Chapter Two

Monopolistic Choice of Product Specifications when Lower End Product Specifications Provide Imperfect Signals of the Performance of the High End Product

2.1. Introduction

Consider the case of a firm that offers different varieties of the same basic product to customers. The varieties differ in the levels of sophistication of their technical features. In such cases, it is fairly commonplace for customers to refer to different varieties as high end and low end products, where high end products are the more technically sophisticated products that are expected to perform better than the less technically sophisticated low end products. Ostensibly, firms resort to such business strategies because they face a heterogeneous customer base with variations in individual incomes and valuation of quality of performance. Thus, loosely speaking, less sophisticated products are designed for customers who have low incomes and/or are unwilling to pay much for superior levels of performance; and sophisticated products are designed for customers who have higher incomes and/or attach greater value to product performance. The firm is thus faced with the following decision problem: what is the optimal menu of products (and corresponding prices) that it should offer to its customers? The decision problem becomes more complex once one recognizes that a firm may have a fairly good idea about the distribution of the types of customers it faces, but may not have a foolproof way of identifying a customer’s type a priori. In that case, a mistake in quality selection and pricing by the firm can result in customers who value quality more opting to purchase products designed for customers who have a lower valuation of quality. The net result from the firms’ perspective is that it would earn a lower level of profit than it could have because it failed to induce people who are willing to pay more for higher quality of
performance actually pay for higher performance. At the heart of the literature on monopolistic quality discrimination lie two questions: what is the menu of “quality” and corresponding prices that the firm should select to induce buyers of each type to buy the product that was designed for them, and also maximize its total profit? And how does this menu differ from the one that a completely informed social planner would offer if he were seeking to maximize social welfare?

In the standard model of quality discrimination by a monopolistic seller, no distinction, however, is made between the level of technical sophistication of a product and its performance. The two are lumped together under a catch-all term, “quality”. In practice, buyers observe technical specifications and may use other information – brand names, feedback from previous users, advertisement, market shares, etc. – to arrive at a judgement about the kind of performance that can be expected from the product. This in turn helps to determine a buyer’s willingness to pay for a unit of the product. In this chapter, we introduce the possibility that customers can only observe technical specifications and are unable to identify the exact performance level of the product a priori. In that sense, the goods in our model are experience goods: a customer may form a priori beliefs about the level of performance of the good based on a study of its technical quality specification, hearsay evidence, etc., but is able to form a clear idea of performance only after it is consumed. Advertisements for cars, electronic equipments like computers, audio and video systems provide technical details that are often confusing for the average customer. For example, a washing machine manufacturer in India issued a series of advertisements for one of its machines that claimed that the machine’s electronics used “fuzzy logic”. It was unclear why this would, in practice, lead to a superior performance, but the idea behind the advertisement appears to have been to convey the message that the machine with its “superior” technology was designed to do a better job of cleaning than regular machines. In particular, we assume following Cabral (2000) that buyers can form an idea (more specifically, a probability distribution) of performance levels based on the quality specification such that a “higher” technical specification generates a “better” probability distribution of performance in the First-
Order Stochastic Dominance (FSD) sense. This, as we shall see in the next section, implies that the amount a customer is willing to pay for a product increases with improvements in the quality specification of the product.\(^4\)

A well known result of the theory of monopolistic quality discrimination says that in a world where there are say two types of customers,\(^6\) a profit maximizing monopolist who cannot identify each customer’s type, \textit{a priori}, distorts the quality spectrum in the sense that buyers who have a higher preference for quality are offered the socially optimal quality level, i.e. the quality level that would have been provided by a fully informed social planner, and those who value performance less are offered a quality below the socially optimal level. Since the price that can be extracted from the high valuation customer increases as the distortion in the quality offered to the low end customer rises, this strategy is a way of extracting a larger price from the high end customer than he would have parted with had the socially optimal low end quality been available. A side effect of this strategy is that the monopolist is able to extract a smaller profit from the low end customer than what it could have had it offered the socially optimal quality to customers who have a relatively lower willingness to pay \cite[e.g., Mussa and Rosen (1978), Maskin and Riley (1984), Tirole (1988), Acharyya (1998), etc.]{7}. At the optimum, the rise in revenue, collected from the high valuation customers as a result of a marginal increase in the distortion of the quality offered to the low valuation customer, is exactly offset by the loss it generates in revenue collected from the latter.

Incorporation of the distinction between quality specifications and actual performance by itself does not alter the basic theoretical result, as we shall see. What it does is that it

\(^5\) Our formal specification draws heavily on Tadelis (1999) and Cabral (2000).
\(^6\) We retain this assumption throughout the paper.
\(^7\) This assumes that customers who have a lower valuation for quality buy the low end quality product, and those who attach a higher valuation to quality buy the high end product. According to Srinagesh and Bradburd (1989) it is the customer group’s relative levels of marginal utility of quality that determines whether they buy the low or high quality good. Because of their assumption of negative association between total and marginal utility of quality distortion takes place even at the high end.
allows us to extend the theory to ask what happens if the quality specification choice at one end affects prospective buyers’ beliefs about the likely performance of the product positioned at the other end of the spectrum? In particular, suppose that the low end product specification acts as a signal about the firm’s ability to produce high quality (measured in terms of performance) high end products: a firm that produces very basic low end products may be seen as one that is incapable of producing high quality top end products. It took Japanese car and electronic devices manufacturers a long time to establish that they were not just producers of “cheap” goods who were incapable of producing good quality high end products. The Indian automobile company Tata Motors, which initially used to manufacture trucks and low end cars, has attracted a lot of publicity over the production of ‘Nano’, another low-budget car. Very recently, the company has acquired luxury auto brands – Jaguar and Land Rover from Ford Motor. This has generated speculation that the brand image of these high end cars would be adversely affected because Tata Motors is a producer of low end cars and has no experience in manufacturing technically sophisticated cars. We explore the possibility that firms that produce relatively (technically) sophisticated low end products induce the belief among customers that the high end products they produce are likely to perform better because of their higher technical skill. That is, customers expect that firms that have the technical skill to make technically sophisticated low end products are also likely to make high end products that perform well.

