Chapter Four

Effects of Increased Competition on the Choice of Quality in an Oligopolistic Framework

4.1. Introduction

Firms in Newly Industrialised Countries that produce for the international market often tend to concentrate on low quality products and this product choice appears to be stationary over long periods. This appears to be the case in Taiwan in 1970s [e.g., Chiu and Lai (1982), Little (1979), Yu (1983)]. Chiang and Masson (1988) offer an explanation for this pattern based on an adverse selection argument. They argue that buyers are often unable to distinguish between firms producing high quality products from those producing low quality goods, and so offer the same price to all suppliers irrespective of the quality that they produce individually. This is the classic lemons problem associated with experience goods, where firms have no credible means of signalling their product quality [Akerlof (1970)].

In Chiang and Masson (1988) the industry of a ‘small’ country is assumed to produce only for a world market. The customers in the world market do not have perfect information about each product’s quality and cannot judge individual firm’s quality. However, it is assumed that they can correctly perceive the average quality of the country’s products in the industry. The small country, producing for a world market, faces a given world price function, which varies positively with the perceived average quality level. Labour is the only input used in the production and homogeneous firms produce their outputs by using CRS technology. Given the number of firms, there is a single quality level in equilibrium since no firm gains by unilaterally deviating from the common quality level. The market is competitive and so price equals marginal cost at equilibrium. The main findings of the Chiang and Masson (1988) give two comparative static results. The most important one is the inverse relationship that is found to exist
between the number of firms in the industry and the equilibrium level of quality. The other one relates the wage rate, equilibrium level of quality and the number of firms in the industry. It states that the wage rate rises along with equilibrium quality as number of firms is reduced.

The analysis of Chiang and Masson (1988) is restrictive in the sense that the number of firms in the industry has to be restricted in order to get improved quality and that is why they prescribe consolidation of industries. Secondly, the formulation of the labour market is such that either the export sector of the domestic country has to be very large or the entire industry produces only one product. In our analysis we do not need to make any such restrictive assumptions. We concentrate on internal trade only where firms simultaneously choose the quality of the product in the first stage and the price in the second stage. We show that an increase in competition in the market does not necessarily lead to a secular shift from a high quality output norm to a low quality output standard as found in Chiang and Masson (1988). Change in the number of firms must be quite significant to change the equilibrium strategy profile: a small increase in the number of firms leaves the quality standard unchanged.

The chapter is organised as follows: Section 4.2 presents the basic model. Section 4.3 offers some concluding remarks.

### 4.2. The Model

Consider an ‘experience’ good market in which there are \( n > 1 \) firms that produce at most one unit each of a good that is not divisible. There are \( m \) identical consumers, \( m > n \), who buy at most one unit of the good. Each firm can choose to produce a high quality (\( Q^G \)) product or a bad quality (\( Q^B \)) product. Since, the good is an experience good, this quality choice is known only to the firm before actual consumption occurs. However, we assume that there is some amount of market information about qualities that is available to
consumers before they transact. In particular, we assume that each consumer knows the distribution of product qualities across firms before transaction takes place.

In formal language, we construct a two-stage game in which each firm chooses its quality level in the first stage, the distribution of quality levels is announced at the beginning of stage 2. Firms pick output prices after observing this distribution, and market transactions take place after that. Let the market price that a buyer is willing to pay for quality level Q be \( P(Q) \) with \( P(Q^G) > P(Q^B) \). Since there are \( m \) identical consumers, the market demand for the good product is given by

\[
D(P, Q^G) = m \quad \text{if } P \leq P(Q^G) \\
= 0 \quad \text{otherwise}
\]

and that for the bad product is

\[
D(P, Q^B) = m \quad \text{if } P \leq P(Q^B) \\
= 0 \quad \text{otherwise}
\]

Suppose, fraction \( \alpha \) of the firms produce quality level \( Q^G \) and the rest produce quality level \( Q^B \). Under incomplete information, a consumer facing a seller does not know the quality level chosen by the firm \( a \ priori \) and can expect that it will be good with probability \( \alpha \) and bad with probability \( (1- \alpha) \). Hence, the maximum expected value to the consumer from the product is \( \alpha P(Q^G) + (1- \alpha)P(Q^B) \). For notational convenience, we shall write this as \( P(\alpha Q^G + (1- \alpha)Q^B) \). In what follows we shall refer to \( \alpha Q^G + (1- \alpha)Q^B \) as the average quality of the product. Since \( Q^G \) and \( Q^B \) need not have specific numerical values, this is not an average in the usual sense of the term. It means that each consumer, given the information he has, expects the product to be of high quality with probability \( \alpha \) and low quality with probability \( (1- \alpha) \).
As firms can sell at most one unit, no firm makes any extra gain by undercutting the price of other firms. So all firms charge the same price, which is equal to the maximum expected value to the consumer from the product:

\[ P = P(\alpha), \text{ with } P \text{ being monotonically increasing in } \alpha \]  

(4.1)

Let \( y_j^Q \) be the output of firm \( j \) producing goods of quality grade \( Q \). It is assumed for simplicity, that labour is the only input used in the production. The firm produces its output by using CRS technology:

\[ y_j^Q = a(Q) L_j, \quad a(Q^G) < a(Q^B) \]  

