6.1 Introduction

The study of flow and heat transfer over a stretching/shrinking sheet receives considerable attention from many researchers due to its variety of application in industries such as extrusion of a polymer in a melt spinning process, manufacturing of plastic films, wire drawing, hot rolling and glass fibre production. Crane [107] first considered the steady laminar boundary layer flow of a Newtonian fluid caused by a linearly stretching flat sheet and found an exact similarity solution in closed analytical form. Chiam [202] investigated the steady two-dimensional stagnation point flow towards a stretching surface in the case when the stretching velocity is identical to the free stream velocity. Hiemenz [98] studied the steady two-dimensional boundary layer flows near the forward stagnation point on an infinite wall using a similarity transformation. Sakiadis ([26], [27]) reported the flow field analysis where the stretched surface was assumed to move with uniform velocity, and similarity solutions were obtained for the governing equations. The solution is later improved by Howrath [106]. Gupta and Gupta [148] have studied the effect of mass transfer on a stretching sheet with suction or blowing for linear surface velocity subject to uniform temperature. Wang [41] studied the three-dimensional flow problem over a stretching surface and Crane [107] obtained the uniqueness of the solution which was further established by McLeod and Rajgopal [79]. Some important contributions in stretching sheet flow were made by Chakrabarti and Gupta [6], Andersson [72], Pop [36], Jat and Chaudhary [160-163] and Bhattacharyya and Layek [94] and many other authors.

The flow of incompressible fluid due to a shrinking sheet is gaining attention of modern day researchers because of its increasing application to many engineering problems. Lok et al.[230] studied oblique stagnation point flow of Newtonian flow towards a stretching surface. They reported that the position of the stagnation point
depends on stretching sheet parameters and the angle of incidence. Hayat et al. [209] reported an analytical solution of MHD flow of a second grade fluid over a shrinking sheet. Fang and Zhang [203] obtained a closed form analytical solution for steady MHD flow over a porous shrinking sheet subjected to mass-suction. Wang [40] studied the stagnation point flow towards a shrinking sheet. Unsteady two-dimensional hydromagnetic flow and heat transfer of an incompressible viscous fluid were investigated by Adhikary and Mishra [173]. Ali et al. [59] investigated the unsteady viscous flow over a shrinking sheet with mass transfer in a rotating fluid. Recently, Bhattacharyya [93] studied the MHD flow and heat transfer over a shrinking sheet in the presence of heat source/sink with mass suction.

Realizing the increasing technical application of MHD effects, the present chapter studies the effects of magnetic parameter on the flow and heat transfer over a porous shrinking sheet in the presence of uniform heat source / sink with mass suction and viscous dissipation. The physical quantities of interest such as skin friction \( C_f \) and Nusslet number \( Nu \) are calculated. The variation of pertinent physical parameters characterizing the flow phenomena are presented through graphs and the numerical results for skin friction coefficient and Nusselt number are given through tables and then discussed in details.

### 6.2 Formulation of the problem

Consider a steady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid over a permeable shrinking sheet which coincides with the plane \( y = 0 \) and the flow is confined in the region \( y > 0 \). The \( x \) and \( y \) axes are taken along and perpendicular to the sheet, respectively. Two equal and opposite forces are applied along the \( x- \)axis so that the sheet is stretched keeping the origin fixed. A uniform magnetic field of strength \( B_0 \) is assumed to be applied normal to the sheet. The magnetic Reynolds number is taken to be small and therefore, the induced magnetic field is neglected. All the fluid properties are assumed to be constant throughout the motion. Under the usual boundary layer approximations the basic governing equations with viscous dissipation are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (6.2.1)
\]
\[
\frac{u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\nu}{\rho} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho},
\]
(6.2.2)

\[
\frac{u}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + Q_0 (T - T_w) + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2 u^2}{\rho c_p},
\]
(6.2.3)

where \( u \) is the velocity component along \(-x\) direction and \( v \) along \(-y\) direction, \( \mu \), the coefficient of viscosity, \( \rho \), the density, \( \nu = \frac{\mu}{\rho} \), the kinematic viscosity, \( \sigma \), the electrical conductivity of the fluid, \( T \) is the fluid temperature, \( T_w \) is the ambient temperature, \( \kappa \), the thermal conductivity of the fluid, \( C_p \), the specific heat at constant pressure and \( Q_0 \), the volumetric rate of heat generation / absorption.