In our model, the technical sophistication of the low end product acts as a bearer of information. The higher is the technical sophistication of the lower end product, the higher is the customer’s willingness to pay for the high end product. In the standard monopolistic quality discrimination model, where the quality specification at the lower end does not affect the customer’s willingness to pay for the product located at the other end of the quality spectrum, the distortion of the former is used as an instrument to extract a higher price from the customers who buy the high end product. Once spillovers are introduced, as in our model, a lowering of the lower end quality specification also has
the contrary effect of reducing the customers’ willingness to pay for the high end product. The firm has to keep this in mind when selecting its product quality specifications.

In Wernerfelt (1988), Tadelis (1999) and Cabral (2000) brand names are bearers of information. In Tadelis (1999), each product is sold under a different name and these names can be traded. The model looks at equilibrium in the market for names and studies the reputational effects that characterise such markets. Assuming that transactions carried out in the market for names are hidden from the potential clients, the paper shows that names must be actively traded in all equilibria. Wernerfelt (1988) and Cabral (2000) focus on the firm’s decision to “brand stretch” given fixed qualities. In Cabral (2000), firms introduce new products whose qualities are known to them, but are unknown to buyers a priori. Ex post, buyers can update their beliefs about the quality of the product based on their experience about the performance of the product. A firm that introduces a new good using its established brand name faces the possibility that its reputation may suffer if the buyers’ experience is not good.

Our problem is different from the ones addressed in these papers in that we explore the effects of quality specifications acting as bearers of information on quality choices made by a seller under second degree quality discrimination. In our model, a firm that produces low end products knows that its low end quality specification affects its customers’ valuation of its higher end products. The question is: will a firm facing two types of customers provide a quality level that is different from the socially optimal level to its high valuation customers in equilibrium? We show in this chapter that in the presence of the spillover effect of the low end product’s quality specification, the standard result of monopolistic quality discrimination breaks down. It is possible that in equilibrium, high end customers are provided a product with a quality specification level above or below the socially optimal level. However, the low end customers are always provided a product whose quality specification is below the socially optimal level. In a closely related paper, Kim and Kim (1996) introduced technical spillover effects into the model by assuming that the cost of production of the low end quality falls as the quality of the
high end product increases. They found that under some conditions the monopolist supplies a high end quality level that is different from the socially optimal level.

The chapter is organized as follows: In section 2.2 we present the formal model and derive the principal result. Section 2.3 offers some concluding remarks.

2.2. The Model

Consider a monopolist producing a vertically differentiated good. In the standard formulation of the model, there are (say) two types of customers indexed by \( i = 1, 2 \) and there are \( n_i \) customers of type \( i \), where \( n_i > 0 \). Each customer buys at most one unit of the good. The monopolist chooses characteristics \((q_1, q_2)\) from the interval \([\underline{q}, \overline{q}]\) and a corresponding set of prices \((p_1, p_2)\) for the product. Potential customers observe the monopolist’s choice of \((q_1, q_2)\) and \((p_1, p_2)\) before deciding the type of product they wish to purchase. What this presumes is that the announced quality specification provides an exact (degenerate distribution) signal to the buyer of the nature of performance that he can expect from the product. In other words, the goods in the standard model are not experience goods. The critical feature of our model is that our goods are experience goods: buyers can observe quality specifications \textit{a priori}, but these specifications provide a noisy signal of the quality of performance that they can expect when they use it.

Let \( r \in [\underline{r}, \overline{r}] \) designate the actual performance of the product. This is not directly observable before the product is used. However, customers can form beliefs about the likelihood of different performance levels based on the announced quality specification. Such beliefs are captured by a cumulative distribution function, say \( F(r| q_j, q_k) \), \( q_j, q_k = 1, 2, q_j \neq q_k \), where the first element \( q_j \) designates the quality purchased and the second element \( q_k \) is the other available quality. The standard monopolistic quality discrimination model then reduces to the special case where there is a monotonically
increasing function $x: [q, \bar{q}] \rightarrow [\underline{r}, \bar{r}]$ such that $F(r| q_j, q_k) = 0$ if $r < x(q_j)$ and equals 1 otherwise. Notice that $x(.)$ does not depend upon $q_k$.

