Chapter 2

COMPUTATIONAL PROCEDURE OF SPECTRAL PROPERTIES OF THE TCAF MODEL

In this Chapter, we shall discuss the algorithm of the serial Monte Carlo code and its parallelization process. Next, we shall discuss how this code is used to study the spectral properties of TCAF in presence of outflows.

2.1 Monte Carlo code for Comptonization and its parallelization

Here we briefly discuss the algorithm of computation of Comptonized spectrum using a Monte Carlo code. The algorithm is similar to the one used by Laurent & Titarchuk (1999) and Ghosh et al. (2009). Then, the implementation of parallelization will be described in detail.

In our simulation, each photon is tracked beginning from its origin till its detection. The photon source may be a point source or any disk (e.g., a standard SS73 disk) or distributed within the electron cloud (e.g., bremsstrahlung, synchrotron etc.). Its initial energy may be drawn from the Planck’s distribution function (for blackbody type source) or from any other given distribution e.g., power-law etc. It can be given any initial preferential direction or it may be emitted isotropically. So, once a photon is generated within the accretion disk, we know its location (coordinate), initial energy and initial direction of motion. Certain scattering condition is associated with each photon. For the present case, we set a target optical depth $\tau_c$ at which a particular photon will suffer scattering. This value is computed from the exponential law $P(\tau) = \exp(-\tau)$ (Laurent & Titarchuk 1999). Next, the op-
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Figure 2.1: Flowchart of the serial code (left) and its parallelization (right).

...thickness $\tau = n_e \sigma l$ along the photon path is integrated taking into account the variation in the electron number density $n_e$ and the scattering cross-section $\sigma$. If integrated $\tau$ for that particular photon becomes $\tau_c$ before leaving the electron cloud, a Compton scattering is simulated following PSS83. After the scattering, the direction as well as energy of the photon change and it is again tracked in the same way as mentioned above. On the other hand, if $\tau < \tau_c$ before leaving the Compton cloud, the photon remains unscattered. This way, each photon is tracked till its escape from the electron cloud or it is absorbed by the black hole. Once a photon is out, it is captured and used for spectrum determination.

In the Monte Carlo code, the initial properties of a photon (e.g., its location, energy etc.), scattering criteria (e.g., target optical depth, mean free path of photons etc.) and the scattering event (e.g., selection of scattering electron, its momentum and energy etc.) are modelled using Monte Carlo technique (PSS83). The pseudo-random number generator that has been used here is taken from Wichmann & Hill (1982).

In a serial run, a single processor processes photons one by one and at the end, we get the full spectrum. For a steady state system, the behavior of one photon...
inside the accretion disk is independent of another one, i.e., processing of photons are mutually exclusive. So, it does not matter if we process more than one photon simultaneously. In a parallelized code, that is what is done. Depending on the number of available processors, the number of simultaneously injected photons is decided. So, if we wish to inject $N_{\text{ph}}$ number of photons and we have $p$ number of available processors, then each processor processes $N_{\text{ph}}/p$ number of photons simultaneously in the same electron cloud. In Fig. 2.1, we show the flowcharts of the serial code and after its parallelization. In the serial code, 'Block A' is the main computational block which is called $N_{\text{ph}}$ times under a 'Do' loop. In the parallel code, we have broken this single 'Do' loop in $N_{\text{ph}}/p$ loops and run the same block A in $p$ number of processors. At the end, we collect the results from each processors and sum them to get the final spectrum.

One important practical problem of parallelization in this case is the choice of set of seed values for the pseudo-random number generator used here. In a serial run, the initial seed values are chosen such that the cycle length is very long, and the cycle length depends on the initial seed values. In parallelized case, since the same program with the same input files runs on different processors, same seed values will produce exactly same results. For this, we have to be careful about the initial seed values for individual processors. We cannot choose any arbitrary seed values for each processors since that may hamper the cycle length. To circumvent this problem, we pick up some numbers from the series that is generated when the pseudo-random number generator is run. We use these numbers as the initial seed values for different processors. We have to make sure that these selected numbers are far away from each other in the series so that no repetition occurs and we do not get same results from different processors.

2.1.1 Parallelization technique

The parallelization has been done using Message-Passing Interface (MPI). As the name suggests, the communication between the multiple processors is done by message-passing. MPI is a library of functions and macros that can be used in C, FORTRAN, and C++ programs. Here, we have used MPI FORTRAN functions for parallelizing our Monte Carlo code written in FORTRAN.

