Chapter Three

Monopolistic Choice of Product Specifications when Higher End Product Specifications Provide Imperfect Signals of the Performance of the Low End Product

3.1. Introduction

As in Chapter Two, we assume here that customers can only observe technical specifications and are unable to identify the exact performance level of the product \textit{a priori}. In that sense, the goods in our model are experience goods: a customer may form \textit{a priori} beliefs about the quality of performance of the good based on a study of its technical specification, hearsay evidence, etc., but is able to form a clear idea of performance only after consumption. Advertisements for cars, electronic equipments like computers, audio and video systems provide technical details that are often largely meaningless for the average customer. In particular, we assume that buyers can form an idea (more specifically, a probability distribution) of performance levels based on the quality specification such that a “higher” quality specification generates a better distribution of performance in the First-Order Stochastic Dominance (FSD) sense. This implies that a customer with a given valuation of quality is willing to pay more for a better quality specification\textsuperscript{12}. This per se does not affect the basic result of the theory of monopolistic quality discrimination in any meaningful way. The result breaks down once we introduce some spillover effects as in Kim and Kim (1996) who introduced technical spillover effects into the model by assuming that the cost of production of the low end quality falls as the quality of the high end product increases.

In this chapter we explore the possibility that firms making technically sophisticated high end products induce the belief among customers that the low end products they produce

\textsuperscript{12} Our formal specification draws heavily on Tadelis (1999) and Cabral (2000).
are likely to perform better because of their higher technical skill. Automobile manufacturers, high fashion apparel and accessory manufacturers, etc. produce low end products that are often valued more by customers than comparable products made by companies that do not make the high end products. While part of the reason could lie in brand snobbery, a likely alternative explanation could be that customers expect firms, that have the technical skill to make high quality products, are also likely to make good quality low end products. In that sense, brand names are bearers of information as in Wernerfelt (1988), Tadelis (1999) and Cabral (2000). Our problem is different from the ones addressed in these papers in that we explore the effects of quality spillovers on quality choices made by a seller under second degree quality discrimination. In Tadelis (1999) each product is sold under a different name and these names can be traded. The model looks at equilibrium in the market for names. Wernerfelt (1988) and Cabral (2000) focus on the firm’s decision to brand stretch, given fixed qualities. In Cabral (2000), firms introduce new products whose qualities are known to them but are unknown to buyers \textit{a priori}. \textit{Ex post}, buyers can update their beliefs about the quality of the product based on their experience about the performance of the product. A firm that introduces new good using its established brand name faces the possibility that its reputation may suffer if the buyers’ experience is not good. In our analysis brand stretching is definitional: a firm that produces high end products knows that its high end quality specification affects customers’ valuation of its lower end products. The question is: will a firm facing two types of customers provide a quality level above the socially optimal level to its high valuation customers in equilibrium? We show in this chapter that in the presence of a quality spillover effect, the standard result of monopolistic quality discrimination breaks down. Among a host of possibilities, it is possible that in equilibrium, customers are provided quality levels above or below the socially optimal level irrespective of whether they belong to high end group or the low end group.

The chapter is organized as follows: In section 3.2 we present the formal model and derive the principal result. Section 3.3 offers some concluding remarks.
3.2. The Model

Consider a monopolist producing a vertically differentiated good. In the standard formulation of the model, there are (say) two types of customers indexed by \( i = 1, 2 \) and there are \( n_i \) customers of type \( i \), where \( n_i > 0 \). Each customer buys at most one unit of the good. The monopolist chooses characteristics \((q_1, q_2)\) from the interval \([q, \bar{q}]\) and a corresponding set of prices \((p_1, p_2)\) for the product. Potential customers observe the monopolist’s choice of \((q_1, q_2)\) and \((p_1, p_2)\) before deciding the type of product they wish to purchase. What this presumes is that the announced quality specification provides an exact (degenerate distribution) signal to the buyer of the nature of performance that he can expect from the product. In other words, the goods in the standard model are not experience goods. The critical feature of our model is that our goods are experience goods: buyers can observe quality specifications \emph{a priori}, but these specifications provide a noisy signal of the quality of performance that they can expect when they use it.