The utility that a type-$i$ customer derives from the consumption of the good is a function of the performance level of the product and a type specific taste parameter $\alpha_i$:

$$u^i = u(r, \alpha_i), \text{ with } \frac{\partial u(r, \alpha_i)}{\partial r} > 0 \text{ and } \frac{\partial^2 u(r, \alpha_i)}{\partial r^2} < 0, \; i = 1, 2 \quad (2.1)$$

Hence, the willingness of customer $i$ to pay for a product of quality specification $q_j$ when $q_k$ is the other quality that is available, is

$$V^i_j(q_j, q_k) = \int_{\underline{r}}^{\bar{r}} u(r, \alpha_i) dF(r| q_j, q_k), \; i = 1, 2, \; j, k = 1, 2, \; j \neq k \quad (2.2)$$

$$= u(\tilde{r}, \alpha_i) F(\tilde{r} | q_j, q_k) - u(r, \alpha_i) F(r | q_j, q_k) - \int_{\underline{r}}^{\bar{r}} F(r | q_j, q_k) du(r, \alpha_i) \quad (2.2')$$

$$i = 1, 2, \; j, k = 1, 2, \; j \neq k$$

Since $u(r, \alpha_i)$ is differentiable in $r$ and since $F(\tilde{r} | q_j, q_k) = 1$ and $F(\tilde{r} | q_j, q_k) = 0$,

$$V^i_j(q_j, q_k) = u(\tilde{r}, \alpha_i) - \int_{\underline{r}}^{\bar{r}} (\frac{\partial u(r, \alpha_i)}{\partial r}) F(r | q_j, q_k) dr \quad (2.2')$$

We assume that the total utility as well as the marginal utility is higher for the type-2 customers than for the type-1 ones:

$$u(r, \alpha_2) > u(r, \alpha_1) \quad \forall r \in [\underline{r}, \bar{r}] \quad (2.3)$$

$$u_t(r, \alpha_2) > u_t(r, \alpha_1) \quad \forall r \in [\underline{r}, \bar{r}] \quad (2.4)$$

It is easy to see that equation (2.2) and inequality (2.3) imply $V^2_1 > V^1_1$ and $V^2_2 > V^2_1$.

---

Consider now a customer of type $i$ who is considering purchasing the good with quality specification $q_j$. If the value of $q_j$ increases, i.e. the quality specification of the good under consideration improves, then this generates the belief in the customer’s mind that the probability of getting a better level of performance from the product is higher than before. A change in the quality specification of the other (quality) good available, i.e. a change in the value of $q_k$, has asymmetric effects. In the presence of the one-sided spillover effect, mentioned earlier, the probability distribution of performance at the higher end “improves” as the quality specification at the lower end increases, while the probability distribution of performance levels at the lower end remains unaffected by changes in the higher end quality specification. If the good being considered by the customer is the one with the higher quality specification then an improvement in the quality specification of the lower end product has a “positive” impact on the customer’s beliefs about the performance he can expect from the product he is looking at. On the other hand, if the good under consideration is at the lower end of the quality spectrum being offered, a change in the quality specification of the high end product has no impact on the customer’s beliefs about the performance that he can expect from the low end product. To capture this intuition we assume that in each of the first two cases, the higher quality specification generates a “better” distribution of performance in the First-Order Stochastic Dominance (FSD) sense. In the last case, a change in $q_k$ does not have any effect on the probability distribution of performances.

The Distribution $F(x, y)$ first-order stochastically dominates the distribution $F(x, y')$, $y \neq y'$, if, for every non-decreasing function $u: \mathbb{R} \to \mathbb{R}$ we have

$$\int_x u(x) \ dF(x, y) \geq \int_x u(x) \ dF(x, y')$$

(2.5)

A well-known result is that

$$\int_x u(x) \ dF(x, y) \geq \int_x u(x) \ dF(x, y') \iff F(x, y) \leq F(x, y') \text{ for every } x.$$
Thus, in formal language, we assume\footnote{It is to be noted that $F(r|q_j, q_k)$ is defined as the cumulative distribution of performance where $q_j$ is the quality purchased and $q_k$ is the other alternative quality.}: 

(I) for all $q_k, q_j, q'_k \in [q, \bar{q}]$, and $q_j > q'_j$, $F (r| q_j, q_k) \leq F (r| q'_j, q_k)$ for every $r \in [\underline{r}, \bar{r}]$ with the strict inequality holding at some $r$; 

(II) for all $q_j$ and $q_k, q'_k \in [q, \bar{q}]$, $q_j > q_k > q'_k$, $F (r| q_j, q_k) \leq F (r| q_j, q'_k)$ for every $r \in [\underline{r}, \bar{r}]$ with the strict inequality holding at some $r$; and 

(III) for all $q_j$ and $q_k, q'_k \in [q, \bar{q}]$, $q_k > q'_k > q_j$, $F (r| q_j, q_k) = F (r| q_j, q'_k)$ for every $r \in [\underline{r}, \bar{r}]$.

If $F(.)$ is differentiable with respect to $q_j, q_k$ then assumptions (I'), (II') and (III') stated below are equivalent to assumptions (I), (II) and (III) respectively:

for any $q_j, q_k \in [q, \bar{q}]$, 

(I') $\partial F (r| q_j, q_k) / \partial q_j \leq 0$ for every $r \in [\underline{r}, \bar{r}]$ with a strict inequality holding at some $r$; 

(II') if $q_j > q_k$, $\partial F (r| q_j, q_k) / \partial q_k \leq 0$ for every $r \in [\underline{r}, \bar{r}]$ with a strict inequality holding at some $r$; and 

(III') if $q_j \leq q_k$, $\partial F (r| q_j, q_k) / \partial q_k = 0$ for every $r \in [\underline{r}, \bar{r}]$.