There are many functions in MPI, out of which only a handful have been utilized here. Here, in brief, we describe the functions that we have used. More details can be found in Pacheco & Ming (1997).

**MPI\_init and MPI\_finalize** – The first function must be called before any
other MPI function is called. After a program has finished using MPI library, the second function must be called. The syntaxes are as follows:

\[
\begin{align*}
\text{MPI.init}(\text{Ierr}) \\
\text{MPI.finalize}(\text{Ierr}) \\
\end{align*}
\]

The argument contains an error code. This argument is generally used in every FORTRAN MPI routines.

**MPI.COMM.Rank** - This function gives rank to the processors being used. Its syntax is as follows:

\[
\text{MPI.COMM.Rank} (\text{Comm}, \text{Myrank}, \text{Ierr})
\]

The first argument is a communicator. A communicator is a collection of processors that communicate among themselves and take part in message passing when the program begins execution. In our program, we have used the communicator 'MPI.COMM.WORLD'. It is pre-defined in MPI and consists of all the processors running when program execution begins.

**MPI.COMM.SIZE** - This function is used to know the number of processors that is executing the program. Its syntax is:

\[
\text{MPI.COMM.SIZE} (\text{Comm}, \text{P}, \text{Ierr})
\]

'P' gives the number of processors. This number may be used for various purposes. In our program, we have explicitly used this number while message-passing.

**MPI.send and MPI.recv** - These are the two functions we find mostly used in message-passing between different processors. The syntaxes are as follows:

\[
\begin{align*}
\text{MPI.Send} (\text{Message}, \text{Count}, \text{Datatype}, \text{Dest}, \text{Tag}, \text{Comm}, \text{Ierr}) \\
\text{MPI.Recv} (\text{Message}, \text{Count}, \text{Datatype}, \text{Source}, \text{Tag}, \text{Comm}, \text{Status}, \text{Ierr})
\end{align*}
\]

The first one sends a message to a designated processor, whereas the second one receives from a processor. The contents of 'Message' is stored in a block of memory referenced by the argument message. 'Message' may be a single number, an array of numbers or characters. 'Count' and 'Datatype' are the count values and the MPI type datatype of 'Message', respectively. This type is not the Fortran type. In the following list, some of the MPI types and their corresponding Fortran types are listed.
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<table>
<thead>
<tr>
<th>MPI datatype</th>
<th>Fortran datatype</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPLinteger</td>
<td>Integer</td>
</tr>
<tr>
<td>MPLreal</td>
<td>Real</td>
</tr>
<tr>
<td>MPLdouble_precision</td>
<td>Double precision</td>
</tr>
<tr>
<td>MPLcomplex</td>
<td>Complex</td>
</tr>
</tbody>
</table>

'Dest' and 'Source' are two integer variables to mark the rank of the destination and the source processors of 'Message', respectively. The 'Tag' is also an integer that is used to distinguish messages received from a single processor.

MPI_Barrier – This function provides a mechanism for synchronizing all the processors in the communicator. Each processor pauses until every processors in communicator have called this function. It has the following syntax:

MPI_Barrier(Comm, ierr)

MPI_Bcast – This is a collective communication function, meaning the communication where usually all the processors are involved. Using this command a single processor can send the same data to every processors in a single call. The 'Send-Recv' commands usually involve two processors - one sender and other receiver, whereas in collective communications like broadcast, number of sender is one but receivers are all the other processors in the communicator. The syntax is as follows:

MPI_Bcast(Message, Count, Datatype, Root, Comm, ierr)

All the processors in a communicator have to call this function with the same argument 'Root'. The contents of 'Message' in processor 'Root' is broadcasted to all the processors.

MPI_Reduce – This is another collective communication function. This is a global reduction operation in which all the processors contribute data which is combined using a binary operation. The typical binary operations are addition, max, min, product etc. The syntax is as follows:

MPI_Reduce(Operand, Result, Count, Datatype, Operation, Root, Comm, ierr)

MPI_Reduce combines 'Operand' stored in different processors to 'Results' in 'Root' using operation 'Operation'. In the following table, we present some predefined operations.
### Operation Name and Meaning

<table>
<thead>
<tr>
<th>Operation Name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPI_sum</td>
<td>Addition</td>
</tr>
<tr>
<td>MPI_max</td>
<td>Maximum</td>
</tr>
<tr>
<td>MPI_min</td>
<td>Minimum</td>
</tr>
<tr>
<td>MPI_prod</td>
<td>Product</td>
</tr>
</tbody>
</table>

**MPI_Gather** - This collective communication function is used to gather data in one processor from all other processors. The syntax is as follows:

```c
MPIGather(Send_Val, Send_Count, Send_Type, Recv_Val, Recv_Count, Recv_Type, Root, Comm, ierr)
```

Each processor sends the contents of 'Send_Val' to processor 'Root' and the 'Root' concatenates the received data in processor rank order, i.e., data from processor 0 is followed by data from processor 1, which is followed by processor 2 and so on.