Let \( r \in [\underline{r}, \overline{r}] \) designate the actual performance of the product. This is not directly observable before the product is used. However, customers can form beliefs about the likelihood of different performance levels based on the announced quality specification. Such beliefs are captured by a cumulative distribution function, say \( F(r|q_j, q_k)\), \( q_j, q_k = 1, 2 \), \( q_j \neq q_k \), where the first element \( q_j \) designates the quality purchased and the second element \( q_k \) is the other available quality. The standard monopolistic quality discrimination model then reduces to the special case where there is a monotonically increasing function \( x:[q, \bar{q}] \rightarrow [\underline{r}, \overline{r}] \) such that \( F(r|q_j, q_k) = 0 \) if \( r < x(q_j) \) and equals 1 otherwise. Notice that \( x(.) \) does not depend upon \( q_k \).

The utility that a type-\( i \) customer derives from the consumption of the good is a function of the performance level of the product and a type specific taste parameter \( \alpha_i \):

\[
u^i = u( r, \alpha_i), \text{ with } \partial u( r, \alpha_i) / \partial r > 0 \text{ and } \partial^2 u( r, \alpha_i) / \partial r^2 < 0, \text{ i = 1, 2} \tag{3.1}\]
Hence, the willingness of customer $i$ to pay for a product of quality specification $q_j$ when $q_k$ is the other quality that is available, is

$$V_{ij}(q_j, q_k) = \int u(r, \alpha_i)dF(r| q_i, q_k), \quad i = 1, 2, \quad j, k = 1, 2, j \neq k$$

$$= u(r, \alpha_i)F(r| q_j, q_k) - u(r, \alpha_i)F(r| q_j, q_k) - \int F(r| q_j, q_k)du(r, \alpha)$$

$$i = 1, 2, \quad j, k = 1, 2, j \neq k$$

Since $u(r, \alpha_i)$ is differentiable in $r$ and since $F(r| q_j, q_k) = 1$ and $F(r| q_j, q_k) = 0,

$$V_{ij}(q_j, q_k) = u(r, \alpha_i) - \int (\partial u(r, \alpha_i)/\partial r)F(r| q_j, q_k)dr$$

We assume that the total utility as well as the marginal utility is higher for the type-2 customers than for the type-1 ones:

$$u(r, \alpha_2) > u(r, \alpha_1) \quad \forall r \in [r, \tilde{r}]$$  \hspace{1cm} (3.3)

$$u_r(r, \alpha_2) > u_r(r, \alpha_1) \quad \forall r \in [r, \tilde{r}]$$  \hspace{1cm} (3.4)

It is easy to see that equation (3.2) and inequality (3.3) $V_{12}^2 > V_{11}^1$ and $V_{22}^2 > V_{21}^1$.

Consider now a customer of type $i$ who is considering purchasing the good with quality specification $q_j$. If the value of $q_j$ increases, i.e. the quality specification of the good under consideration improves, then this generates the belief in the customer’s mind that the probability of getting a better level of performance from the product is higher than before. A change in the quality specification of the other (quality) good available, i.e. a change in the value of $q_k$, has asymmetric effects. In the presence of the one-sided spillover effect, mentioned earlier, the probability distribution of performance at the lower end “improves” as the quality specification at the higher end increases, while the probability distribution of performance levels at the higher end remains unaffected by

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changes in the lower end quality specification. If the good being considered by the customer is the one with the lower quality specification then an improvement in the quality specification of the higher end product has a “positive” impact on the customer’s beliefs about the performance he can expect from the product he is looking at. On the other hand, if the good under consideration is at the higher end of the quality spectrum being offered, a change in the quality specification of the low end product has no impact on the customer’s beliefs about the performance that he can expect from the high end product. To capture this intuition we assume that in each of the first two cases, the lower quality specification generates a “better” distribution of performance in the First-Order Stochastic Dominance (FSD) sense. In the last case, a change in \( q_k \) does not have any effect on the probability distribution of performances.