\textbf{Proposition 2.1:}

\begin{enumerate}
\item[(i)] $\forall q_j, q_k \in [q, \bar{q}]$, $\partial u (r, \alpha_i) / \partial r > 0$ and (I') imply $\partial V^i_j (q_j, q_k) / \partial q_j \geq 0$; 
\item[(ii)] $\forall q_j, q_k \in [q, \bar{q}]$ such that $q_j > q_k$, $\partial u (r, \alpha_i) / \partial r > 0$ and (II') imply $\partial V^i_j (q_j, q_k) / \partial q_k \geq 0$; 
\item[(iii)] $\forall q_j, q_k \in [q, \bar{q}]$ such that $q_j < q_k$, $\partial u (r, \alpha_i) / \partial r > 0$ and (III') imply $\partial V^i_j (q_j, q_k) / \partial q_k = 0$.
\end{enumerate}
Proof: (i) Applying Leibniz Rule to differentiate (2.2') with respect to $q_j$

$$\frac{\partial V_j^i}{\partial q_j} = 0 - \int_{\bar{r}} \left( \frac{\partial u(r, \alpha_i)}{\partial r} \frac{\partial F(r| q_j, q_k)}{\partial q_j} \right) dr$$

Since, marginal utility $\partial u(r, \alpha_i) / \partial r > 0$ it follows from assumption (I') that 
$$\frac{\partial V_j^i(q_j, q_k)}{\partial q_j} \geq 0.$$ 

(ii) Applying Leibniz Rule to differentiate (2.2') with respect to $q_k$ we get 

$$\frac{\partial V_j^i}{\partial q_k} = 0 - \int_{\bar{r}} \left( \frac{\partial u(r, \alpha_i)}{\partial r} \frac{\partial F(r| q_j, q_k)}{\partial q_k} \right) dr$$

Since, marginal utility $\partial u(r, \alpha_i) / \partial r > 0$ it follows from assumption (II') that 
$$\frac{\partial V_j^i(q_j, q_k)}{\partial q_k} \geq 0.$$ 

(iii) Since, marginal utility $\partial u(r, \alpha_i) / \partial r > 0$ it follows from assumption (III') that 
$$\frac{\partial V_j^i(q_j, q_k)}{\partial q_k} = 0. \Box$$

In the special case where $q_1 < q_2$, Proposition 2.1 says that:

$$\frac{\partial V_1^i(q_1, q_2)}{\partial q_1} > 0 \quad i = 1, 2 \quad (2.6a)$$

$$\frac{\partial V_1^i(q_1, q_2)}{\partial q_2} = 0 \quad i = 1, 2 \quad (2.6b)$$

$$\frac{\partial V_2^i(q_2, q_1)}{\partial q_1} > 0 \quad i = 1, 2 \quad (2.6c)$$

$$\frac{\partial V_2^i(q_2, q_1)}{\partial q_2} > 0 \quad i = 1, 2 \quad (2.6d)$$

Proposition 2.2:

$q_1 < q_2$ and (III') imply 
$$\frac{\partial^2 V_1^i(q_1, q_2)}{\partial q_1 \partial q_2} = 0, \quad i = 1, 2.$$ 

Proof: 
$$\frac{\partial^2 V_1^i(q_1, q_2)}{\partial q_1 \partial q_2} = \frac{\partial}{\partial q_1} \left( \frac{\partial V_1^i(q_1, q_2)}{\partial q_2} \right) = \frac{\partial^2 V_1^i(q_1, q_2)}{\partial q_2} = 0 \ (\text{using (III')}). \ Box$$
Finally, assume that the total cost of production, C(q), and the marginal cost vary positively with the quality:

\[ \frac{dC(q)}{dq} > 0 \text{ and } \frac{d^2C(q)}{dq^2} > 0. \]  

(2.7)

2.2.1. The Social Planner’s Solution

To establish the benchmark qualities against which we shall measure the distortion that the profit-maximizing monopolist introduces in the quality spectrum we first look at the quality choices of the Social Planner who chooses quality levels to maximize social welfare. In accordance with analytical custom, we assume that the social planner, unlike the monopolist, is fully informed about each potential customer’s preference pattern.

Let W be the social welfare level. Hence the problem of the social planner is:

\[
\begin{align*}
\text{Max } & W = n_1[V_1^1(q_1, q_2) - C(q_1)] + n_2[V_2^2(q_2, q_1) - C(q_2)] \\
( & (q_1, q_2))
\end{align*}
\]

Let \( (q_1^*, q_2^*) \) be the unique interior extremum of the welfare-maximization problem, that is, \( q_1^* \in [\underline{q}, \overline{q}] \) and \( q_2^* \in [\underline{q}, \overline{q}] \). Then \( (q_1^*, q_2^*) \) satisfy the following first-order conditions:

\[
\frac{\partial W}{\partial q_1} = n_1[\frac{\partial V_1^1(q_1, q_2)}{\partial q_1} - \frac{dC(q_1)}{dq_1}] + n_2[\frac{\partial V_2^2(q_2, q_1)}{\partial q_1} - \frac{dC(q_2)}{dq_2}] = 0 \quad (2.8)
\]

\[
\frac{\partial W}{\partial q_2} = n_2[\frac{\partial V_2^2(q_2, q_1)}{\partial q_2} - \frac{dC(q_2)}{dq_2}] = 0 \quad (2.9)
\]

Assume further that \( q_1^* < q_2^* \). Let the locus of \( (q_1, q_2) \) satisfying equations (2.8) and (2.9) be denoted by \( M_1 \) and \( M_2 \) respectively. The slopes of \( M_1 \) and \( M_2 \) are given by:
\[
\begin{align*}
\frac{dq_2}{dq_1} M_1 &= - \frac{\partial^2 W / \partial q_1^2}{\partial^2 W / \partial q_1 \partial q_2} \\
&= - \left[ n_1 \partial^2 V_1^1(q_1, q_2) / \partial q_1^2 - n_1 \partial d^2 C(q_1) / \partial q_1^2 + n_2 \partial^2 V_2^2(q_2, q_1) / \partial q_1^2 \right] \\
&= \frac{n_2 \partial^2 V_2^2(q_2, q_1) / \partial q_1 \partial q_2}{\partial^2 W / \partial q_1 \partial q_2} \\
\frac{dq_2}{dq_1} M_2 &= - \frac{\partial^2 W / \partial q_1 \partial q_2}{\partial^2 W / \partial q_2^2} \\
&= - \left[ \partial^2 V_2^2(q_2, q_1) / \partial q_1 \partial q_2 \right] / \left[ \partial^2 V_2^2(q_2, q_1) / \partial q_2^2 - d^2 C(q_2) / \partial q_2^2 \right] \\
\end{align*}
\]