The boundary conditions are:

\[
y = 0: \quad u = U_w = -cx, \quad v = -v_w; \quad T = T_w, \]
\[
y \to \infty: \quad u \to 0; \quad T \to T_\infty
\]
(6.2.4)

where \( c > 0 \), \( 0 < c < 1 \) is the shrinking constant, \( T_w \) is temperature of the sheet, \( v_w (>0) \) is a prescribed distribution of wall mass suction through the porous sheet.

**6.3 Mathematical analysis**

The equation of continuity (6.2.1) is identically satisfied, if we take the stream function \( \psi(x, y) \) such that

\[
u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}
\]
(6.3.1)

The momentum and energy equations (6.2.2) and (6.2.3) can be transformed into the corresponding ordinary differential equations by introducing the following similarity transformations:

\[
\psi(x, y) = \sqrt{c \nu x f(\eta)}, \quad \frac{T - T_w}{T_w - T_w} = \theta(\eta),
\]
(6.3.2)
\[ \eta = y \frac{\sqrt{c}}{v} \]  

(6.3.3)

The momentum and energy equations (6.2.2) and (6.2.3) are transformed to

\[ f'' + ff' - f'^2 - Mf' = 0 \]  

(6.3.4)

\[ \theta' + Pr(f\theta' + \lambda \theta) + Pr Ec f'^2 + M \Pr Ec f'^2 = 0 \]  

(6.3.5)

where \( M = \frac{\sigma c B_0^2}{\rho c} \) is the magnetic parameter, \( \Pr = \frac{\mu C_p}{\kappa} \) is the Prandtl number, \( \lambda = \frac{Q_0}{\rho C_p c} \) is the heat source \((\lambda < 0)\) or sink \((\lambda > 0)\) parameter and \( Ec = \frac{U_w^2}{C_p (T_w - T_\infty)} \) is the Eckert number.

The corresponding boundary conditions are:

\[ \begin{align*}
  f &= S, \quad f' = -1, \quad \theta = 1, \quad \text{at} \quad \eta = 0 \\
  f' &\to 0, \quad \theta \to 0, \quad \text{as} \quad \eta \to \infty
\end{align*} \]  

(6.3.6)

where \( S = \frac{v_w}{\sqrt{c v}} \) is the mass suction parameter.

The physical quantities of interest of the problem are the skin-friction coefficient \( C_f \) and the Nusselt number \( Nu \), can be expressed, respectively as

\[ C_f = \frac{\mu \left( \frac{\partial u}{\partial \eta} \right)_{\eta=0}}{\frac{\rho U_w^2}{2}} = \frac{2}{\sqrt{\text{Re}}} f'(0) \]  

(6.3.7)

\[ Nu = \frac{\chi \left( \frac{\partial T}{\partial \eta} \right)_{\eta=0}}{\left( T_w - T_\infty \right)} = -\sqrt{\text{Re}} \theta'(0) \]  

(6.3.8)

where \( \text{Re} = \frac{U_w x}{v} \) is the local Reynolds number.
6.4 Results and discussion

The set of non-linear differential equations (6.3.4) and (6.3.5) with boundary conditions (6.3.6) were solved numerically using Runge-Kutta fourth order algorithm with a systematic guessing of shooting technique until the boundary condition at infinity is satisfied. The computations were done by a programme which uses a symbolic and computational computer language Matlab. Numerical computations for different physical parameters involved e.g. the magnetic parameter $M$, the mass suction parameter $S$, the Prandtl number $Pr$, the Eckert number $Ec$ and the heat source/sink parameter $\lambda$ are performed. The accuracy of the applied numerical scheme, the numerical computation of skin-friction coefficient are compared with the earlier published results of Muhaimin et al. [134] in table-6.1 and it is to note that the present result is in good agreement. The rate of heat transfer from the sheet $-\theta'(0)$, is presented in table-6.2 and 6.3.

In Fig.6.1, the variations in velocity field for several values of $M$ are presented. The impact of the magnetic parameter $M$ on the velocity distribution is significant. It is observed that magnetic parameter enhances the velocity profile. Subsequently, the thickness of the momentum boundary layer decreases. This is due to the Lorentz forces, which act as a resistive force and resists the thickness of the momentum boundary layer.