### 2.2 Spectral properties of TCAF in presence of a jet

We use the above mentioned Monte Carlo code to study the spectral properties of a TCAF in presence of a jet around a galactic black hole (Ghosh, Garain, Chakrabarti & Laurent 2010, hereafter GGCL10). Computation of the spectral characteristics have so far concentrated only on the advective accretion flows (CT95; Chakrabarti & Mandal 2006; Dutta & Chakrabarti 2010) and the jet was not included. In the Monte Carlo simulations of Laurent & Titarchuk (2007) outflows in isolation were considered and not in conjunction with inflows. In the following work, we obtain the outgoing spectrum in presence of both inflows and outflows (GGCL10). We also include a Keplerian disk inside an advective flow which is the source of the soft photons. We show how the spectrum depends on the flow parameters of the inflow, such as the accretion rates and the shock strength. These results, as such, were anticipated earlier (C99; Das, Chattopadhyay, Nandi & Chakrabarti 2001). The post-shock region being denser and hotter, behaves as the so-called 'Compton cloud' in the classical model of Sunyaev and Titarchuk (1980) and is known as the CENtrifugal pressure supported Boundary Layer or CENBOL. The variation of the size of the Compton cloud, and therefore the basic Comptonized component of the spectrum is thus a function of the basic parameters of the flow, such as the energy, accretion rate and the angular momentum. Since the intensity of soft...
2.2.1 Simulation set up

In Fig. 2.2, we present a schematic diagram of our simulation set up (GGCL10). The components of the hot electron clouds, namely, the CENBOL, the jet and the sub-Keplerian flow, intercept the soft photons emerging out of the Keplerian disk and reprocess them via inverse-Compton scattering. A photon may undergo a single, multiple or no scattering at all with the hot electrons in between its emergence from the Keplerian disk and its detection by the telescope at a large distance. The photons which enter the black holes are absorbed. The CENBOL, though toroidal in nature (CT95), is chosen to be of spherical shape for simplicity. The sub-Keplerian inflow in the pre-shock region is assumed to be of wedge shape of a constant angle. The outflow, which emerges from the CENBOL in this picture is also assumed to be of constant conical angle.
2.2.2 Temperature, velocity and density profiles inside the Compton cloud

We assume the black hole to be non-rotating and we use the pseudo-Newtonian potential (PW80) to describe the geometry around a black hole (Chapter 1). This potential is \[-\frac{1}{2(r-1)}\] (\(r\) is in the unit of Schwarzschild radius \(r_g = 2GM/c^2\)). Velocities and angular momenta are measured accordingly.

As a simple example, we use the Bondi accretion and wind solutions to compute the density, velocity and temperature in the inflowing (inside sub-Keplerian inflow and CENBOL) and outflowing regions of the CENBOL, respectively (GGCL10). Bondi solution (Bondi 1952) was originally done to describe the accretion of matter which is at rest at infinity onto a star at rest. The motion of matter is steady and spherically symmetric. The equation of the motion of this matter around the black hole in the steady state is given by (C90),

\[
\frac{du}{dr} + \frac{1}{\rho} \frac{d\rho}{dr} + \frac{1}{2(r-1)^2} = 0.
\]

Here, \(u\) is the velocity, \(\rho\) is the density and \(P\) is the thermal pressure. This is the Eulerian equation written in the spherical polar coordinate system \([r, \theta, \phi]\). \(\theta\) and \(\phi\) derivatives have been removed because of spherical symmetry and the time derivative has been removed since we consider the steady state. Integrating this equation, we get the expression of the conserved specific energy as (C90),

\[
e = \frac{u^2}{2} + \frac{na^2}{2} - \frac{1}{2(r-1)}.
\] (2.1)