Second-order conditions require that the principal minors of the relevant Hessian determinant evaluated at \((q_1^*, q_2^*)\) alternate in sign:

\[
i.e., \partial^2 W / \partial q_1^2 < 0, \partial^2 W / \partial q_2^2 < 0
\]

and

\[
\begin{vmatrix}
\partial^2 W / \partial q_1^2 & \partial^2 W / \partial q_2 \partial q_1 \\
\partial^2 W / \partial q_1 \partial q_2 & \partial^2 W / \partial q_2^2
\end{vmatrix} > 0
\]

The following Lemma describes the slopes of \(M_1\) and \(M_2\).

**Lemma 2.1:**

Slopes of \(M_1\) and \(M_2\) depend on the sign of \(\partial^2 W / \partial q_1 \partial q_2\). However, \(M_1\) is “steeper” than \(M_2\) irrespective of the sign of \(\partial^2 W / \partial q_1 \partial q_2\).

**Proof:** It follows immediately from equations (2.10), (2.11) and condition (2.12) that if \(\partial^2 W / \partial q_1 \partial q_2\) is positive then \(M_1\) and \(M_2\) are both positively sloped; and they are both negatively sloped when \(\partial^2 W / \partial q_1 \partial q_2\) is negative.
Again equations (2.10) and (2.11) imply that

\[
(dq_2 / dq_1)_{M_1} - (dq_2 / dq_1)_{M_2}
\]

\[
= - [(\partial^2 W / \partial q_1^2) / (\partial^2 W / \partial q_1 \partial q_2) - (\partial^2 W / \partial q_1 \partial q_2) / (\partial^2 W / \partial q_2^2)]
\]

\[
= -[((\partial^2 W / \partial q_1^2) (\partial^2 W / \partial q_2^2) - (\partial^2 W / \partial q_1 \partial q_2)^2) / ((\partial^2 W / \partial q_1 \partial q_2) (\partial^2 W / \partial q_2^2))]
\]

\[
= 1/ (\partial^2 W / \partial q_1 \partial q_2) [\frac{1}{2} ((\partial^2 W / \partial q_1^2) (\partial^2 W / \partial q_2^2) - (\partial^2 W / \partial q_1 \partial q_2)^2) / \partial^2 W / \partial q_2^2]
\]

The second order conditions for maximum (2.12) and (2.13) imply that the bracketed term is positive. Hence,

if \( \partial^2 W / \partial q_1 \partial q_2 > 0 \) then at \((q_1^*, q_2^*)\), \((dq_2 / dq_1)_{M_1} > (dq_2 / dq_1)_{M_2} > 0 \) and

if \( \partial^2 W / \partial q_1 \partial q_2 < 0 \) then at \((q_1^*, q_2^*)\), \((dq_2 / dq_1)_{M_1} < (dq_2 / dq_1)_{M_2} < 0 \)

i.e., \( M_1 \) is “steeper” than \( M_2 \) in both the cases. □

From Proposition 2.2 it follows that \( \partial^3 W / \partial q_1 \partial q_2 = n_2 \partial^2 V_2^2(q_2, q_1) / \partial q_1 \partial q_2 \). This can be either positive or negative. Now \( \partial^2 V_2^2(q_2, q_1) / \partial q_1 \partial q_2 = \partial / \partial q_2 (\partial V_2^2(q_2, q_1) / \partial q_1) > 0 \) implies that the impact of the spillover effect, \((\partial V_2^2(q_2, q_1) / \partial q_1)\), increases as the quality specifications at the higher end improves. On the other hand, a negative value of \( \partial / \partial q_2 (\partial V_2^2(q_2, q_1) / \partial q_1) \) implies that the impact of the spillover effect decreases as the quality specifications at the higher end improves.

Figures (2.1.a) and (2.1.b) show how the impact of the spillover changes with the change in the quality specifications at the higher end.
Figure 2.1.a: Change in the impact of the spillover effect with respect to $q_2$ when $\partial^2W / \partial q_1 \partial q_2 > 0$

Figure 2.1.b: Change in the impact of the spillover effect with respect to $q_2$ when $\partial^2W / \partial q_1 \partial q_2 < 0$

Figures (2.2.a) and (2.2.b) show the optimum choice of quality of the Social Planner in the two cases\textsuperscript{10}.

