal force produces an equal and an opposite reaction at the gear tooth. As the gear is mounted on a shaft, the radial force ‘$F_r$’ acts at the centre of the shaft and is equal in magnitude but opposite in direction to the normal force ‘$F_n$’. For the given data, the force ‘$F_t$’ known as the tangential force or transmitting load was derived from the standard Equation (4.5):
\[ F_t = \frac{2000 \times T}{d} \]  
\[ \text{where, Torque } (T) = \frac{9550 \times P}{n} \]

Figure 4.3  Force diagram of spur gear

The tangential force ‘\( F_t \)’ constitutes a couple which produces the torque on the pinion which in turn drives the mating gears. The tangential force bends the tooth and the radial force compresses it. The magnitude of the radial force (\( F_r \)) is arrived using the Equation (4.6).

\[ F_r = F_n \times \sin \alpha \]  
\[ \text{(4.6)} \]

where,

\[ F_n = \frac{F_r}{\cos \alpha \times \cos \beta} \]
As far as the transmission power is concerned the tangential force is really the useful component, because the radial component serves no useful purpose. Irrespective of the value of the contact ratio, the gear forces are taken to be effective on a single pair of teeth in the mesh. The components of force i.e. $F_t$, $F_r$ and $F_n$ are computed for a power (P) value of 10 hp at different gear speed (n) 1500 rpm, 2000 rpm, 2500 rpm and 3000 rpm and presented in Table 4.2.

### Table 4.2 Force components

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>Torque (N-mm)</th>
<th>Force in Newton</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_t$</td>
<td>$F_n$</td>
</tr>
<tr>
<td>1500</td>
<td>47480</td>
<td>3652.3</td>
</tr>
<tr>
<td>2000</td>
<td>35600</td>
<td>2739.0</td>
</tr>
<tr>
<td>2500</td>
<td>28490</td>
<td>2191.2</td>
</tr>
<tr>
<td>3000</td>
<td>23740</td>
<td>1826.1</td>
</tr>
</tbody>
</table>

### 4.5 FINITE ELEMENT ANALYSIS

A finite element model with a single tooth is considered for bending stress analysis and a gear pair is chosen for contact shear stress analysis. The gear material strength is a major consideration for the operational loading and environment. Generally cast iron is used in normal loading and higher wear resisting conditions. In modern practice, the heat treated alloy steels are used to overcome the wear resistance. In this work, heat treated alloy steel is taken for analysis. The material properties chosen for finite element analysis are presented in Table 4.3. The fatigue and the yielding of the gear tooth as a result of excessive bending stress are the two important gear design considerations. In order to predict the fatigue and the yielding, the maximum stresses on the tensile and the compressive sides of the tooth are essential. The gear tooth surface is nonlinear and in this analysis,
the tooth forces are applied normal to the profile at the Highest Point of
Single Tooth Contact (HPSTC) to estimate the bending stressess during single
tooth analysis and the contact shear stress of a gear pair during non-linear
contact analysis. The following are the conditions assumed during the
analysis:

i. There is no sliding in the contact zone between the two bodies.

ii. The contact surface is continuous and smooth.

Table 4.3 Material properties

<table>
<thead>
<tr>
<th>Gear material</th>
<th>Alloy steel (18Cr NiMo7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (E)</td>
<td>2.1×10^5 N/mm^2</td>
</tr>
<tr>
<td>Density</td>
<td>7.85×10^-6 kg/m</td>
</tr>
<tr>
<td>Poison’s ratio (ν)</td>
<td>0.3</td>
</tr>
<tr>
<td>Yield strength (R_p)</td>
<td>366 to 1798 MPa</td>
</tr>
</tbody>
</table>

Since the Solid 92 element has a quadratic displacement behavior
and is well suited to model irregular meshes produced from various
CAD/CAM systems, this Solid 92 element type with 10 nodes is selected to
describe the gear and the tooth deflection in ANSYS software version 11.0.
This element also has plasticity, creep, swelling, stress stiffening, large
deflection, and large strain capabilities. The 10 node 3-D solid elements with
three degree of freedom per node (UX, UY, and UZ) are stacked to model
through the thickness discontinuities. As the gears are made out of heat
treated alloy steel, carburized and case hardened alloy steel (18CrNiMo7) is
taken for analyzing the root stresses. For non-linear contact analysis,
CONTA174 element was introduced between the involute profiles of mating
gear to have surface-to-surface contact. This element has three degree of
freedom at each node; translations in the nodal x, y, and z directions.
Chien Hsing Li et al (2002) have developed a simple and practical method, by
which this module was enabled to search for contact nodes and elements and to automatically define the contact surfaces for contact analysis.