Here, \(a\) is the adiabatic sound speed, given by \(a = \sqrt{\gamma P/\rho}\), \(\gamma\) being the adiabatic index and is equal to \(\frac{4}{3}\) in our case. The conserved mass flux equation, as obtained from the continuity equation, is given by (C90),

\[
\dot{M} = \Omega \rho u r^2,
\] (2.2)

where, \(\Omega\) is the solid angle subtended by the flow. For an inflowing matter, \(\Omega\) is given by,

\[
\Omega_{in} = 4\pi \sin \Psi,
\]

where, \(\Psi\) is the half-angle of the conical inflow (see. Fig. 2.2). For the outgoing matter, the solid angle is given by,

\[
\Omega_{out} = 4\pi (1 - \cos \Psi),
\]
where, $\Phi$ is the half-angle of the conical outflow (see, Fig. 2.2). From Eq. (2-2), we get

$$\dot{\mu} = \alpha^{2\eta} \Omega^2.$$  \hspace{1cm} (2-3)

The quantity $\dot{\mu} = \delta r^2 K^2$ is called the entropy accretion rate (Chakrabarti 1989b; C90), $K$ being the constant measuring the entropy of the flow, and $\eta = \frac{1}{r-1}$ is called the polytropic index. We take derivative of Eq. (2-1) and (2-3) with respect to $r$ and eliminating $\frac{\dot{\mu}}{\Omega^2}$ from both the equations (C90), we get the gradient of the velocity as,

$$\frac{du}{dr} = \frac{\frac{1}{2(r-1)^2} - \frac{2\eta}{r}}{\frac{a^2}{u} - u}. \hspace{1cm} (2-4)$$

This equation is solved numerically using 4th order Runge Kutta method. Solving these equations, we obtain the radial variations of $u$, $\alpha$ and finally the temperature profile using $T(r) = \frac{m_p k_B}{\mu m_e} \frac{\rho}{\kappa_B}$, where $\mu = 0.5$ is the mean molecular weight, $m_p$ is the proton mass and $k_B$ is the Boltzmann constant. Using Eq. (2-2), we calculate the mass density $\rho$, and hence, the number density variation of electrons inside the Compton cloud. We ignore the electron-positron pair production inside the cloud.

The flow is supersonic in the pre-shock region and sub-sonic in the post-shock (CENBOL) region. The shock forms at the location of the CENBOL surface (CT95). We chose this surface at a location where the Mach number $M = 2$. This location depends on the specific energy $\epsilon$. In our simulation, we have chosen $\epsilon = 0.015$ so that we get $R_s = 10$ (GGCL10). We simulated a total of six cases: for Cases 1(a-c), we chose the halo accretion rate $\dot{\mu}_h = 1$ and the disk accretion rate $\dot{\mu}_d = 0.01$, and for Cases 2(a-c), the values are listed in Table 2 (GGCL10). The velocity variation of the sub-Keplerian flow is the inflowing Bondi solution (pre-shock point). The density and the temperature of this flow have been calculated according to the above mentioned formulas. Inside the CENBOL, both the Keplerian and the sub-Keplerian components are merged. The velocity variation of the matter inside the CENBOL is assumed to be the same as the Bondi accretion flow solution reduced by the compression ratio $R$ due to the shock. The compression ratio (i.e., the ratio between the post-shock and pre-shock densities) $R$ is also used to compute the density and the temperature profile.

When the outflow is adiabatic, the ratio of the outflow to the inflow rate is given by (Das et al., 2001),

$$R_{in} = \frac{\Omega_{out}}{\Omega_{in}} \left( \frac{f_0}{47} \right)^\frac{3}{2} \frac{R}{2} \left[ \frac{4}{3} \left( \frac{8(R-1)}{R^2} - 1 \right) \right]^{3/2} \hspace{1cm} (2-5)$$
where, $f_0 = \frac{R^2}{K-1}$ and we have used $n = 3$ for a relativistic flow. Using this and the velocity variation obtained from the wind branch of the Bondi solution, we compute the density variation inside the jet. In our simulation, we have used $\Phi = 58^\circ$ and $\Psi = 32^\circ$ (GGCL13). Figure 2.3 shows the variation of the percentage of matter in the outflow for these particular parameters.