\textsuperscript{10} If the Jacobean of the Welfare function has a dominant diagonal, then $M_2$ in Figure (2.2.a) will have a slope less than 1.
Figure 2.2.a: The Social Planner’s optimum choice of quality when $\frac{\partial^2 W}{\partial q_1 \partial q_2} > 0$

Figure 2.2.b: The Social Planner’s optimum choice of quality when $\frac{\partial^2 W}{\partial q_1 \partial q_2} < 0$
2.2.2. Quality Choice under Asymmetric Information: The Monopolist’s Solution

Unlike the Social planner, the monopolist suffers from an informational deficiency: he is unable to identify the type of each customer \( a \ priori \). Under this condition, the monopolist’s problem is to select a profit maximizing pair of customer type specific contracts \((p_1, q_1)\) and \((p_2, q_2)\) such that for each type of customer there is at least one acceptable contract (individual rationality); and no customer is better off accepting a contract designed for a customer whose type is different from his own (incentive compatibility). Formally, the monopolist’s problem is:

\[
\text{Max } \Pi = \sum_{i=1}^{2} [p_i - C(q_i)]n_i
\]

(subject to:

(i) \( p_1 \leq V_1^1(q_1, q_2) \)
(ii) \( p_2 \leq V_2^2(q_2, q_1) \)
(iii) \( V_1^1(q_1, q_2) - p_1 \geq V_2^1(q_2, q_1) - p_2 \)
(iv) \( V_2^2(q_2, q_1) - p_2 \geq V_1^2(q_1, q_2) - p_1 \)

Conditions (i) and (ii) are the individual rationality constraints for the first and second customer types respectively; and conditions (iii) and (iv) are their respective incentive compatibility constraints. It is possible, of course, that no such separating menu of contracts exists and the profit maximizing strategy is to offer the same contract for all types of customers (pooling contract). A second possibility is that it is best for the monopolist to serve only one type of customer (partial market coverage). In what follows we assume that a separating menu of contracts exists. It is fairly straightforward to show that only constraints (i) and (iv) bind in equilibrium\(^{11}\) and so after some straightforward substitutions the monopolist’s problem reduces to:

\(^{11}\)If (i) is satisfied, i.e., the net surplus of the low demand customers is positive then the high demand customers are automatically willing to purchase, i.e., (ii) is also satisfied. Moreover, the monopolist, who benefits from higher prices, is able to extract the entire surplus from the lower end. Hence, constraint (i)
\[
\text{Max} \quad [V_1^1(q_1, q_2) - C(q_1)]n_1 + [V_2^2(q_2, q_1) + V_1^1(q_1, q_2) - V_1^2(q_1, q_2) - C(q_2)]n_2
\]
\[(q_1, q_2)\]

The first-order conditions for an interior maximum are:

\[n_1[\partial V_1^1(q_1, q_2) / \partial q_1 - dC(q_1) / dq_1] + n_2[\partial V_2^2(q_2, q_1) / \partial q_1 + \partial V_1^1(q_1, q_2) / \partial q_1 - \partial V_1^2(q_1, q_2) / \partial q_1 ] = 0 \quad (2.14)\]

and \[n_2[\partial V_2^2(q_2, q_1) / \partial q_2 - dC(q_2) / dq_2] = 0 \quad (2.15)\]

The second-order conditions are assumed to hold. Let \((q_1^m, q_2^m)\) be the unique profit-maximizing quality levels of the monopolist.

We are now in a position to answer the fundamental question that we ask in this chapter: if improvements in the lower end quality specification have a positive effect on customers’ beliefs about the performance that they can expect from products at the higher end of the quality spectrum, how do the quality levels chosen by the monopolist differ from those of the social planner? Figures 2.2.a and 2.2.b provide an intuitive idea of the answer to this question. Consider first the case where \(\partial^2 W / \partial q_1 \partial q_2 > 0\) [Figure 2.2.a]. As equations (2.9) and (2.15) are identical, the monopolist’s profit maximizing choice of \((q_1, q_2)\) must lie on \(M_2\). Since \((q_1^*, q_2^*)\) is by assumption a unique extremum, \(\partial^2 W / \partial q_1^2 < 0\) for all \((q_1, q_2)\) in the interior of \([\underline{q}, \overline{q}] \times [\underline{q}, \overline{q}]\). This means that to the left of \(M_1, \partial W / \partial q_1 > 0\) and to the right of \(M_1, \partial W / \partial q_1 > 0\). Again, with \(\partial^2 W / \partial q_1 \partial q_2 > 0\), to the left of \(M_2, \partial W / \partial q_2 < 0\) and to the right of \(M_2, \partial W / \partial q_2 > 0\). However, if \(\partial^2 W / \partial q_1 \partial q_2 < 0\), to

---

binds in equilibrium. The incentive compatibility constraint (iii) is not relevant as the monopolist is more interested in inducing the high demand customers to reveal their true type.
the left of $M_2$, $\partial W / \partial q_2 > 0$ and to the right of $M_2$, $\partial W / \partial q_2 < 0$. Now, using equations (2.8) and (2.14):

$$\partial W (q_1^m, q_2^m) / \partial q_1 = -n_2 [\partial V_1^1(q_1^m, q_2^m) / \partial q_1 - \partial V_1^2(q_1^m, q_2^m) / \partial q_1 ] > 0$$

It follows immediately then that $(q_1^m, q_2^m)$ must lie on the segment of $M_2$ that lies to the south west of $(q_1^*, q_2^*)$. In other words, $q_1^m < q_1^*$ and $q_2^m < q_2^*$. An identical argument using Figure 2.2.b shows that if $\partial^2 W / \partial q_1 \partial q_2 < 0$, $q_1^m < q_1^*$ and $q_2^m > q_2^*$.

Before we formally state and prove this result, consider two numerical examples, which corroborate this intuition about the relationship between the monopolist’s optimal quality levels and those of the social planner.

**Example 1:** Let $F(r | q_1, q_2) = r/(q_1 q_2)$, $0 \leq r \leq r^2$.

$u(r, \alpha_i) = \alpha_i \sqrt{r}$

$c(q_i) = cq_i^2/2$

Let $n_1 = 100$, $n_2 = 200$, $\alpha_1 = 3$, $\alpha_2 = 4$, $c = 5$, $r = 20$.