The Distribution \( F(x, y) \) first-order stochastically dominates the distribution \( F(x, y') \), \( y \neq y' \), if, for every non-decreasing function \( u: \mathbb{R} \to \mathbb{R} \) we have

\[
\int x u(x) \, dF(x, y) \geq \int x u(x) \, dF(x, y')
\]

(3.5)

A well-known result is that

\[
\int x u(x) \, dF(x, y) \geq \int x u(x) \, dF(x, y') \iff F(x, y) \leq F(x, y') \text{ for every } x.
\]

Thus, in formal language, we assume\(^{14}\):

(I) for all \( q_k, q_j, q_j' \in [q, \bar{q}] \), and \( q_j > q_j', F(r| q_j, q_k) \leq F(r| q_j', q_k) \) for every \( r \in [\underline{r}, \bar{r}] \) with the strict inequality holding at some \( r \);

(II) for all \( q_k, q_j, q_j' \in [q, \bar{q}], q_j' > q_j > q_k, F(r| q_j', q_k) \leq F(r| q_j, q_k) \) for every \( r \in [\underline{r}, \bar{r}] \) with the strict inequality holding at some \( r \); and
(III) for all \( q_k, q_j, q'_j \in [q, \bar{q}] \), \( q_k > q'_j > q_j \), \( F (r| q_j, q_k) = F (r| q'_j, q_k) \) for every
\( r \in [\underline{r}, \bar{r}] \).

If \( F(.) \) is differentiable with respect to \( q_j, q_k \) then assumptions (I'), (II') and (III') stated below are equivalent to assumptions (I), (II) and (III) respectively:

- for any \( q_j, q_k \in [q, \bar{q}] \),
  - (I') \( \partial F (r| q_j, q_k) / \partial q_j \leq 0 \) for every \( r \in [\underline{r}, \bar{r}] \) with a strict inequality holding at some \( r \);
  - (II') if \( q_j > q_k \), \( \partial F (r| q_j, q_k) / \partial q_j \leq 0 \) for every \( r \in [\underline{r}, \bar{r}] \) with a strict inequality holding at some \( r \); and
  - (III') if \( q_j \leq q_k \), \( \partial F (r| q_j, q_k) / \partial q_j = 0 \) for every \( r \in [\underline{r}, \bar{r}] \).

**Proposition 3.1:**

- (iv) \( \forall q_j, q_k \in [q, \bar{q}] \), \( \partial u (r, \alpha_i) / \partial r > 0 \) and (I') imply \( \partial V^i_j (q_j, q_k) / \partial q_j \geq 0 \);
- (v) \( \forall q_j, q_k \in [q, \bar{q}] \) such that \( q_j > q_k \), \( \partial u (r, \alpha_i) / \partial r > 0 \) and (II') imply \( \partial V^i_j (q_j, q_k) / \partial q_j \geq 0 \);
- (vi) \( \forall q_j, q_k \in [q, \bar{q}] \) such that \( q_j < q_k \), \( \partial u (r, \alpha_i) / \partial r > 0 \) and (III') imply \( \partial V^i_j (q_j, q_k) / \partial q_j = 0 \).

**Proof:** (i) Applying Leibniz Rule to differentiate (3.2') with respect to \( q_j \)
\[
\partial V^i_j / \partial q_j = 0 - \int_{\underline{r}}^{\bar{r}} \left( \partial u (r, \alpha_i) / \partial r \right) \left( \partial F (r| q_j, q_k) / \partial q_j \right) dr \quad i = 1, 2
\]

\(^{14}\) It is to be noted that \( F(r| q_j, q_k) \) is defined as the cumulative distribution of performance where \( q_j \) is the quality purchased and \( q_k \) is the other alternative quality.
Since, marginal utility $\partial u (r, \alpha_i) / \partial r > 0$ it follows from assumption (I’)
that $\partial V_1^i (q_i, q_k) / \partial q_j \geq 0$.

(ii) Since, marginal utility $\partial u (r, \alpha_i) / \partial r > 0$ it follows from assumption (II’)
that $\partial V_1^i (q_i, q_k) / \partial q_j \geq 0$.

(iii) Since, marginal utility $\partial u (r, \alpha_i) / \partial r > 0$ it follows from assumption (III’)
that $\partial V_1^i (q_i, q_k) / \partial q_k = 0$. □

In the special case where $q_1 < q_2$, Proposition 3.1 says that:

$$\frac{\partial V_1^i (q_1, q_2)}{\partial q_i} > 0 \quad i = 1, 2 \quad (3.6a)$$

$$\frac{\partial V_1^i (q_1, q_2)}{\partial q_2} > 0 \quad i = 1, 2 \quad (3.6b)$$

$$\frac{\partial V_2^i (q_2, q_1)}{\partial q_1} = 0 \quad i = 1, 2 \quad (3.6c)$$

$$\frac{\partial V_2^i (q_2, q_1)}{\partial q_2} > 0 \quad i = 1, 2 \quad (3.6d)$$

**Proposition 3.2:**

$q_1 < q_2$ and (III’) imply $\partial^2 V_2^i (q_2, q_1) / \partial q_1 \partial q_2 = 0, \ i = 1, 2$.