### 2.2.3 Keplerian disk

The soft photons are produced from a Keplerian disk whose inner edge coincides with the CENBOL surface, while the outer edge is located at $500r_p$. The source of soft photons has a multi-color blackbody spectrum coming from a standard disk (SS73). We assume the disk to be optically thick and the opacity due to free-free absorption is more important than the opacity due to scattering. The emission is blackbody type with the local surface temperature (Eq. 1-1):

$$T(r) = 5 \times 10^7 (M_{bh})^{-1/2}(\dot{m}_d_{17})^{1/4}(2r)^{-3/4} \left[1 - \sqrt{\frac{3}{r}}\right]^{1/4} K.$$

Photons are emitted from both the top and bottom surfaces of the disk at each radius. Total number of photons emitted from the disk surface at a radius $r$ is
obtained by integrating over all frequencies ($\nu$) and is given by,

$$n_\nu(r) = \frac{4\pi}{c^3} \left( \frac{kT}{\hbar} \right)^3 \times 1.202 \text{ cm}^{-2} \text{ s}^{-1}. \quad (2-6)$$

Thus, the disk between radius $r$ to $r + \delta r$ produces $dN(r)$ number of soft photons:

$$dN(r) = 4\pi r^2 n_\nu(r) \delta r \text{ s}^{-1}. \quad (2-7)$$

The soft photons are generated isotropically between the inner and the outer edges of the Keplerian disk. Their positions are randomized using the above distribution function (Eq. 2-7). All the results of the simulations presented here have used the number of injected photons as $6.4 \times 10^8$. We chose $M_\text{in} = 10M_\odot$ and $\delta r = 0.5r_g$ (GGCL10).

### 2.2.4 Simulation procedure

The simulation procedure is the same as used in Ghosh et al. (2009) and GGCL10. To begin a Monte Carlo simulation, we generate photons from the Keplerian disk with randomized locations as mentioned in the earlier Section. The energy of the soft photons at radiation temperature $T(r)$ is calculated using the Planck's distribution formula, where the number density of the photons $[n_\nu(E)]$ having an energy $E$ is expressed by (PSS83),

$$n_\nu(E) = \frac{1}{2\zeta(3)} b^3 E^2 (e^{bE} - 1)^{-1},$$

where, $b = 1/k_B T(r)$ and $\zeta(3) = \sum_{i=1}^{\infty} i^{-3} = 1.202$, the Riemann zeta function. Using another set of random numbers, we obtain the direction of the injected photon and with yet another random number, we obtain a target optical depth $\tau_c$ at which the scattering takes place. The photon is followed within the sub-Keplerian matter till the total optical depth ($\tau$) reached $\tau_c$. The increase in optical depth ($d\tau$) during its traveling of a path of length $dl$ inside the sub-Keplerian matter is given by:

$$d\tau = \rho_e dl,$$

where $\rho_e$ is the electron number density.

The total scattering cross section $\sigma$ is given by Klein-Nishina formula:

$$\sigma = \frac{2\pi r_g^2}{x} \left[ \left(1 - \frac{4}{x} - \frac{8}{x^2}\right) \ln(1+x) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(1+x)^2} \right],$$

where, $x$ is given by,

$$x = \frac{2E}{m_e c^2} \left(1 - \frac{\mu}{c} \right).$$
Figure 2.4: (a-c): Velocity (left), density (middle) and the temperature (right) profiles of the Cases 1(a-c) as described in Table 1 are shown with solid \((R = 2)\), dotted \((R = 4)\) and dashed \((R = 6)\) curves. \(m_d = 0.01\) and \(m_h = 1\) were used (GGCL10).

Here, \(r_e = e^2/m_e c^2\) is the classical electron radius and \(m_e\) is the mass of the electron.

We have assumed here that a photon of energy \(E\) and momentum \(\frac{2}{c}\Omega\) is scattered by an electron of energy \(\gamma m_e c^2\) and momentum \(p = \gamma m_e v\), with \(\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\) and \(\mu = \Omega \cdot v\). At the point where a scattering is allowed to take place, the photon selects an electron and the energy exchange is computed using the Compton or inverse-Compton scattering formula. The electrons are assumed to obey relativistic Maxwell distribution inside the sub-Keplerian matter. The number \(dN(p)\) of Maxwellian electrons having momentum between \(p\) to \(p + dp\) is expressed by,

\[
dN(p) \propto \exp\left[-\frac{(p^2 c^2 + m_e^2 c^4)^{1/2}}{kT_e}\right] dp.
\]