(4.2)

where \( a(Q) \) is a quality dependent labour productivity parameter. Let \( a(Q) \) take up two values \( a(Q^G) = a^G \) for \( Q = Q^G \) and \( a(Q^B) = a^B \) for \( Q = Q^B \). \( L_j \) is the labour input used by the \( j \)th firm. Since each firm produces one unit of the product, it follows that

\[ y_j^Q = a(Q) L_j = 1 \]

i.e., \( L_j = \frac{1}{a(Q)} \)

The profit of firm \( j \) producing goods of quality \( Q_j \) is:

\[ \Pi_j = P(\alpha) y_j^Q - w L_j \]

\[ = P(\alpha) - \frac{w}{a(Q)} \]  

(4.3)

where \( w \) is the fixed wage rate (unlike Chiang and Masson (1988)).

A Two Firm Example:
To get an intuitive feel for the nature of equilibrium in this model we look at an example with two firms. Since the firm can be either a good quality firm or a bad quality firm we
can think of the following four possibilities:

1) Both firms produce good quality products. The payoff of each of the firm in this case is \( P(Q^G) - \frac{w}{a^G} \), since the maximum expected value to the consumer is \( Q^G \).

2) Both firms produce bad quality products. The payoff of each of the firm in this case is \( P(Q^B) - \frac{w}{a^B} \), since the maximum expected value to the consumer is \( Q^B \).

3) and 4) One firm produces good quality product and the other one produces bad quality product. The payoff of the good quality firm is \( P \left( \frac{Q^G + Q^B}{2} \right) - \frac{w}{a^G} \) and that of the bad quality firm is \( P \left( \frac{Q^G + Q^B}{2} \right) - \frac{w}{a^B} \), since the maximum expected value to the consumer in each of these two cases is \( \frac{Q^G + Q^B}{2} \).

\((Q^G, Q^G)\) is a Nash Equilibrium strategy profile if unilateral deviation is not profitable to a particular firm when the other firm produces good quality product. In other words, it must satisfy the following condition: \( P \left( \frac{Q^G + Q^B}{2} \right) - \frac{w}{a^G} < P(Q^G) - \frac{w}{a^G} \).

\((Q^B, Q^B)\) is a Nash Equilibrium strategy profile if it satisfies the following condition:
\[ P \left( \frac{Q^G + Q^B}{2} \right) - \frac{w}{a^G} < P(Q^B) - \frac{w}{a^B}; \] i.e., unilateral deviation does not pay.

\((Q^G, Q^B)\) or \((Q^B, Q^G)\) is a Nash Equilibrium strategy profile if the following two conditions are satisfied:
\[ P(Q^G) - \frac{w}{a^G} < P \left( \frac{Q^G + Q^B}{2} \right) - \frac{w}{a^B} \]
and $P(Q^B) - \frac{w}{a_B} < P\left(\frac{Q^G + Q^B}{2}\right) - \frac{w}{a_G}$.

So, for $(Q^G, Q^G)$ to be a unique Nash Equilibrium all the other three strategies, namely, $(Q^G, Q^B)$, $(Q^B, Q^G)$ and $(Q^B, Q^B)$ should be ruled out as Nash Equilibrium strategy profiles. Hence, it must satisfy the following conditions:

\[
P\left(\frac{Q^G + Q^B}{2}\right) - \frac{w}{a_B} < P(Q^G) - \frac{w}{a_G} \quad (4.4a)
\]

\[
P(Q^B) - \frac{w}{a_B} < P\left(\frac{Q^G + Q^B}{2}\right) - \frac{w}{a_G} \quad (4.4b)
\]

While condition (4.4b) rules out $(Q^B, Q^B)$ as Nash Equilibrium strategy profile, condition (4.4a) rules out $(Q^G, Q^B)$ or $(Q^B, Q^G)$ as a Nash Equilibrium strategy profile. For $(Q^B, Q^B)$ to be a unique Nash Equilibrium, inequalities in conditions (4.4a) and (4.4b) must be held in the opposite direction.

**Example With Three Firms:**

What happens if the number of firms increases?

To get a sense of the change, let us look at an example with three firms. With three firms we get eight possible combinations of good firms and bad firms in the market. Proceeding in the same way as that of two firms we derive the conditions to attain Nash Equilibrium. For $(Q^B, Q^B)$ to be a unique Nash Equilibrium we need the following three conditions to be satisfied:
While condition (4.5a) is required to have \((Q^G, Q^G, Q^G)\) as a Nash Equilibrium, conditions (4.5b) and (4.5c) rule out seven other possibilities. The three conditions together establish \((Q^G, Q^G, Q^G)\) as a unique Nash Equilibrium.

For \((Q^B, Q^B, Q^B)\) to be a unique Nash Equilibrium the three conditions (4.5a), (4.5b) and (4.5c) need to be satisfied with their inequalities holding in the reverse order.

By applying the method of Induction we derive the conditions for Nash Equilibrium in the following Propositions when there are \(n\) number of firms in the market.

**Proposition 4.1:**

If \(P \left( \frac{(n-i)Q^G + iQ^B}{n} \right) - \frac{w}{a^B} < P \left( \frac{(n-i+1)Q^G + (i-1)Q^B}{n} \right) - \frac{w}{a^G}, \ i = 1, ..., n \)

then \((Q^G,...,Q^G)\) is a unique Nash Equilibrium.