**Proof:** $\partial^2 V_2^i (q_2, q_1) / \partial q_1 \partial q_2 = \partial (\partial V_2^i (q_2, q_1) / \partial q_2) / \partial q_1$

$= \partial (\partial V_2^i (q_2, q_1) / \partial q_1) / \partial q_2 = 0$ (using (III’)). □

Finally, assume that the total cost of production, $C(q)$, and the marginal cost vary
positively with the quality:

$$\frac{dC(q)}{dq} > 0 \text{ and } d^2C(q) / dq^2 > 0. \quad (3.7)$$
3.2.1 The Social Planner’s Solution

To establish the benchmark qualities against which we shall measure the distortion that the profit-maximizing monopolist introduces in the quality spectrum we first look at the quality choices of the Social Planner who chooses quality levels to maximize social welfare. In accordance with analytical custom, we assume that the social planner, unlike the monopolist, is fully informed about each potential customer’s preference pattern.

Let $W$ be the social welfare level. Hence the problem of the social planner is:

$$\text{Max } W = n_1[V_1(q_1, q_2) - C(q_1)] + n_2[V_2(q_2, q_1) - C(q_2)]$$

Let $(q_1^*, q_2^*)$ be the unique interior extremum of the welfare-maximization problem, that is, $q_1^* \in [\underline{q}, \bar{q}]$ and $q_2^* \in [\underline{q}, \bar{q}]$. Then $(q_1^*, q_2^*)$ satisfy the following first-order conditions:

$$\frac{\partial W}{\partial q_1} = n_1[\frac{\partial V_1(q_1, q_2)}{\partial q_1} - \frac{dC(q_1)}{dq_1}] = 0$$

(3.8)

$$\frac{\partial W}{\partial q_2} = n_1[\frac{\partial V_1(q_1, q_2)}{\partial q_2}] + n_2[\frac{\partial V_2(q_2, q_1)}{\partial q_2} - \frac{dC(q_2)}{dq_2}] = 0$$

(3.9)

Let $q_1 = h(q_2)$ be the solution to (3.8) and $q_1 = g(q_2)$ be the solution to (3.9).

Assume further that $q_1^* < q_2^*$. Let the locus of $(q_1, q_2)$ satisfying equations (3.8) and (3.9) be denoted by $M_1$ and $M_2$ respectively. The slopes of $M_1$ and $M_2$ are given by:

$$(dq_2 / dq_1)_{M1} = - \left(\frac{\partial^2 W}{\partial q_1^2}\right) / \left(\frac{\partial^2 W}{\partial q_1 \partial q_2}\right)$$

$$= - \left[\frac{\partial^2 V_1(q_1, q_2)}{\partial q_1^2} - \frac{d^2 C(q_1)}{dq_1^2}\right] / \left[\frac{\partial^2 V_1(q_1, q_2)}{\partial q_1 q_2}\right]$$

(3.10)
\[ \frac{dq_2}{dq_1}M_2 = -\frac{\partial^2 W / \partial q_1 \partial q_2}{\partial^2 W / \partial q_2^2} \]

\[ = -n_1\frac{\partial^2 V_1^1(q_1, q_2) / \partial q_1 \partial q_2}{\partial^2 V_2^2(q_2, q_1) / \partial q_2^2} - n_2 \frac{d^2 C(q_2)}{dq_2^2} \]

where \( A = n_1\frac{\partial^2 V_1^1(q_1, q_2) / \partial q_1 \partial q_2}{\partial^2 V_1^1(q_1, q_2) / \partial q_2^2} + n_2\frac{\partial^2 V_2^2(q_2, q_1) / \partial q_2^2}{\partial^2 V_2^2(q_2, q_1) / \partial q_2^2} \)

Second-order conditions require that the principal minors of the relevant Hessian determinant evaluated at \((q_1^*, q_2^*)\) alternate in sign:

i.e., \( \partial^2 W / \partial q_1^2 < 0, \partial^2 W / \partial q_2^2 < 0 \) (3.12)

and \[
\begin{vmatrix}
\partial^2 W / \partial q_1^2 & \partial^2 W / \partial q_2 \partial q_1 \\
\partial^2 W / \partial q_1 \partial q_2 & \partial^2 W / \partial q_2^2 \\
\end{vmatrix} > 0 \] (3.13)

