ABSTRACT

The production and properties of nonclassical states continues to be of great interest. Nonclassical properties like antibunching, sub-Poissonian photon counting statistics and squeezing have been studied and experimentally observed in many systems. Central to these concepts is the quantum harmonic oscillator. In the second quantisation formalism of the electromagnetic field, it is seen that the electromagnetic field is a collection of harmonic oscillators. The main aim of this thesis is to study the generation of nonclassical states and their properties in the generic system of a quantum harmonic oscillator. A harmonic oscillator whose frequency is time dependent is considered which is then shown to manifest nonclassical properties. The results are then applied to specific quantum systems for which the harmonic oscillator forms a representative. Two specific systems are studied: (a) the quantised motion of an ion in a Paul trap and (b) production of photons in a one-dimensional cavity, one boundary of which executes an “in-out” motion.

Another fundamental system of great current interest and activity is that of a micromaser. The micromaser field exhibits many novel features which are not seen in conventional masers and lasers. The micromaser field is also rich in exhibiting nonclassical features. An extensive study of these properties of the micromaser field is carried out.

The outline of the thesis is as follows:

To begin with in the first chapter, the meanings of classicality and nonclassicality are defined, following which an introduction and overview of the nonclassical properties of radiation like antibunching, sub-Poissonian photon-counting statistics and squeezing is provided.

In the second chapter, a linear system with externally controllable parameters is considered. The system consists of a quantum harmonic oscillator Hamiltonian with its frequency being time dependent. The time dependence of the frequency is achieved through the restoring force being time dependent. The Heisenberg equations of motion for the quadrature operators X and P are solved and the exact analytical solutions for the
time dependence of the operators are obtained. The time dependence of the annihilation and creation operators defined in terms of the quadratures $X$ and $P$ is then determined. It is found that the time dependence of the annihilation and creation operators defines a Bogoliubov transformation, the co-efficients of which are functions of time. Using this Bogoliubov transformation, the time evolution operator and the time dependence of the density matrix are obtained. The probability of the system to make a transition to the state $|n\rangle$ at time $i$ given that at time $t = 0$ the system was in state $|m\rangle$ is calculated from the time evolution operator.

The quantum statistical characteristics of a system are also studied by quasiprobability distributions. A very large class of states of the harmonic oscillator have a Gaussian form for the Wigner function. The time evolution of the Wigner function of the system is obtained by using the Bogoliubov transformation. It is found that the Gaussian nature of the Wigner function remains intact even if the frequency of the oscillator is changed. The major finding of this study is that the system exhibits strong nonclassical nature when the frequency of the oscillator is changed suddenly, whereas, for adiabatic changes of the frequency there is no appreciable nonclassical nature. A linear sweep of the restoring force is considered and is solved as an example.

In the third chapter, as an application of the above generic system to a specific situation, the quantised motion of an ion in a Paul trap is considered. Paul trap is a device to trap ions in an effective attractive potential that is formed with a combination of a high frequency $rf$ field and a dc field. The quantised motion of the ion in a Paul trap is described by an equation that is classically a Mathieu equation. By an exact solution of the Heisenberg equations of motion for the position and momentum of the ion, the time evolution operator, the density matrix and the Wigner function are determined. Explicit forms of the wave functions for the ground and excited harmonic oscillator states in the co-ordinate representation are also obtained. It is found that various initial states, as they evolve in time, show nonclassical properties like squeezing of fluctuations in the quadratures. It is shown that this approach is also useful in determining the strengths of
the sidebands in the fluorescence spectrum of the trapped ion. They are calculated from the consideration of Raman transitions in which the centre of mass motion is excited to a higher level by an external electromagnetic field.

Another related problem that is considered is that of the production of particles in the vicinity of a moving mirror. This problem has received considerable attention in the recent past in the context of particle creation. The present aim is to study the quantum statistical properties of the field so produced due to accelerated mirror motion and to look for nonclassical nature of the field. Thus in the fourth chapter, a simple model which consists of a quantised scalar field in a region bounded by two mirrors, one of which has an "in-out" motion is considered. Nonclassical properties of the field so produced inside such a cavity are studied. The field so produced shows squeezing and the modes inside the cavity are found to be correlated.

In the fifth chapter, a simple but extremely important and practically viable quantum system — a micromaser is considered. The micromaser is a practical realisation of the simplest model in Quantum Optics, viz., the Jaynes-Cummings Model (JCM). The field produced in a micromaser is highly nonclassical. The sub-Poissonian nature of the field was theoretically predicted and was later experimentally verified. The experimental observation of collapse and revival phenomena has also been reported. The phase sensitive properties of the micromaser field also shows very many interesting features. Recently a proposal has been made regarding the measurement of the linewidth of the micromaser field. In this chapter the intensity-intensity correlations of the micromaser field are calculated. Two types of intensity-intensity correlation functions are defined and then using the steady state photon statistics of the micromaser field and the quantum regression theorem these functions are calculated. These functions are also obtained numerically by using the standard continued-fraction method and by an equivalent eigenvalue approach. It is found that the two-photon linewidth increases as a function of the pump parameter and after a certain value starts decreasing. It also shows resonances which are associated with the existence of trapping states for those pump parameter values. From the
eigenvalue approach it is found that various eigenvalues contribute to the linewidth. In particular, the multi-exponential character of the correlation function and the antibunching character of the micromaser field are demonstrated. Finally, a proposal as to how one can probe such intensity-intensity correlations in a typical micromaser setup is given.

In the sixth and final chapter, a short description of quasiprobability distributions is given. Quasiprobability distributions, in addition to being computational tools, also provide insight into the quantum statistical aspects of a system. Two of the most important quasiprobability distributions, viz., the Q-function and the Wigner function are calculated for the micromaser field. Contrary to the equally important Glauber-Sudarshan P-function, these two distributions always exist as ordinary functions for any state of the system. Further, the Q-function has the property that it is strictly positive definite for any state. The Wigner function does not, however, share this property. Since the micromaser field density matrix remains diagonal the off-diagonal density matrix elements are zero. Hence the Q- and the Wigner functions are phase independent.

The micromaser has an unique feature unlike conventional masers and lasers. In addition to the initial maser transition the micromaser shows many abrupt jumps at approximately integer multiples of 2π of the pump parameter. The initial maser transition shows the characteristics of a continuous (second-order) phase transition whereas the subsequent transitions have the characteristics of a first-order phase transition. In these regimes the micromaser has very interesting bistable and hysteretic nature. The quasiprobability distributions are studied for the first order phase transition regimes of the micromaser. The very low temperature behaviour of the micromaser exhibits very sharp resonances due to the occurrence of trapping states. Following a brief description on trapping states the quasiprobability functions are studied for the pump parameter values corresponding to these trapping states.