**Proof:** With \(n\) number of firms there are \(2^n\) possible combinations of good and bad firms in the market. For \(i = 1\) we get the above condition as

\[ P \left( \frac{(n-1)Q^G + Q^B}{n} \right) - \frac{w}{a^B} < P \left( Q^G \right) - \frac{w}{a^G}, \]

i.e., unilateral deviation from the strategy profile \((Q^G,...,Q^G)\) is unprofitable.
Hence, \((Q^G, ..., Q^G)\) is a Nash Equilibrium strategy profile.

Consider next the case where \(i > 1\).

If \(\overline{Q}\) is a strategy profile where \((n - i)\) firms produce \(Q^G\) and \(i\) firms produce \(Q^B\) then the condition of the Proposition says that unilateral deviation by a firm producing \(Q^B\) is profitable. Hence, \(\overline{Q}\) is not a Nash Equilibrium strategy profile.

Hence, \((Q^G, ..., Q^G)\) is a unique Nash Equilibrium strategy profile. \(\square\)

**Proposition 4.2:**

If \(P \left( \frac{(n-i)Q^G + iQ^B}{n} \right) \frac{w}{a_B} > P \left( \frac{(n-i+1)Q^G + (i-1)Q^B}{n} \right) \frac{w}{a_B} \), \(i = 1, ..., n\)

then \((Q^B, ..., Q^B)\) is a unique Nash Equilibrium.

**Proof:** For \(i = n\) we get the above condition as

\[
P \left( Q^B \right) \frac{w}{a_B} > P \left( \frac{Q^G + (n-1)Q^B}{n} \right) \frac{w}{a_B},
\]

i.e., unilateral deviation from the strategy profile \((Q^B, ..., Q^B)\) is unprofitable.

Hence, \((Q^B, ..., Q^B)\) is a Nash Equilibrium strategy profile.

Consider next the case where \(i < n\).

If there is a strategy profile \(\overline{Q}\) where \((n - i +1)\) firms produce \(Q^G\) and \((i -1)\) firms produce \(Q^B\) then it is profitable for a good firm to deviate. Hence, \(\overline{Q}\) is not a Nash Equilibrium strategy profile. Hence, \((Q^B, ..., Q^B)\) is a unique Nash Equilibrium strategy profile. \(\square\)

**Proposition 4.3:**

If \((i) P \left( \frac{(n-1)Q^G + Q^B}{n} \right) \frac{w}{a_B} < P \left( Q^G \right) \frac{w}{a_G}\)

Then \((Q^B, ..., Q^B)\) is a Nash Equilibrium strategy profile.
(ii) \( P (Q^B) - \frac{w}{a_B} > P \left( \frac{Q^G + (n-1)Q^B}{n} \right) - \frac{w}{a_G} \)

then \((Q^G, \ldots, Q^G)\) and \((Q^B, \ldots, Q^B)\) are both Nash equilibrium strategy profiles.

**Proof:** Following the arguments used in the first part of Proposition 4.1 it is easy to show that if (i) holds, then \((Q^G, \ldots, Q^G)\) is a Nash Equilibrium strategy profile. Again it follows from first part of Proposition 4.2 that if (ii) holds then \((Q^B, \ldots, Q^B)\) is a Nash Equilibrium strategy profile. Also, if conditions (i) and (ii) both hold then \(2^n - 2\) other strategies cannot be Nash Equilibrium strategy profiles. Hence, \((Q^G, \ldots, Q^G)\) and \((Q^B, \ldots, Q^B)\) are both Nash Equilibrium strategy profiles. \(\square\)

Let us define \(f(i, n) = P \left( \frac{(n-i)Q^G + iQ^B}{n} \right) - P \left( \frac{(n-i+1)Q^G + (i-1)Q^B}{n} \right), i = 1, \ldots, n\)

It may be noted that \(f(i, n)\) is negative, since we have assumed \(P\) to be a monotonically increasing function in the proportion of good firms in the market. We check the monotonicity of the above function with respect to \(i\) in the following proposition.

**Proposition 4.4:**
\(f(i, n)\) is a monotonic function in \(i\).

**Proof:** \(f(i + 1, n) - f(i, n)\)
\[
= \{P \left( \frac{(n-i-1)Q^G + (i+1)Q^B}{n} \right) - P \left( \frac{(n-i)Q^G + iQ^B}{n} \right)\}
- \{P \left( \frac{(n-i)Q^G + iQ^B}{n} \right) - P \left( \frac{(n-i+1)Q^G + (i-1)Q^B}{n} \right)\}

= \{P(Q^*) - P(\overline{Q})\} - \{ P(\overline{Q}) - P(\overline{Q})\}

where \(Q^*\) is a strategy profile with \((n - i - 1)\) firms producing \(Q^G\) and \((i + 1)\) firms producing \(Q^B\).
Clearly, \( \overline{Q} > \overline{Q} > Q^* \), since the weight given to \( Q^b \) in the strategy profile \( \overline{Q} \) is lowest and that given to \( Q^b \) in the strategy profile \( Q^* \) is highest.