From Lemma 2.1 of Chapter Two it follows that slopes of \( M_1 \) and \( M_2 \) depend on the sign of \( \partial^2 W / \partial q_1 \partial q_2 \). If \( \partial^2 W / \partial q_1 \partial q_2 \) is positive then \( M_1 \) and \( M_2 \) are both positively sloped; and they are both negatively sloped when \( \partial^2 W / \partial q_1 \partial q_2 \) is negative. However, in both cases \( M_1 \) is “steeper” than \( M_2 \).

Again, from Proposition 3.2 it follows that \( \partial^2 W / \partial q_1 \partial q_2 = n_1\partial^2 V_1^1(q_1, q_2) / \partial q_1 \partial q_2 \). This can be either positive or negative. Now \( \partial^2 V_1^1(q_1, q_2) / \partial q_1 \partial q_2 = \partial / \partial q_1 (\partial V_1^1(q_1, q_2) / \partial q_2) > 0 \) implies that the impact of the spillover effect, \( (\partial V_1^1(q_1, q_2) / \partial q_2) \), increases as the quality specifications at the lower end improves. On the other hand, a negative value of \( \partial / \partial q_1 (\partial V_1^1(q_1, q_2) / \partial q_2) \) implies that the impact of the spillover effect decreases as the quality specifications at the lower end improves.

Figures 3.1.a and 3.1.b show how the impact of the spillover changes with the change in the quality specifications at the lower end.
Figures 3.1.a and 3.1.b show the change in the impact of the spillover effect with respect to $q_1$ when $\partial^2 W / \partial q_1 \partial q_2 > 0$.

Figures 3.2.a and 3.2.b show the optimum choice of quality of the Social Planner in the two cases.
Figure 3.2.a: The Social Planner’s optimum choice of quality when $\partial^2 W / \partial q_1 \partial q_2 > 0$

Figure 3.2.b: The Social Planner’s optimum choice of quality when $\partial^2 W / \partial q_1 \partial q_2 < 0$
3.2.2 Quality Choice under Asymmetric Information: The Monopolist’s Solution

Unlike the Social planner, the monopolist suffers from an informational deficiency: he is unable to identify the type of each customer \textit{a priori}. Under this condition, the monopolist’s problem is to select a profit maximizing pair of customer type specific contracts \((p_1, q_1)\) and \((p_2, q_2)\) such that for each type of customer there is at least one acceptable contract (individual rationality); and no customer is better off accepting a contract designed for a customer whose type is different from his own (incentive compatibility). Formally, the monopolist’s problem is:

\[
\max \quad \Pi = \sum_{i=1}^{2} [p_i - C(q_i)]n_i
\]

\((p_i, q_i), i = 1, 2\)

subject to:

\(v\) \quad p_1 \leq V_1^1(q_1, q_2) \]

\(vi\) \quad p_2 \leq V_2^2(q_2, q_1) \]

\(vii\) \quad V_1^1(q_1, q_2) - p_1 \geq V_2^1(q_2, q_1) - p_2 \]

\(viii\) \quad V_2^2(q_2, q_1) - p_2 \geq V_1^2(q_1, q_2) - p_1

Conditions (i) and (ii) are the individual rationality constraints for the first and second customer types respectively; and conditions (iii) and (iv) are their respective incentive compatibility constraints. It is possible, of course, that no such separating menu of contracts exists and the profit maximizing strategy is to offer the same contract for all types of customers (pooling contract). A second possibility is that it is best for the monopolist to serve only one type of customer (partial market coverage). In what follows we assume that a separating menu of contracts exists. It is fairly straightforward to show that only constraints (i) and (iv) bind in equilibrium\(^15\) and so after some straightforward substitutions the monopolist’s problem reduces to:

\(^15\)If (i) is satisfied, i.e., the net surplus of the low demand customers is positive then the high demand customers are automatically willing to purchase, i.e., (ii) is also satisfied. Moreover, the monopolist, who benefits from higher prices, is able to extract the entire surplus from the lower end. Hence, constraint (i)
Max \[ V_1^1(q_1, q_2) - C(q_1) \] _n_1 + [\[V_2^2(q_2, q_1) + V_1^1(q_1, q_2) - V_1^2(q_1, q_2) - C(q_2)\]]_n_2