In this case, $\partial^2 W / \partial q_1 \partial q_2 = n_2 \partial^2 V_2(q_2, q_1) / \partial q_1 \partial q_2 = n_2 (-5 \alpha_2 r^6 / (6q_1^{7/2} q_2^{7/2})) < 0$.

$(q_1^*, q_2^*) = (5.5602, 2.1176)$

$(q_1^m, q_2^m) = (4.692, 2.3271)$

i.e., the monopolist provides sub-optimal quality at the lower end and the high end customers get a quality level which lies above the socially optimal level.

**Example 2:** Let the performance level, $r$, takes up two distinct values –

a high performance denoted by $r = h$ and a low performance denoted by $r = l$.

Let $P(h) = \sqrt{q_1 q_2}$, $q_1, q_2 \in [0, 1]$. 42
\[ P(l) = 1 - \sqrt{(q_1 q_2)} \]
\[ u(r, \alpha_i) = \alpha_i \sqrt{r} \]
\[ c(q_i) = cq_i^2/2 \]

Let \( n_1 = 100, n_2 = 200, \alpha_1 = 30, \alpha_2 = 40, c = 5, h = 1, l = 0. \)

In this case, \( \frac{\partial^2 W}{\partial q_1 \partial q_2} = n_2 \frac{\partial^2 V_2^2(q_2, q_1)}{\partial q_1 \partial q_2} = n_2/4(\alpha_2/\sqrt{(q_1 q_2)}) > 0. \)

\((q_1^*, \ q_2^*) = (13.992, 6.0721)\)

\((q_1^m, \ q_2^m) = (7.0123, 4.8231)\)
i.e., the monopolist provides sub-optimal quality at both ends.

Proposition 2.3 states and formalises the relationship between the monopolist’s optimal quality levels and those of the social planner.

In the second example, \( r \) is a discrete variable. In the proof of Proposition 2.3, we assume that \( r \) is a continuous variable, but an analogous proof can be written down for the discrete case as well.

**Proposition 2.3:**

Let \((q_1^m, \ q_2^m)\) be the unique profit-maximizing quality vector of the monopolist.

(i) If \( \frac{\partial^2 W}{\partial q_1 \partial q_2} > 0 \) then \( q_1^m < q_1^* \) and \( q_2^m < q_2^* \); and

(ii) If \( \frac{\partial^2 W}{\partial q_1 \partial q_2} < 0 \) then \( q_1^m < q_1^* \) and \( q_2^m > q_2^* \).

**Proof:** Let \( q_1 = g(q_2) \) be the solution value of \( q_1 \) (in terms of \( q_2 \)) in equation (2.9).

Substituting for \( q_2 \) in equation (2.8) we get:

\[ \frac{\partial W}{\partial g(q_2) \partial q_2} / \partial q_1 = 0 \quad (2.8') \]

Clearly, \( q_2 = q_2^* \) is the unique solution to equation (2.8'). Further, using equations (2.8) and (2.14):
\( \partial W (q_1^m, q_2^m) / \partial q_1 = - n_2 [\partial V_1^1(q_1^m, q_2^m) / \partial q_1 - \partial V_1^2(q_1^m, q_2^m) / \partial q_1] > 0 \) \hspace{1cm} (2.16)

Differentiating the left hand side of (2.8') with respect to \( q_2 \) we get:

\[
\begin{align*}
  h(q_2) &= [\partial^2 W (g(q_2), q_2) / \partial q_2^2] [\partial g(q_2) / \partial q_2] + [\partial^2 W (g(q_2), q_2) / \partial q_1 \partial q_2] \\
  &\hspace{1cm} [\partial^2 W (g(q_2), q_2) / \partial q_2] \] \hspace{1cm} (2.17)
\end{align*}
\]

From equation (2.9),

\[
\begin{align*}
  \partial W (g(q_2), q_2) / \partial q_2 &= 0 \hspace{1cm} (2.9')
\end{align*}
\]

Differentiating both sides of equation (2.9') with respect to \( q_2 \) we get:

\[
\begin{align*}
  \partial g(q_2) / \partial q_2 &= [\partial^2 W (g(q_2), q_2) / \partial q_2^2] \] \hspace{1cm} (2.18)
\end{align*}
\]

Hence, substituting for \( \partial g(q_2) / \partial q_2 \) from equation (2.18) in equation (2.17) we get:

\[
\begin{align*}
  h(q_2) &= [(\partial^2 W (g(q_2), q_2) / \partial q_1 \partial q_2)^2 - \] \hspace{1cm} (2.19)
\end{align*}
\]

Since, by assumption, \( (q_1^*, q_2^*) \) is the unique interior maximum of the Social Planner’s optimization programme, the numerator is negative for all \( (q_1, q_2) \) in the interior of \( [\bar{q}_1, \bar{q}_1] \times [\bar{q}_2, \bar{q}_2] \). Hence the sign of \( h(q_2) \) depends on the sign of \( [\partial^2 W (g(q_2), q_2) / \partial q_1 \partial q_2] \).

If \( [\partial^2 W (g(q_2), q_2) / \partial q_1 \partial q_2] > 0 \) then \( h(q_2) < 0 \). Notice that \( h(q_2) \) is the slope of the function \( \partial W (g(q_2), q_2) / \partial q_2, \partial W (g(q_2^*), q_2^*) / \partial q_2 = 0 \) and \( \partial W (g(q_2^m), q_2^m) / \partial q_2 > 0 \).