By (1) \( P \) is assumed to be monotonically increasing in the proportion of good firms, \( \alpha \). This implies
\[
\{P(Q^*) - P(\overline{Q})\} < 0
\]
\[
\{P(\overline{Q}) - P(\overline{Q})\} < 0
\]

Let us further assume that \( P \) is a strictly concave function. i.e., \( \{P(Q^*) - P(\overline{Q})\} > \{P(\overline{Q}) - P(\overline{Q})\} \)
i.e., \( f(i + 1, n) - f(i, n) < 0 \)
Hence, \( f(i, n) \) is monotonically decreasing in \( i \).

Let us now assume that \( P \) is a strictly convex function. Then
\[
\{P(Q^*) - P(\overline{Q})\} < \{P(\overline{Q}) - P(\overline{Q})\}
\]
i.e., \( f(i + 1, n) - f(i, n) > 0 \)
Hence, \( f(i, n) \) is monotonically increasing in \( i \).

The following Propositions now describe the equilibrium strategies of the firms in single period game and explain how the equilibrium strategies change with the change in the number of firms. However, we make the analysis in two parts. In the first part, we assume that price is a strictly convex function. In the second part, we assume that price is a strictly concave function so that \( f(i, n) \) is monotonically decreasing in \( i \).

4.2.1. Price is a strictly convex function

Since \( P(\alpha) \) is assumed to be a strictly convex function, it follows from Proposition 4.4 that \( f(i, n) \) is monotonically increasing in \( i \).
**Proposition 4.5:**
Let $f(i, n)$ be monotonically increasing in $i$, $i = 1, 2, ..., n$

If $f(n, n) < \frac{w}{a^B} - \frac{w}{a^G}$

then there is a *unique* Nash Equilibrium: $(Q^G, ..., Q^G)$.

**Proof:** Since $f(i, n)$ is monotonically increasing in $i$,

$$f(n, n) < \frac{w}{a^B} - \frac{w}{a^G} \Rightarrow f(i, n) < \frac{w}{a^B} - \frac{w}{a^G} \forall i = 1, ..., n$$

i.e.,

$$P \left( \frac{(n-i)Q^G + iQ^B}{n} \right) - \frac{w}{a^B} < P \left( \frac{(n-i+1)Q^G + (i-1)Q^B}{n} \right) - \frac{w}{a^G}, \ i = 1, ..., n \quad (4.1)$$

Now it follows directly from Proposition 4.1 that if (4.1) holds then $(Q^G, ..., Q^G)$ is the *unique* Nash Equilibrium strategy profile. □

At $f(1, n)$ of Figure 4.1 all the existing firms produce good quality product with no incentive to deviate and the market reaches the *unique* Nash Equilibrium strategy $(Q^G, ..., Q^G)$.

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**Figure 4.1:** *Unique* Nash Equilibrium: $(Q^G, ..., Q^G)$ when $f(i, n)$ is monotonically increasing in $i$
Proposition 4.6:
Let \( f(i, n) \) be monotonically increasing in \( i, i = 1, 2, \ldots, n \)
and \( \bar{n} = \max \{ n \colon f(n, n) < \frac{w}{a^B} - \frac{w}{a^G} \} \)

Then for all \( n < \bar{n} \), a rise in the number of firms leaves quality standard unchanged, provided that the change in the number of firms is bounded above by \( (\bar{n} - n + 1) \).

Proof: We prove the proposition in three steps.

(i) \( f(n+1, n+1) > f(n, n) \forall n = 2, 3, \ldots \)
Proof: \( f(n+1, n+1) - f(n, n) \)
\[
= P\left(\frac{1}{n} Q^G + \frac{n-1}{n} Q^B\right) - P\left(\frac{1}{n+1} Q^G + \frac{n}{n+1} Q^B\right)
\]
Since the weight of \( Q^G \) in second part is smaller than that in the first part, it follows that
\[
\frac{1}{n} Q^G + \frac{n-1}{n} Q^B > \frac{1}{n+1} Q^G + \frac{n}{n+1} Q^B
\]
Hence, \( P\left(\frac{1}{n} Q^G + \frac{n-1}{n} Q^B\right) - P\left(\frac{1}{n+1} Q^G + \frac{n}{n+1} Q^B\right) > 0 \), \( P \) being an increasing function of the proportion of good firms.
Hence, \( f(n+1, n+1) > f(n, n) \)

(ii) If \( n \leq \bar{n} \) then average quality is \( Q^G \).
Proof: For \( n \leq \bar{n} \), \( f(n, n) < \frac{w}{a^B} - \frac{w}{a^G} \) and the result follows directly from Proposition 4.5.

(iii) Let \( n < \bar{n} \) and let \( k \) be a positive integer. Then repeated application of (i) yields \( f(n+k, n+k) > f(n, n) \). From (ii) it follows that the average quality is \( Q^G \) provided that \( n + k \leq \bar{n} \). Hence, average quality is \( Q^G \) so long as \( k < (\bar{n} - n + 1) \). \( \square \)
Proposition 4.7:

Let $f(i, n)$ be monotonically increasing in $i$, $i = 1, 2, ..., n$

If

1) $f(1, n) < \frac{W}{a^B} - \frac{W}{a^G}$

2) $f(n, n) > \frac{W}{a^B} - \frac{W}{a^G}$

then there are exactly two Nash Equilibrium strategy profiles: $(Q^G, ..., Q^G)$ and $(Q^B, ..., Q^B)$

Proof: Let $i^* = \max_i \{ i: f(i, n) < \frac{W}{a^B} - \frac{W}{a^G} \}$

and $i^{**} = \min_i \{ i: f(i, n) > \frac{W}{a^B} - \frac{W}{a^G} \}$

Since $n$ is finite, $i^*$ and $i^{**}$ exist.