2.2.5 Results and discussions

In a given simulation, we assume one Keplerian disk rate \((m_d)\) and one sub-Keplerian halo rate \((m_h)\) (GGCL10). The specific energy of the halo provides hydrodynamic (such as number density of the electrons and the velocity variation) and the thermal properties of matter. The shock location of the CENBOL is chosen where the Mach number \(M = 2\) for simplicity and the compression ratio \(R\) (i.e., jump in density) at the shock is assumed to be a free parameter.
In Fig. 2.4(a-c), we present the velocity, electron number density and temperature variations as a function of the radial distance from the black hole for specific energy $\epsilon = 0.015$. $m_\text{d} = 0.01$ and $m_\text{h} = 1$ were chosen. Three cases were run by varying the compression ratio $R$. These are given in Column 2 of Table 1. The corresponding percentage of matter going in the outflow is also given in Column 2. In the left panel, the bulk velocity variation is shown. Since we chose the pseudo-Newtonian potential, the radial velocity is not exactly unity at $r = 1$, the horizon, but it becomes unity just outside. In order not to overestimate the effects of bulk motion Comptonization which is due to the momentum transfer of the moving electrons to the photons, we shift the horizon just outside $r = 1$ where the velocity is unity. The solid, dotted and dashed curves are the velocity for $R = 2$ (Case 1a), 4 (Case 1b) and 6 (Case 1c) respectively. The same line style is used in other panels. The velocity variation within the jet does not change with $R$, but the density (in the unit of $cm^{-3}$) does (middle panel). The double dot-dashed line gives the velocity variation of the matter within the jet for all the above cases. The arrows show the direction of the bulk velocity (radial direction in accretion, vertical direction in jets). The last panel gives the temperature (in keV) of the electron cloud in the CENBOL, jet, sub-Keplerian and Keplerian disk. Big dash-dotted line gives the temperature profile inside the Keplerian disk.
In Fig. 2.5(a-c), we show the velocity (left), number density of electrons (middle) and temperature (right) profiles of Cases 2(a-c) as described in Table 2. Here we have fixed $n_{e,0} = 1.5$ and $n_{h,0}$ is varied: $n_{h,0} = 0.5$ (solid), 1 (dotted) and 1.5 (dashed). No jet is present in this case ($R = 1$). To study the effects of bulk motion Comptonization, the temperature of the electron cloud has been kept low for these cases. The temperature profile of the Keplerian disk for the above cases has been marked as ‘Disk’.

### Table 1 (GGCL10)

<table>
<thead>
<tr>
<th>Case</th>
<th>$R, P_m$</th>
<th>$N_{int}$</th>
<th>$N_{e,0}$</th>
<th>$N_{cenbol}$</th>
<th>$N_{jet}$</th>
<th>$N_{subkep}$</th>
<th>$N_{cap}$</th>
<th>$p$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>2, 5.8</td>
<td>2.7e+8</td>
<td>4.03e+8</td>
<td>1.35e+7</td>
<td>7.48e+7</td>
<td>8.39e+8</td>
<td>3.34e+5</td>
<td>63</td>
<td>0.43</td>
</tr>
<tr>
<td>1b</td>
<td>4, 97</td>
<td>2.7e+8</td>
<td>4.14e+8</td>
<td>2.38e+6</td>
<td>1.28e+8</td>
<td>8.58e+8</td>
<td>3.27e+5</td>
<td>65</td>
<td>1.05</td>
</tr>
<tr>
<td>1c</td>
<td>6, 37</td>
<td>2.7e+8</td>
<td>3.09e+8</td>
<td>5.35e+7</td>
<td>4.75e+7</td>
<td>8.26e+8</td>
<td>3.07e+5</td>
<td>62</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

In Table 1, we summarize the results of all the cases in Fig. 2.4(a-c). In Column 1, various cases are marked. In Column 2, the compression ratio ($R$) and percentage $P_m$ of the total matter that is going out as outflow (see, Fig. 2.4) are listed. In Column 3, we show the total number of photons (out of the total injection of $6.4 \times 10^8$) intercepted by the CENBOL and jet ($N_{int}$) combined. Column 4 gives the number of photons ($N_{e,0}$) that have suffered Compton scattering inside the flow. Column 5, 6 and 7 show the number of scatterings which took place in the CENBOL ($N_{cenbol}$), in the jet ($N_{jet}$) and in the pre-shock sub-Keplerian halo ($N_{subkep}$) respectively. A comparison of them will give the relative importance of these three sub-components of the sub-Keplerian disk. The number of photons captured ($N_{cap}$) by the black hole is given in Column 8. In Column 9, we give the percentage $p$ of the total injected photons that have suffered scattering through CENBOL and the jet. In Column 10, we present the energy spectral index $\alpha$ [$I(E) \sim E^{-\alpha}$] obtained from our simulations.