(q_1, q_2)

The first-order conditions for an interior maximum are:

\[ n_1 [\partial V_1^1(q_1, q_2) / \partial q_1 - \partial C(q_1) / \partial q_1] + n_2 [\partial V_1^1(q_1, q_2) / \partial q_1 - \partial V_1^2(q_1, q_2) / \partial q_1] = 0 \]

(3.14)

and

\[ n_1 \partial V_1^1(q_1, q_2) / \partial q_2 + n_2 [\partial V_2^2(q_2, q_1) / \partial q_2 + \partial V_1^1(q_1, q_2) / \partial q_2 - \partial V_1^2(q_1, q_2) / \partial q_2 - \partial C(q_2) / \partial q_2] = 0 \]

(3.15)

The second-order conditions are assumed to hold. Let \((q_1^m, q_2^m)\) be the unique profit-maximizing quality levels of the monopolist.

We are now in a position to answer the fundamental question that we ask in this paper: if improvements in the higher end quality specification have a positive effect on customers’ beliefs about the performance that they can expect from products at the lower end of the quality spectrum, how do the quality levels chosen by the monopolist differ from those of the social planner? Figures 3.1.a and 3.1.b provide an intuitive idea of the answer to this question. Consider first the case where \(\partial^2 W / \partial q_1 \partial q_2 > 0\) [Figure 3.2.a]. Since \((q_1^*, q_2^*)\) is by assumption a unique extremum, \(\partial^2 W / \partial q_1^2 < 0\) for all \((q_1, q_2)\) in the interior of \([q, \bar{q}] \times [\bar{q}, \bar{q}]\). This means that to the left of \(M_1\), \(\partial W / \partial q_1 > 0\) and to the right of \(M_1\), \(\partial W / \partial q_1 < 0\). Again, since by assumption \(\partial^2 W / \partial q_1 \partial q_2 = \partial / \partial q_1 (\partial W / \partial q_2) > 0\), \(\partial W / \partial q_2 > 0\) to the right of \(M_2\) and \(\partial W / \partial q_2 < 0\) to the left of \(M_2\). Now, using equations (3.8) and (3.14):

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binds in equilibrium. The incentive compatibility constraint (iii) is not relevant as the monopolist is more interested to induce the high demand customers to reveal their true type.
\[ \frac{\partial W}{\partial q_1} = -n_2 \left[ \frac{\partial V_1^1}{\partial q_1} - \frac{\partial V_1^2}{\partial q_1} \right] > 0 \]

Similarly, using equations (3.9) and (3.15):

\[ \frac{\partial W}{\partial q_2} = -n_2 \left[ \frac{\partial V_1^1}{\partial q_2} - \frac{\partial V_1^2}{\partial q_2} \right] > 0 \]

It follows immediately then that \((q_1^m, q_2^m)\) must lie within the region ‘ABC’ of Figure 3.2.a that lies to the south west of \((q_1^*, q_2^*)\). In other words, \(q_1^m < q_1^*\) and \(q_2^m < q_2^*\).

However, if \(\frac{\partial^2 W}{\partial q_1 \partial q_2} < 0, \frac{\partial W}{\partial q_2} > 0\) to the left of \(M_2\) and \(\frac{\partial W}{\partial q_2} < 0\) to the right of \(M_2\). Other arguments remaining same, Figure 3.2.b shows that \((q_1^m, q_2^m)\) may lie in any of the shaded regions ‘A’, ‘B’ or ‘C’. In region ‘A’, \(q_1^m < q_1^*\) and \(q_2^m > q_2^*\), in region ‘B’, \(q_1^m < q_1^*\) and \(q_2^m < q_2^*\), in region ‘C’, \(q_1^m > q_1^*\) and \(q_2^m < q_2^*\).

Before we formally state and prove this result, consider two numerical examples which corroborate this intuition about the relationship between the monopolist’s optimal quality levels and those of the social planner.

**Example 1:** Let the performance level, \(r\), takes up two distinct values –

a high performance denoted by \(r = h\) and a low performance denoted by \(r = l\).