Hence, \( q_2^m < q_2^* \). It then follows from equation (2.18) and that \( [\partial^3 W (g(q_2), q_2) / \partial q_2^2] < 0 \) for all \( (q_2, q_2) \) in the interior of \( [\bar{q}_1, \bar{q}_1] \times [\bar{q}_2, \bar{q}_2] \) that \( q_1^m = g(q_2^m) < g(q_2^*) = q_1^* \). This proves part (i) of Proposition 2.3.
Observing only that if \( \frac{\partial^2 W (g(q_2), q_2)}{\partial q_1 \partial q_2} < 0 \) then \( h(q_2) > 0 \), the proof of part (ii) of the proposition is analogous.

So, unlike the standard quality discrimination model distortion occurs at both ends of the quality levels. However, while distortion at the lower end occurs in the downward direction, it may go either way at the high end. While enhanced level of quality is provided at the higher end, it may not be possible to provide the maximum level of quality, \( \bar{q} \), at the higher end. Since we have assumed the cost function to be increasing and convex in nature with respect to the quality of the product, the cost may increase at a faster rate than the quality and hence may not be profitable for the firm to increase the quality of the product beyond a certain level. Proposition 2.4 provides sufficient conditions under which monopolistic quality choice leads to a smaller degree of differentiation than that which would have been offered by an informed Social Planner.

**Proposition 2.4:**
If slope of \( M_2 \) is non-decreasing in \( q_1 \) and bounded below by 1
then \( q_2^m - q_1^m < q_2^* - q_1^* \).

**Proof:** Along \( M_2 \), \( q_2 \) can be expressed in terms of \( q_1 \); i.e., \( q_2 = f(q_1) \).
This implies, along \( M_2 \), \( dq_2 = f'(q_1) dq_1 \).
Hence, if \( f'(q_1) > 1 \) then \( dq_2 > dq_1 \).
Since slope of \( M_2 \) is non-decreasing in \( q_1 \), from Proposition 2.3 it follows that
\( q_2^* - q_2^m > q_1^* - q_1^m \) which implies
\( q_2^m - q_1^m < q_2^* - q_1^* \).

It may be noted at this point that the absence of technical spillover effect takes us back to the standard Mussa–Rosen result.
**Observation 2.1:**
If $\frac{\partial V_2^2(q_2, q_1)}{\partial q_1} = 0$ then $q_1^m < q_1^*$ and $q_2^m = q_2^*$.

**Proof:** Given in Appendix.

**2.3. Conclusion**

In this chapter we have studied the impact of quality spillover effects in a vertically differentiated goods model. For analytical simplicity, we have restricted the analysis to the case where spillover effects are one-sided in the sense that changes in the lower end quality specification affect the customers’ perceptions about the performance of the higher quality good whereas changes in the specification of the higher end product have no effect on the customers’ perceptions about the performance of the lower quality good. The principal result of our analysis is that in a situation where quality specifications do not provide an exact (degenerate) signal of expected performance, and a firm’s decision to produce relatively more sophisticated low quality products has the effect of signalling that the performance of its higher end products is likely to be better than that of similar products had it chosen to produce less technically sophisticated lower end products, the standard result breaks down. We show that unlike the standard result of the monopolistic imperfect quality discrimination model, quality distortion can take place even at the higher end: it is possible that the monopolist will provide an enhanced level of quality at the higher end and a lower quality specification at the lower end, thereby enlarging the quality spectrum. However, we also demonstrate that it is also possible to provide a sub-optimal level of quality at both ends. In this case, however, the impact on the quality spectrum is unclear, i.e., product differentiation may be higher, lower or the same as in the benchmark case.
Appendix

Proof of Observation 2.1:

Putting $\partial V_2^2(q_2, q_1) / \partial q_1 = 0$ in conditions (2.8) and (2.9) we get the modified first-order conditions for welfare maximization as

$$\partial W / \partial q_1 = n_1 [dV_1^1(q_1, q_2) / dq_1 - dC(q_1) / dq_1] = 0 \quad (2.8''')$$

$$\partial W / \partial q_2 = n_2 [dV_2^2(q_2, q_1) / dq_2 - dC(q_2) / dq_2] = 0 \quad (2.9'')$$

Similarly, the first-order conditions for profit maximization of the monopolist, (2.14) and (2.15) change to

$$n_1 [dV_1^1(q_1, q_2) / dq_1 - dC(q_1) / dq_1] + n_2 [dV_1^1(q_1, q_2) / dq_1 - dV_1^2(q_1, q_2) / dq_1] = 0 \quad (2.14'')$$

and $n_2 [dV_2^2(q_2, q_1) / dq_2 - dC(q_2) / dq_2] = 0 \quad (2.15'')$

Using conditions (2.8''') and (2.14''):

$$\partial W (q_1^m, q_2^m) / \partial q_1 = - n_2 [dV_1^1(q_1^m, q_2^m) / dq_1 - dV_1^2(q_1^m, q_2^m) / dq_1] > 0$$

Similarly, Using conditions (2.9'') and (2.15''):

$$\partial W (q_1^m, q_2^m) / \partial q_2 = 0.$$
Now from \( \partial W (q_1^*, q_2^*) / \partial q_1 = 0, \partial W (q_1^m, q_2^m) / \partial q_1 > 0 \) and \( \partial^2 W (q_1, q_2) / \partial q_1^2 < 0 \) it follows that \( q_1^m < q_1^* \).

Also, \( \partial W (q_1^*, q_2^*) / \partial q_2 = \partial W (q_1^m, q_2^m) / \partial q_2 = 0 \).

Hence, \( q_2^m = q_2^* \).  \( \square \)