### Table 2 (GGCL10)

<table>
<thead>
<tr>
<th>Case</th>
<th>$n_{h,0}, n_{e,0}$</th>
<th>$N_{int}$</th>
<th>$N_{e,0}$</th>
<th>$N_{cenbol}$</th>
<th>$N_{jet}$</th>
<th>$N_{subkep}$</th>
<th>$N_{cap}$</th>
<th>$p$</th>
<th>$\alpha_1, \alpha_2$</th>
</tr>
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<tbody>
<tr>
<td>2a</td>
<td>0.5, 1.5</td>
<td>1.05e+6</td>
<td>2.13e+8</td>
<td>7.41e+5</td>
<td>3.12e+8</td>
<td>1.66e+5</td>
<td>33.34</td>
<td>33.34</td>
<td>-0.09, 0.1</td>
</tr>
<tr>
<td>2b</td>
<td>1.0, 1.5</td>
<td>1.22e+6</td>
<td>3.37e+8</td>
<td>1.01e+6</td>
<td>6.82e+8</td>
<td>2.33e+5</td>
<td>32.72</td>
<td>32.72</td>
<td>-0.13, 0.75</td>
</tr>
<tr>
<td>2c</td>
<td>1.5, 1.5</td>
<td>1.34e+6</td>
<td>4.15e+8</td>
<td>1.28e+6</td>
<td>1.11e+9</td>
<td>2.29e+5</td>
<td>64.87</td>
<td>64.87</td>
<td>-0.13, 1.3</td>
</tr>
</tbody>
</table>

In Table 2, we summarize the results of simulations where we have varied $n_{h,0}$ for a fixed value of $n_{e,0}$. In all of these cases no jet comes out of the CENBOL (i.e., $R = 1$). In the last column, we list two spectral slopes $\alpha_1$ (from 10 to 100keV) and $\alpha_2$ (due to the bulk motion Comptonization). Here, $N_{ms}$ represents the photons that have suffered scattering between $R_g = 3$ and the horizon of the black hole.
In Fig. 2.6, we show the variation of the spectrum in the three simulations presented in Fig. 2.4(a-c). The dashed, dash-dotted and double dot-dashed lines are for $R = 2$ (Case 1a), $R = 4$ (Case 1b) and $R = 6$ (Case 1c), respectively. The solid curve gives the spectrum of the injected photons. Since the density, velocity and temperature profiles of the pre-shock, sub-Keplerian region and the Keplerian flow are the same in all these cases, we find that the difference in the spectrum is mainly due to the CENBOL and the jet. In the case of the strongest shock (compression ratio $R = 6$), only 37% of the total injected matter goes out as the jet. At the same time, due to the shock, the density of the post-shock region increases by a factor of 6. Out of the three cases, the effective density of the matter inside CENBOL is the highest and that inside the jet is the lowest in this case. Due to the shock, the temperature increases inside the CENBOL and hence the spectrum is the hardest. Similar effects are seen for moderate shock ($R = 4$) and to a lesser extent, the low strength shock also ($R = 2$). When $R = 4$, the density of the post-shock region increases by the factor of 4 while almost 97% of total injected matter (Fig. 2.4) goes out as the jet reducing the matter density of the CENBOL significantly. From Table 1, we find that the $N_{\text{cendot}}$ is the lowest and $N_{\text{jet}}$ is the highest in this case (Case 1b). This decreases the up-scattering and increases the down-scattering of the photons.
Figure 2.7: (a-c): Variation of the components of the emerging spectrum with the shock strength ($R$). The dashed curves correspond to the photons emerging from the CENBOL region and the dash-dotted curves are for the photons coming out of the jet region. The solid curve is the spectrum for all the photons that have suffered scatterings (GGCL10). See, the text for details.

This explains why the spectrum is the softest in this case (Chakrabarti 1998b). In the case of low strength shock ($R = 2$), 57% of the inflowing matter goes out as jet, but due to the shock the density increases by factor of 2 in the post-shock region. This makes the density similar to a case as though the shock did not happen except that the temperature of CENBOL is higher due to the shock. So the spectrum with the shock would be harder than when the shock is not present. The disk and the halo accretion rates used for these cases are $\dot{m}_d = 0.01$ and $\dot{m}_h = 1$.