As $f(i, n)$ is monotonically increasing in $i$,

$$f(i, n) < \frac{W}{a^B} - \frac{W}{a^G} \forall i \leq i^* \quad (4.II)$$

$$f(i, n) > \frac{W}{a^B} - \frac{W}{a^G} \forall i \geq i^{**} \quad (4.III)$$

Rewriting Equation (4.II) we get:

$$P \left( \frac{(n-i)Q^G + iQ^B}{n} \right) - \frac{W}{a^B} < P \left( \frac{(n-i+1)Q^G + (i-1)Q^B}{n} \right) - \frac{W}{a^G}, \quad i \leq i^* \quad (4.IIa)$$

Following the arguments used in Proposition 4.1 it is easy to show that for $i \leq i^*$ strategy profile $(Q^G, ..., Q^G)$ is a Nash Equilibrium, since deviation does not pay for the good firm.

Further, it is profitable for any of the bad firms to deviate to $Q^B$. That is, $(Q^G, ..., Q^G)$ is a locally unique Nash Equilibrium.

Similarly, rewriting Equation (4.III) we get
\[
P \left( \frac{(n-i)Q^G + iQ^B}{n} \right) - \frac{w}{a^B} > P \left( \frac{(n-i+1)Q^G + (i-1)Q^B}{n} \right) - \frac{w}{a^G}, \ i \geq i^* \tag{4.IIIa}
\]

It follows directly from Proposition 4.2 that for \( i \geq i^* \) strategy profile \((Q^B, ..., Q^B)\) is a Nash Equilibrium, since deviation does not pay for the bad firm. Further, it is profitable for any of the good firms to deviate to \( Q^B \). That is, \((Q^B, ..., Q^B)\) is a locally unique Nash Equilibrium.

Let there exist \( i = i_0 \) such that

\[
f(i_0, n) = \frac{w}{a^B} - \frac{w}{a^G}
\]

i.e., \( P \left( \frac{(n-i_0)Q^G + i_0Q^B}{n} \right) - \frac{w}{a^B} = P \left( \frac{(n-i_0 + 1)Q^G + (i_0 - 1)Q^B}{n} \right) - \frac{w}{a^G} \)

i.e., if there are \((i_0 - 1)\) bad firms and \((n - i_0 + 1)\) good firms, unilateral deviation does not pay for the good firms. From the monotonicity of \( f(i, n) \) with respect to \( i \), we get

\[
f(i_0 - 1, n) < \frac{w}{a^B} - \frac{w}{a^G}
\]

i.e., \( P \left( \frac{(n - i_0 + 1)Q^G + (i_0 - 1)Q^B}{n} \right) - \frac{w}{a^B} < P \left( \frac{(n - i_0 + 2)Q^G + (i_0 - 2)Q^B}{n} \right) - \frac{w}{a^G} \)

i.e., with \((i_0 - 1)\) bad firms and \((n - i_0 + 1)\) good firms, unilateral deviation pays for the bad firms. So any configuration of \((i_0 - 1)\) bad firms and \((n - i_0 + 1)\) good firms is not a Nash Equilibrium.

Hence, there are exactly two Nash Equilibrium strategy profiles: \((Q^G, ..., Q^G)\) and \((Q^B, ..., Q^B)\). \( \square \)

Figure 4.2 describes the multiple equilibrium strategies of the firms. There are exactly two Nash Equilibrium strategy profiles: \((Q^G, ..., Q^G)\) and \((Q^B, ..., Q^B)\).
Proposition 4.8:
Let $f(i, n)$ be monotonically increasing in $i$, $i = 1, 2, \ldots, n$

If $f(1, n) > \frac{w}{a^B} - \frac{w}{a^G}$

then there is a unique Nash Equilibrium: $(Q^G, \ldots, Q^G)$

Proof: Since $f(i, n)$ is monotonically increasing in $i$,

$$f(1, n) > \frac{w}{a^B} - \frac{w}{a^G} \Rightarrow f(i, n) > \frac{w}{a^B} - \frac{w}{a^G} \quad \forall i = 1, \ldots, n$$

i.e., $P \left( \frac{(n-i)Q^G + iQ^B}{n} \right) - \frac{w}{a^B} > P \left( \frac{(n-i+1)Q^G + (i-1)Q^B}{n} \right) - \frac{w}{a^G}, i = 1, \ldots, n$

It follows directly from Proposition 4.2 that $(Q^B, \ldots, Q^B)$ is a unique Nash Equilibrium Strategy profile. □

The market attains its unique Nash Equilibrium $(Q^B, \ldots, Q^B)$ in Figure 4.3 where all the firms gain by producing bad quality products with no incentive to deviate.
Figure 4.3: Unique Nash Equilibrium: \((Q^B, \ldots, Q^B)\)
when \(f(i, n)\) is monotonically increasing in \(i\)

**Proposition 4.9:**

Let \(f(i, n)\) be monotonically increasing in \(i\), \(i = 1, 2, \ldots, n\)

and \(n = \min \{ n: f(1, n) > \frac{w}{a^B} - \frac{w}{a^G} \}\)

Then for all \(n \geq n\), a rise in the number of firms leaves quality standard unchanged.