### 4.5.1 Bending Stress Analysis

In order to facilitate the finite element analysis, the gear tooth was considered as a cantilever beam. All degrees of freedom were constrained at the root circle and the tooth force was applied normal to the profile at their Highest Point of Single Tooth Contact (HPSTC) that is at the tip of the gear tooth to the entire face width. The total force was distributed to an individual node on the line of contact. The line of contact in spur gear is described in detail in Figure 4.3. The meshed model of gear tooth having trochoidal root fillet and circular root fillet is depicted in Figures 4.4 and 4.5. The finite element analysis is carried out for the gear speed of 2000 rpm. The induced bending stress and the corresponding tooth deflection (Figure 4.6) in 13 teeth gear having different root fillets are presented in Table 4.4.
Figure 4.4 FEA meshed model of tooth with trochoidal root fillet

Figure 4.5 FEA meshed model of tooth with circular root fillet

Table 4.4 FEA results at 2000 rpm - Bending stress

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>Deflection (mm) for 13 teeth</th>
<th>Bending stress (N/mm(^2)) for 13 teeth</th>
<th>Stiffness (N/mm) for 13 teeth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trochoidal root fillet</td>
<td>Trochoidal root fillet</td>
<td>Trochoidal root fillet</td>
</tr>
<tr>
<td></td>
<td>Circular root fillet</td>
<td>Circular root fillet</td>
<td>Circular root fillet</td>
</tr>
<tr>
<td>2000</td>
<td>0.013089</td>
<td>447.435</td>
<td>279035.83</td>
</tr>
<tr>
<td></td>
<td>0.010323</td>
<td>442.457</td>
<td>353802.18</td>
</tr>
</tbody>
</table>
Figure 4.6 ANSYS results
Further the maximum tooth bending stress ($\sigma$) is calculated using the Lewis formula (Equation (4.7)) (Shigley 2008) for all the four speeds and are compared with the ANSYS results.

$$\sigma = \frac{K_y \times F_t}{b \times m \times Y}$$  \hspace{1cm} (4.7)

where, $K_y = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}}$ and

‘V’ is in meters per second (m/s)

The induced bending stresses presented in Table 4.4 for both the circular root fillet and the trochoidal root fillet designs are found to be within the permissible limit for 2000 rpm (Table 4.5).

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>Maximum bending stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>602.904</td>
</tr>
<tr>
<td>2000</td>
<td>459.690</td>
</tr>
<tr>
<td>2500</td>
<td>372.679</td>
</tr>
<tr>
<td>3000</td>
<td>314.261</td>
</tr>
</tbody>
</table>

4.5.2 Nonlinear Contact Shear Stress Analysis for Single Gear Pair

During contact shear stress analysis, all degrees of freedom are constrained at the root circle but for analysis purpose, the constrained degrees of freedom are transferred to the gear hub surface. Four speeds
1500 rpm, 2000 rpm, 2500 rpm and 3000 rpm are selected for this analysis and the rotation of the gear is limited to 3000 rpm. The tooth force is applied on the gear tooth profile at their highest point of single tooth contact that is at the pitch circle to the entire face width. The total force was distributed to an individual node on the line of contact. Figures 4.7(a) and 4.7(b) show the meshed model of both the circular root fillet and trochoidal root fillet gear pair. Similarly, Figures 4.8(a) and 4.8(b) shows the contact shear stress obtained for the circular root fillet gear pair and the trochoidal root fillet gear pair at 1500 rpm respectively. Figures 4.9(a) and 4.9(b) show the contact shear stress for the circular root fillet gear pair and the trochoidal root fillet gear pair at 2000 rpm. Similar results obtained at 2500 rpm and 3000 rpm are given in Figures 4.10 and 4.11.