In Fig. 2.7, we show the components of the emerging spectrum for all the three cases presented in Fig. 2.6. The solid curve is the intensity of all the photons, which suffered at least one scattering. The dashed curve corresponds to the photons emerging from the CENBOL region and the dash-dotted curve is for the photons coming out of the jet region. We find that the spectrum from the jet region is softer than the spectrum from the CENBOL. As $N_{jet}$ increases and $N_{cenbol}$ decreases, the spectrum from the jet becomes softer because of two reasons. First, the temperature of the jet is lesser than the CENBOL, so the photons get lesser amount of energy from thermal Comptonization. Second, the photons are down-scattered by the outflowing jet which eventually make the spectrum softer. We note that a larger number
Figure 2.8: The spectrum which includes the effects of bulk motion Comptonization. Solid (Injected), dotted ($\dot{M}_h = 0.5$), dashed ($\dot{M}_h = 1$), dash-dotted ($\dot{M}_h = 1.5$). $\dot{M}_d = 1.5$ for all the cases. Keplerian disk extends up to $3.1r_g$. Table 2 summarizes the parameter used and the simulation results for these cases (GGCL10).

of photons are present in the spectrum from the jet than the spectrum from the CENBOL, which shows the photons have actually been down-scattered. The effect of down-scattering is larger when $R = 4$. For $R = 2$ also there is significant amount of down scattered photons. But this number is very small for the case $R = 6$ as $N_{\text{cenbol}}$ is much larger than $N_{\text{jet}}$. So most of the photons are up-scattered. The difference between the total (solid) and the sum of the other two regions gives an idea of the contribution from the sub-Keplerian halo located in the pre-shock region. In our choice of geometry (half angles of the disk and the jet), the contribution of the pre-shock flow is significant. In general it could be much less. This is especially true when the CENBOL is farther out.

In Fig. 2.8, the emerging spectra due to the bulk motion Comptonization is shown when the halo rate is varied. The solid curve is the injected spectrum (modified blackbody). The dotted, dashed and dash-dotted curves are for $\dot{m}_h = 0.5$, 1 and 1.5, respectively. $\dot{m}_d = 1.5$ for all the cases. The Keplerian disk extends up to $3r_g$. Table 2 summarizes the parameters used and the results of the simulations. As the halo rate increases, the density of the CENBOL also increases causing a larger number of scatterings. From Fig. 2.5(a), we notice that the bulk velocity variation of the electron cloud is the same for all the four cases. Hence, the case where the density is maximum, the photons got energized to a very high value due to repeated
scatterings with that high velocity cold matter. As a result, there is a hump in the spectrum around 100 keV energy for all the cases. We find the signature of two power-law regions in the higher energy part of the spectrum. The spectral indices are given in Table 2. It is to be noted that $\alpha_2$ is increased with $\tilde{m}_h$ and becomes softer for higher $\tilde{m}_h$. Our geometry here at the inner edge is conical, unlike a sphere in Laurent and Titarchuk (2001). This may be the reason why our slope is not the same as in Laurent and Titarchuk (2001) where $\alpha_2 = 1.9$. In Fig. 2.9, we present the components of the emerging spectra. As in Fig. 2.7, solid curves are the spectra of all the photons that have suffered scattering. The dashed and dash-dotted curves are the spectra of photons emitted from inside and outside of the marginally stable orbit ($3r_g$), respectively. The photons emitted inside the marginally stable radius are Comptonized by the bulk motion of the infalling matter. Here the jet is absent (GGCL10).

As mentioned earlier, to compute the spectral properties of the TCAF in this Chapter, we have used the pseudo-Newtonian potential to describe the geometry around a non-rotating black hole. On the other hand, if we choose a rotating (Kerr)
black hole, the space-time geometry will change which will affect the structure of the accretion disk, which in turn may affect spectral properties. Computation of topologies of global viscous transonic accretion flow has been done by Chakrabarti (1996a, 1996b) by solving the general relativistic equations and by Chakrabarti & Mondal (2006) and Mondal & Chakrabarti (2006) using pseudo-Kerr potential. These works show that in general, shock in accretion flow forms closer around rotating black holes. Thus, size of CENBOL is smaller compared to that in case of a non-rotating black hole and we expect the number of intercepted soft photons by the CENBOL to be lower. Also, the inner edge of the Keplerian disk will extend to much closer to the black hole when it is rotating for the same set of flow parameters. This may result in increase in number of soft photons, though the overall luminosity may increase. However, as the shock forms closer, post-shock region becomes hotter and may thus contribute to very high energy X-rays and gamma-rays (Chakrabarti 1996a). Therefore, as a result of all these effects, we expect overall softer spectra with powerlaw tail extending to higher energies. However, detailed study of the spectral properties from the accretion disk around a rotating black hole is necessary.