**Proof:** We prove the proposition in two steps.

(i) \(f(1, n+1) > f(1, n) \ \forall \ n = 2, 3, \ldots\)

Proof: \(f(1, n+1) - f(1, n)\)

\[
= P\left( \frac{n}{n+1}Q^G + \frac{1}{n+1}Q^B \right) - P\left( \frac{n-1}{n}Q^G + \frac{1}{n}Q^B \right)
\]

Since the weight of \(Q^G\) in first part is comparatively higher than that in second part, and the weight of \(Q^B\) in first part is smaller than that in second part, it follows that

\[
\frac{n}{n+1}Q^G + \frac{1}{n+1}Q^B > \frac{n-1}{n}Q^G + \frac{1}{n}Q^B
\]

Hence, \(P\left( \frac{n}{n+1}Q^G + \frac{1}{n+1}Q^B \right) - P\left( \frac{n-1}{n}Q^G + \frac{1}{n}Q^B \right) > 0\), since \(P' > 0\).
Hence, \( f(1, n+1) > f(1, n) \)

(ii) If \( n \geq n \) then average quality is \( Q^b \).

Proof: For \( n \geq n \), \( f(1, n) > \frac{w}{a^b} - \frac{w}{a^g} \) and the result follows directly from Proposition 4.8.

Hence, an increase in the number of firms above \( n \) does not change the average quality level of \( Q^b \), provided \( f(1, n) > \frac{w}{a^b} - \frac{w}{a^g} \).

Our next Proposition shows that if the increase in the number of firms is sufficiently high, then the economy may move towards bad quality equilibrium where all the firms gain by producing bad quality products.

**Proposition 4.10:**

Let \( f(i, n) \) be monotonically increasing in \( i \), \( i = 1, 2, \ldots, n \), for all \( n \).

Then as \( n \to \infty \), \((Q^b, \ldots, Q^b)\) is the unique Nash Equilibrium.

**Proof:** \( \lim_{n \to \infty} f(1, n) = \lim_{n \to \infty} \left[ P\left(\frac{n-1}{n} Q^g + \frac{1}{n} Q^b\right) - P(Q^g)\right] = 0 \)

From the first part of Proposition 4.9, we know that \( f(1, n+1) > f(1, n) \) \( \forall n \geq 2 \)
i.e., \( f(1, n) \) is monotonically increasing in \( n \).

It follows then, \( \lim_{n \to \infty} \left[ f(1, n) - \left( \frac{w}{a^b} - \frac{w}{a^g} \right) \right] = -\left( \frac{w}{a^b} - \frac{w}{a^g} \right) > 0 \)

Hence, by Proposition 4.8, the monotonicity of \( f(i, n) \) for all \( n \), implies that there exists a unique Nash equilibrium \((Q^b, \ldots, Q^b)\) for sufficiently high \( n \).

**4.2.2. Price is a strictly concave function**

Let us now assume that \( P(\alpha) \) is a strictly concave function so that \( f(i, n) \) is monotonically decreasing in \( i \). The next few Propositions describe how the equilibrium strategies of the
firms change with change in the number of firms when \( f(i, n) \) is monotonically decreasing in \( i \).

**Proposition 4.11:**
Let \( f(i, n) \) be monotonically decreasing in \( i \), \( i = 1, 2, ..., n \)

If \( f(1, n) < \frac{w}{a^B} - \frac{w}{a^G} \)

then there is a unique Nash Equilibrium: \((Q^G, ..., Q^G)\).

**Proof:** Since \( f(i, n) \) is monotonically decreasing in \( i \),

\[
f(1, n) < \frac{w}{a^B} - \frac{w}{a^G} \quad \Rightarrow \quad f(i, n) < \frac{w}{a^B} - \frac{w}{a^G} \quad \forall \quad i = 1, ..., n
\]

i.e., \( P \left( \frac{(n-i)Q^G + iQ^B}{n} \right) - \frac{w}{a^B} < P \left( \frac{(n-i+1)Q^G + (i-1)Q^B}{n} \right) - \frac{w}{a^G}, \ i = 1, ..., n \)

Now, it follows directly from Proposition 4.1 that if the above condition holds then \((Q^G, ..., Q^G)\) is a unique Nash Equilibrium strategy profile. \( \square \)

In Figure 4.4, all the existing firms produce good quality product with no incentive to deviate and the market reaches the unique Nash Equilibrium strategy: \((Q^G, ..., Q^G)\).

![Figure 4.4: Unique Nash Equilibrium: \((Q^G, ..., Q^G)\) when \( f(i, n) \) is monotonically decreasing in \( i \)
Proposition 4.12:
Let \( f(i, n) \) be monotonically decreasing in \( i \), \( i = 1, 2, \ldots, n \)

And \( n^* = \max \{ n : f(1, n) < \frac{W}{a^B} - \frac{W}{a^G} \} \)

Then for all \( n < n^* \), a rise in the number of firms leaves quality standard unchanged at \( Q^G \), provided that the change in the number of firms is bounded above by \( (n^* - n + 1) \).