Fig.6.2 shows the effects of mass suction parameter $S$ on the velocity profile for a fixed value of magnetic parameter. It is noticed that increase in applied suction increases the velocity profiles significantly and consequently it contributes to the growth of boundary layer.

Fig.6.3 exhibits the effect of magnetic parameter $M$ on the temperature distribution. Here it is noticed that the temperature $\theta(\eta)$ decreases slightly with increasing values of $M$. Fig.6.4 presents the effect of Prandtl number $Pr$ on the temperature profiles. It is clear to mark that with increasing $Pr$ the thermal boundary layer thickness sharply decreases. The temperature field for various values of heat source or sink parameter $\lambda$ is exhibited in Fig.6.5. It is noted that for a fixed value of $\eta$, temperature $\theta(\eta)$ decreases as $\lambda$ decreases. On careful observation it is revealed that for moderately high value of $\lambda$, temperature hike at the plate is marked, afterwards temperature decreases. This is due to the contribution of
higher value of heat source to increase the plate temperature due to heat absorption. Fig. 6.6 shows the effects of Eckert number $Ec$ on temperature $\theta(\eta)$. It is observed that for any fixed value of $\eta$, temperature $\theta(\eta)$ decreases as $Ec$ decreases.

### 6.5 Conclusion

The effect of magnetic parameter $M$ on the boundary layer flow and heat transfer over a porous shrinking sheet subject to strong mass suction are studied. The self similar equations are obtained using similarity transformations. The study shows that due to increase in magnetic parameter $M$ and the mass suction parameter $S$, the momentum boundary layer thickness decreases. Also, the dimensionless temperature profile as well as the thermal boundary layer thickness sharply reduces with increasing $Pr$. Similarly, thermal boundary layer thickness decreases as decreasing $Ec$. For some higher values of heat source parameter heat absorption occurs at the sheet. The rate of heat transfer increases with Prandtl number.

**Table 6.1:** Skin Friction coefficient $f''(0)$ for different values of $S$ with $M=2$

<table>
<thead>
<tr>
<th>$S$</th>
<th>Present study</th>
<th>Muhaimin et al.[134]</th>
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<tbody>
<tr>
<td>2</td>
<td>2.414476</td>
<td>2.414214</td>
</tr>
<tr>
<td>3</td>
<td>3.302813</td>
<td>3.302776</td>
</tr>
<tr>
<td>4</td>
<td>4.236071</td>
<td>4.236068</td>
</tr>
</tbody>
</table>

**Table 6.2:** $-\theta'(0)$ for various values of $Pr$ when $M=2$, $S=2$ and $Ec = 0.1$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$Pr = 0.1$</th>
<th>$Pr = 0.71$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.2</td>
<td>0.428487</td>
<td>1.229785</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.418625</td>
<td>1.179859</td>
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<tr>
<td>0</td>
<td>0.408646</td>
<td>1.127127</td>
</tr>
<tr>
<td>0.1</td>
<td>0.398547</td>
<td>1.071191</td>
</tr>
<tr>
<td>0.2</td>
<td>0.388325</td>
<td>1.011564</td>
</tr>
</tbody>
</table>
Table 6.3: $- \theta'(0)$ for various values of $Ec$ when $M=2$, $S=2$ and $Pr = 0.71$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$Ec = 0.1$</th>
<th>$Ec = 0.71$</th>
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<td>1.011564</td>
<td>0.250881</td>
</tr>
</tbody>
</table>

Fig. 6.1 Velocity profile against $\eta$ for various values of $M$ when $S = 2.0$
Fig. 6.2 Velocity profile against $\eta$ for various values of $S$ when $M=2$

Fig. 6.3 Temperature profile for various values of $M$ when $S=2$, $Pr = 0.71$ and $Ec = 0.1$
Fig. 6.4 Temperature profile for various values of $Pr$ when $M=2$, $S=2$, $Ec=0.1$

Fig. 6.5 Temperature profile against $\eta$ for various values of $\lambda$ when $M=2$, $S=2$, $Pr=0.71$, $Ec=0$
Fig. 6.6 Temperature profile for various values of $Ec$ when $M=2, S=2$, $Pr=0.71$.