**Figure 4.7(a)** FEA meshed model of a gear pair with circular root fillet
Figure 4.7(b) FEA meshed model of a gear pair with trochoidal root fillet

Figure 4.8(a) Contact shear stress in circular root fillet gear at 1500 rpm
Figure 4.8(b) Contact shear stress in trochoidal root fillet gear at 1500 rpm

Figure 4.9(a) Contact shear stress in circular root fillet gear at 2000 rpm
Figure 4.9(b) Contact shear stress in trochoidal root fillet gear at 2000 rpm

Figure 4.10(a) Contact shear stress in circular root fillet gear at 2500 rpm
Figure 4.10(b) Contact shear stress in trochoidal root fillet gear at 2500 rpm

Figure 4.11(a) Contact shear stress in circular root fillet gear at 3000 rpm
Figure 4.11(b) Contact shear stress in trochoidal root fillet gear at 3000 rpm

The induced contact shear stress, tooth deflection over the line of contact and derived tooth stiffness are presented in Table 4.6.

Table 4.6 FEA results - Contact shear stress for 13 teeth gear

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>Deflection (mm)</th>
<th>Contact shear stress (N/mm²)</th>
<th>Stiffness (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trochoidal</td>
<td>Circular</td>
<td>Trochoidal</td>
</tr>
<tr>
<td>1500</td>
<td>0.019601</td>
<td>0.010894</td>
<td>451.594</td>
</tr>
<tr>
<td>2000</td>
<td>0.014161</td>
<td>0.007904</td>
<td>326.113</td>
</tr>
<tr>
<td>2500</td>
<td>0.014204</td>
<td>0.006527</td>
<td>287.021</td>
</tr>
<tr>
<td>3000</td>
<td>0.010903</td>
<td>0.005441</td>
<td>250.904</td>
</tr>
</tbody>
</table>
4.6 RESULTS AND DISCUSSION

The induced bending stress (von Mises) at 2000 rpm corresponding tooth deflection and stiffness of 13 teeth gear provided with circular and trochoidal root fillet are presented in Table 4.4. It is observed from Table 4.4 that the deflection of both the circular and the trochoidal root fillet gear is more or less the same at 2000 rpm, but looking into the bending stress (von Mises) it is 442.457 N/mm$^2$ for circular root fillet gear and it is found to be 447.435 N/mm$^2$ for trochoidal root fillet gear. Table 4.6 gives the contact shear stress and corresponding deflection obtained using ANSYS corresponding to various root fillets considered for analysis. It is evident from Table 4.6 that the deflection (0.005441 mm) for 13 teeth gear having circular root fillet is lesser than that of the gear having trochoidal root fillet (0.010903 mm) at 3000 rpm. The contact shear stress for the given load is less in circular root fillet design irrespective of the speeds when compared to the trochoidal root fillet design.

The contact shear stress for the gear teeth provided with circular root fillet was 206.085 N/mm$^2$ at 3000 rpm and is found to be 250.904 N/mm$^2$ for trochoidal root fillet design. It is well understood from ANSYS results (Table 4.4 and Table 4.6) that the obtained bending stress (von Mises) and contact shear stress values are the least for circular root fillet gear irrespective of the speed than trochoidal root fillet gear. Figure 4.12 shows the deflection judgment for circular and trochoidal root fillet gear at various speeds. Similarly, Figure 4.13 enables to predict how the contact shear stress is varying for the change in root fillet at various speeds. Contact shear stress decreases gradually with increase in speed for circular root fillet design.
Figure 4.12 Speed Vs deflection comparison

Figure 4.13 Speed Vs contact shear stress comparison
4.7 CONCLUSION

The investigation result infers that the gear tooth deflection in the circular root fillet design is less when compared to the trochoidal root fillet design. Further, there is an appreciable reduction in the bending stress and the contact shear stress for circular root fillet design in comparison to that of trochoidal root fillet design. From the foregoing analysis it is found that the circular root fillet design is more apt for less number of teeth and whatever may be the pinion speed. ANSYS results indicate that the gears made of circular root fillet yield better strength (reduced bending and contact shear stress) thereby improving the fatigue life of the gear material.