Proof: We prove the proposition in three steps.

(i) \( f(1, n+1) > f(1, n) \) \( \forall n = 2, 3, \ldots \)
Proof: Given in the first part of Proposition 4.9.

(ii) If \( n \leq n^* \) then average quality is \( Q^G \).
Proof: For \( n \leq n^* \), \( f(1, n) < \frac{W}{a^B} - \frac{W}{a^G} \) and the result follows directly from Proposition 4.11.

(iii) Let \( n < n^* \) and let \( m \) be a positive integer. Then repeated application of (i) yields
\( f(1, n+m) > f(1, n) \). From (ii) it follows that the average quality is \( Q^G \) provided that \( n + m \leq n^* \). Hence, average quality is \( Q^G \) so long as \( m < (n^* - n + 1) \). □

Proposition 4.13:
Let \( f(i, n) \) be monotonically decreasing in \( i \), \( i = 1, 2, \ldots, n \)

and i) \( f(1, n) > \frac{W}{a^B} - \frac{W}{a^G} \)

ii) \( f(n, n) < \frac{W}{a^B} - \frac{W}{a^G} \)
then a Nash Equilibrium quality profile exists if and only if there exists an \( i = i_0 \), such that

\[
f(i_0, n) = \frac{w}{a^G} - \frac{w}{a^G}.
\]

**Proof: (Necessity):** Suppose that there is no \( i = i_0 \) such that \( f(i_0, n) = \frac{w}{a^B} - \frac{w}{a^G} \).

Let \( \bar{i} = \max \{i: f(i, n) > \frac{w}{a^B} - \frac{w}{a^G}\} \)

and \( \underline{i} = \min \{i: f(i, n) < \frac{w}{a^B} - \frac{w}{a^G}\} \)

Since \( n \) is finite, \( \bar{i} \) and \( \underline{i} \) exist.

As \( f(i, n) \) is monotonically decreasing in \( i \)

\[
f(i, n) > \frac{w}{a^B} - \frac{w}{a^G} \quad \forall \ i \leq \bar{i} \tag{4.IV}
\]

and \( f(i, n) < \frac{w}{a^B} - \frac{w}{a^G} \quad \forall \ i \geq \underline{i} \tag{4.V}
\]

Rewriting Equation (4.IV)

\[
P \left( \frac{(n - i)Q^G + iQ^B}{n} \right) - \frac{w}{a^B} > P \left( \frac{(n - i + 1)Q^G + (i - 1)Q^B}{n} \right) - \frac{w}{a^G}, \quad i \leq \bar{i} \tag{4.IVa}
\]

If \( Q \) is a strategy profile where \( n - i + 1 \) firms produce \( Q^G \) and \( i - 1 \) firms produce \( Q^B \), equation (4.IVa) says that unilateral deviation by a firm producing \( Q^G \) is profitable.

Hence, \( Q \) is not a Nash Equilibrium.

Hence, there exists no Nash Equilibrium strategy profile for \( i \leq \bar{i} \).

Similarly, rewriting Equation (4.V) we get:

\[
P \left( \frac{(n - i)Q^G + iQ^B}{n} \right) - \frac{w}{a^B} < P \left( \frac{(n - i + 1)Q^G + (i - 1)Q^B}{n} \right) - \frac{w}{a^G}, \quad i \geq \underline{i} \tag{4.Va}
\]
i.e., if $\vec{Q}$ is the strategy profile with $(n - i)$ firms producing $Q^G$ and $i$ firms producing $Q^B$, then unilateral deviation pays for a firm producing $Q^B$. Hence, $\vec{Q}$ is not a Nash Equilibrium.

Hence, there exists no Nash Equilibrium strategy profile for $i \geq \frac{n}{2}$.

(Sufficiency): Suppose that there exists $i = i_0$ such that $f(i, n) = \frac{w}{a^B} - \frac{w}{a^G}$

i.e., $P \left( \frac{(n - i_0)Q^G + i_0Q^B}{n} \right) - \frac{w}{a^B} > P \left( \frac{(n - i_0 + 1)Q^G + (i_0 - 1)Q^B}{n} \right) - \frac{w}{a^G}$ (4.Vb)

With $\vec{Q}$ strategy profile, equation (4.Vb) says that good firms gain nothing from unilateral deviation. Again from the monotonicity of $f(i, n)$ with respect to $i$ we get

$f(i_0 - 1, n) > \frac{w}{a^B} - \frac{w}{a^G}$

i.e., $P \left( \frac{(n - i_0 + 1)Q^G + (i_0 - 1)Q^B}{n} \right) - \frac{w}{a^B} > P \left( \frac{(n - i_0 + 2)Q^G + (i_0 - 2)Q^B}{n} \right) - \frac{w}{a^G}$

With the strategy profile $\vec{Q}$, the $(i_0 - 1)$ bad firms do not gain from unilateral deviation.

Hence, any configuration with $(i_0 - 1)$ bad firms and $(n - i_0 + 1)$ good firms is a Nash Equilibrium$^{16}$.