Let \(P(h) = 1 - \left[ \frac{1}{q_1 + q_2} \right], q_1, q_2 \in [0, 1] \)

\(P(l) = \frac{1}{q_1 + q_2} \)

\(u(r, \alpha) = \alpha \sqrt{r} \)

\(c(q_i) = cq_i^2/2 \)

In this case, \(\frac{\partial^2 W}{\partial q_1 \partial q_2} = n_1 \frac{\partial^2 V_1^1}{\partial q_2} - \frac{\partial V_1^2}{\partial q_1} = -2\alpha_1 / (q_1 + q_2)^3 < 0. \)

i) Let \(n_1 = 100, n_2 = 20, \alpha_1 = 35, \alpha_2 = 40, c = .25, h = 1, l = 0. \)

\((q_1^*, q_2^*) = (1.3528, 8.8204) \)

\((q_1^m, q_2^m) = (3.2612, 3.1965) \)
i.e., the monopolist provides sub-optimal quality at high end but a quality higher than the optimal level at the lower end (region ‘C’ of Figure 3.2.b).

ii) Let \( n_1 = 100, n_2 = 100, \alpha_1 = 35, \alpha_2 = 40, c = .25, h = 1, l = 0. \)
\[
(q_1^*, q_2^*) = (2.0525, 6.2063)
\]
\[
(q_1^{m}, q_2^{m}) = (1.8732, 6.1305)
\]
i.e., the monopolist provides sub-optimal quality at both ends (region ‘B’ of Figure 3.2.b).

iii) Let \( n_1 = 30, n_2 = 20, \alpha_1 = 35, \alpha_2 = 50, c = .25, h = 1, l = 0. \)
\[
(q_1^*, q_2^*) = (1.829, 6.92)
\]
\[
(q_1^{m}, q_2^{m}) = (1.4794, 7.3754)
\]
Here the monopolist provides sub-optimal quality at lower end but a quality higher than the optimal level at the high end (region ‘A’ of Figure 3.2.b).

**Example 2:** Let the performance level, \( r \), take up two distinct values –

- a high performance denoted by \( r = h \) and a low performance denoted by \( r = l \).

Let \( P(h) = q_1q_2 \), \( q_1, q_2 \in [0, 1] \)
\[
P(l) = 1 - q_1q_2
\]
\[
u(r, \alpha_i) = \alpha_i \sqrt{r}
\]
\[
c(q_i) = cq_i^{2/2}
\]
In this case, \( \frac{\partial^2 W}{\partial q_1 \partial q_2} = n_1 \frac{\partial^2 V_1}{\partial q_1 \partial q_2} = n_1 \alpha_1 > 0. \)

Let \( n_1 = 10, n_2 = 20, \alpha_1 = 3, \alpha_2 = 4, c = 5, h = 1, l = 0. \)
\[
(q_1^*, q_2^*) = (.58537, .97561)
\]
\[
(q_1^{m}, q_2^{m}) = (.16327, .81633)
\]
i.e., the monopolist provides sub-optimal quality at both ends.
Proposition 3.3 states and proves the relationship found graphically and numerically between the monopolist’s optimal quality levels and those of the social planner.

**Proposition 3.3:**

If \( \frac{\partial^2 W}{\partial q_1 \partial q_2} > 0 \) then \( q_1^m < q_1^* \) and \( q_2^m < q_2^* \)

**Proof:** It follows from Lemma 2.1 of Chapter Two that if \( \frac{\partial^2 W}{\partial q_1 \partial q_2} > 0 \) then at \((q_1^*, q_2^*)\),

\[
\frac{dq_2}{dq_1}_{M1} > \frac{dq_2}{dq_1}_{M2} > 0
\]

Since, there is a unique maximum it follows that \( g(q_2) \oplus h(q_2) \) according as \( q_2 \oplus q_2^* \).

Along \( q_1 = h(q_2) \), \( \frac{\partial W(q_1, q_2)}{\partial q_1} = 0 \) and since \( \frac{\partial^2 W}{\partial q_1^2} < 0 \), at any \((q_1, q_2)\) where \( q_1 < h(q_2) \), \( \frac{\partial W(q_1, q_2)}{\partial q_1} > 0 \).

Again, along \( q_1 = g(q_2) \), \( \frac{\partial W(q_1, q_2)}{\partial q_2} = 0 \) and since \( \frac{\partial^2 W}{\partial q_1 \partial q_2} > 0 \), at any \((q_1, q_2)\) where \( q_1 > g(q_2) \), \( \frac{\partial W(q_1, q_2)}{\partial q_2} > 0 \).