Hence, no Nash Equilibrium strategy profile exists except at $i = i_0$, such that

$f(i_0, n) = \frac{w}{a^B} - \frac{w}{a^G}$.

Figure 4.5 shows that if there exists $i = i_0$, such that $f(i_0, n) = \frac{w}{a^B} - \frac{w}{a^G}$, then a Nash Equilibrium exists when there are $(i_0 - 1)$ bad firms and $(n - i_0 + 1)$ good firms.

---

$^{16}$ Such equilibrium has the problem that it is not easy to determine which firm produces what and hence what payoff it receives.
Figure 4.5: Any configuration of \((i_0 - 1)\) bad firms and \((n - i_0 + 1)\) good firms is a Nash Equilibrium when \(f(i, n)\) is monotonically decreasing in \(i\).

**Proposition 4.14:**

Let \(f(i, n)\) be monotonically decreasing in \(i\), \(i = 1, 2, \ldots, n\).

If \(f(n, n) > \frac{w}{a_B} - \frac{w}{a_G}\) \(\forall i = 1, \ldots, n\),

then there is a unique Nash Equilibrium: \((Q^B, \ldots, Q^B)\)

**Proof:** Since \(f(i, n)\) is monotonically decreasing in \(i\),

\[
f(n, n) > \frac{w}{a_B} - \frac{w}{a_G} \quad \Rightarrow \quad f(i, n) > \frac{w}{a_B} - \frac{w}{a_G} \quad \forall i = 1, \ldots, n
\]

i.e., \(P \left( \frac{(n-i)Q^G + iQ^B}{n} \right) - \frac{w}{a_B} > P \left( \frac{(n-i+1)Q^G + (i-1)Q^B}{n} \right) - \frac{w}{a_G}, i = 1, \ldots, n \quad (4.VI)\)

It now follows directly from Proposition 4.2 that \((Q^B, \ldots, Q^B)\) is a unique Nash Equilibrium Strategy profile. \(\Box\)

Figure 4.6 shows that the market attains its unique Nash Equilibrium \((Q^B, \ldots, Q^B)\) where all the firms gain by producing bad quality products and find no incentive to deviate.
Proposition 4.15:
Let \( f(i, n) \) be monotonically decreasing in \( i \), \( i = 1, 2, ...., n \)
and \( n^* = \min_n \{ n : f(n, n) > \frac{w}{a^B} - \frac{w}{a^G} \} \)

Then for all \( n \geq n^* \), a rise in the number of firms leaves quality standard unchanged at \( Q^B \).

Proof: We prove the proposition in two steps.

(i) \( f(n+1, n+1) > f(n, n) \) \( \forall n = 2, 3, ..... \)
Proof: Given in the first part of Proposition 6.

(ii) If \( n \geq n^* \) then average quality is \( Q^B \).
Proof: For \( n \geq n^* \), \( f(n, n) > \frac{w}{a^B} - \frac{w}{a^G} \) and the result follows directly from Proposition 4.14.

Hence, an increase in the number of firms above \( n^* \) does not change the average
quality level of $Q^B$ provided $f(n, n) > \frac{w}{a^B} - \frac{w}{a^G}$. □

The last Proposition of this section shows that if the number of firms is sufficiently high, then it may be profitable for all the firms to produce bad quality products.

**Proposition 4.16:**

Let $f(i, n)$ be monotonically decreasing in $i$, $i = 1, 2, ..., n$, for all $n$. Then as $n \to \infty$, $(Q^B, ..., Q^B)$ is the unique Nash Equilibrium.

**Proof:** $\lim_{n \to \infty} f(n, n) = \lim_{n \to \infty} \{P(Q^B) - P\left(\frac{n-1}{n}Q^B + \frac{1}{n}Q^G\right)\} = 0$

From the first part of Proposition 4.6, we know that $f(n+1, n+1) > f(n, n) \; \forall \; n \geq 2$
i.e., $f(n, n)$ is monotonically increasing in $n$.

It follows then, $\lim_{n \to \infty} \{ f(n, n) - \left( \frac{w}{a^B} - \frac{w}{a^G} \right) \} = - \left( \frac{w}{a^B} - \frac{w}{a^G} \right) > 0$

Hence, by Proposition 4.14, the monotonicity of $f(i, n)$ for all $n$, implies that there exists a unique Nash equilibrium $(Q^B, ..., Q^B)$ for sufficiently high $n$. □

**4.3. Conclusion**

In this chapter, we have considered a single period oligopolistic market structure and studied the impact of increased competition on the quality level of the product. Our analysis shows that when firms are capacity constrained, there may exist multiple equilibrium strategy profiles. The Propositions described in the previous section reveal that, if it is profitable for all the firms to produce good quality product, then an increase in the number of firms does not alter the quality standard in the market, provided the number is bounded above by a critical level. In other words, increased competition may not necessarily result in a low quality output standard. Similarly, if all the firms gain by producing bad quality product, then a reduction in the number of firms does not
guarantee that the market will have a good quality standard. In particular, if the number of firms in the market is bounded below by a critical level, then consolidation of industry does not necessarily imply that the quality standard in the market will improve. However, if the number of firms in the market is sufficiently high, then the economy probably moves towards bad quality equilibrium.