Equations (3.8) and (3.14) imply \( \frac{\partial W (q_1^m, q_2^m)}{\partial q_1} > 0 \).

Similarly, equations (3.9) and (3.15) imply \( \frac{\partial W (q_1^m, q_2^m)}{\partial q_2} > 0 \).

Hence, \( g(q_2^m) < q_1^m < h(q_2^m) \) \hspace{1cm} (3.I)

This implies \( q_2^m < q_2^* \).

Again, since \( h'(q_2) > 0 \), \( h(q_2^m) < h(q_2^*) = q_1^* \) \hspace{1cm} (3.II)

Combining (3.1) and (3.2) we get \( g(q_2^m) < q_1^m < h(q_2^m) < h(q_2^*) = q_1^* \).

Hence, the monopolist provides sub-optimal qualities at both ends. □
Proposition 3.4:

Let $\partial^2 W / \partial q_1 \partial q_2 < 0$

i) If $q_2^m > q_2^*$ then the upper limit of $q_1$ is given by $q_1 = g(q_2^m)$

ii) If $q_2^m < q_2^*$ then the upper limit of $q_1$ is given by $q_1 = h(q_2^m)$

Proof: It follows from Lemma 2.1 that if $\partial^2 W / \partial q_1 \partial q_2 < 0$ then at $(q_1^*, q_2^*)$,

$\frac{dq_2}{dq_1}_{M1} < \frac{dq_2}{dq_1}_{M2} < 0$

Since, there is a unique maximum it follows that $g(q_2) \nless h(q_2)$ according as $q_2 \nless q_2^*$.

i) If $q_2^m > q_2^*$ then $g(q_2^m) < g(q_2^*) = q_1^*$ [since $g' < 0$] \hspace{1cm} (3.III)

Along $q_1 = g(q_2)$, $\partial W(q_1, q_2) / \partial q_2 = 0$ and since $\partial^2 W / \partial q_1 \partial q_2 < 0$, at any $(q_1, q_2)$ where $q_1 < g(q_2), \partial W(q_1, q_2) / \partial q_2 > 0$.

From equations (3.9) and (3.15) we have $\partial W (q_1^m, q_2^m) / \partial q_2 > 0$.

Hence, for $q_2 > q_2^*, q_1^m < g(q_2^m)$ \hspace{1cm} (3.IV)

Combining (3.III) and (3.IV) we get $q_1^m < g(q_2^m) < g(q_2^*) = q_1^*$

ii) If $q_2^m < q_2^*$ then $h(q_2^m) > h(q_2^*) = q_1^*$ [since $h' < 0$] \hspace{1cm} (3.V)

Along $q_1 = h(q_2)$, $\partial W(q_1, q_2) / \partial q_1 = 0$ and since $\partial^2 W / \partial q_1^2 < 0$, at any $(q_1, q_2)$ where $q_1 < h(q_2), \partial W(q_1, q_2) / \partial q_1 > 0$.

Equations (3.8) and (3.14) give $\partial W (q_1^m, q_2^m) / \partial q_1 > 0$.

Hence, for $q_2 < q_2^*, q_1^m < h(q_2^m)$ \hspace{1cm} (3.VI)

Combining (3.V) and (3.VI) we get $q_1^m < h(q_2^m) > h(q_2^*) = q_1^*$.
Hence, the monopolist may provide a product with a quality specification level above or below the socially optimal level at both ends. □

Like the previous chapter it may be noted here that even when enhanced level of quality is provided at the higher end, it may not be profitable for the firm to increase the quality of the product beyond a certain level due to cost conditions.

3.3. Conclusion

In this chapter we have again studied the impact of quality spillover effects in a vertically differentiated goods model. This spillover effect is also one-sided but now we assume that changes in the quality specification at the higher end affect the customers’ perceptions about the performance of the lower quality good. However, changes in the lower end quality specification of the product have no effect on the customers’ perceptions about the performance of the higher quality good. Under this situation, we show that the standard result breaks down. Quality distortion may take place at either end and it may take the form of either enhanced level of quality or sub-optimal level of quality. However, these possibilities do not always give us a clear idea about what exactly happens to the quality spectrum.