CHAPTER 2
MECHANISM OF ULTRASONIC MACHINING

USM is a mechanical material removal process in which the material is removed by repetitive impact of abrasive particles carried in liquid medium (slurry solution) on to the work surface, by a shaped tool, vibrating at ultrasonic frequency with very small amplitude. In the previous chapter, an overview of the fundamental principle of USM and extensive literature reviews of the past research has been presented. The present chapter will focus on the working principle of USM, machining characteristics of USM, i.e. mechanism of material removal and basic influence of some process parameters during USM process.

2.1 WORKING PRINCIPLE OF USM

In ultrasonic machining, a tool of desired shape vibrates at an ultrasonic frequency (19 ~ 25 kHz) with amplitude of around 15 – 50 µm over the workpiece. Generally the tool is pressed vertically downward with a feed force. Between the tool and workpiece, the machining zone is flooded with hard abrasive particles generally in the form of a water based slurry solution. As the tool vibrates over the workpiece, the abrasive particles act as the indenters and indent both the work material and the tool. The abrasive particles, as they indent, the work material, would remove the same, particularly if the work material is brittle, due to crack initiation, propagation and brittle fracture of the material. Fig.2.1 exhibits the mechanism of material removal during ultrasonic machining operations.

2.2 MECHANISM OF MATERIAL REMOVAL IN USM

USM is generally used for machining hard and brittle work materials. Material removal primarily occurs due to direct hammering and indentation of the hard abrasive grits on the brittle work material. Other than this brittle failure of the work material due to indentation some material removal such as micro-chipping by impact of the free moving abrasive particles, cavitation effects from the abrasive slurry and chemical action associated with the fluid employed may occur, but it is estimated to be rather insignificant. As the tool
vibrates, it leads to indentation of the abrasive grits. During indentation, due to Hertzian contact stresses [76], cracks would develop just below the contact site, then as indentation progresses the cracks would propagate due to increase in stress and ultimately lead to brittle fracture of the work material under each individual interaction site between the abrasive grits and the workpiece. Moreover, a small amount of material removal might also be contributed by the mechanical abrasion of the hard microabrasives, chemical actions and cavitation effects from the abrasive slurry between the tool and machined surface as shown in Fig.2.1.

The tool material should be such that indentation by the abrasive grits does not lead to brittle failure. Thus the tools are made of tough, strong and ductile materials like steel, stainless steel and other ductile metallic alloys.

Some researchers have developed several numbers of analytical models of material removal in ultrasonic machining process, considering different basic approaches to removal mechanism. Some developments in the relevant works have been discussed as follows:

Shaw [74] proposed the first theory to confirm the conception on ultrasonic machining process with assumptions for determining the characteristics of ultrasonic machining based on the model as follows:

(i) The work material removal rate is proportional to the volume of work material per impact,
(ii) The work material removal rate is proportional to the number of particles making impact per cycle,
(iii) The rate of work material removal is proportional to the frequency of tool vibration,
(iv) All impacts are identical and
(v) All abrasive grains are identical and spherical in shape, etc.

According to above assumptions, material removal rate due to direct hammering was given by [25]:

\[ \text{Material Removal Rate} = k \times \text{Volume of Work Material} \times \text{Number of Impacts} \times \text{Frequency of Vibration} \]

\[ k \]

...
\[ V_1 = K \left[ \frac{FA}{H_w (1 + \lambda)} \right]^{3/4} C^{1/4} d_m f \]  

(2.1)

where \( F \) is mean static force over the time period \( T \), \( A \) is amplitude of vibration; \( H \) is Brinell hardness (MPa or \( \text{N/mm}^2 \)), \( \lambda \) is indentation ratio or ratio of tool and work material, \( C \) is concentration of abrasive in the slurry by mass, \( d_m \) is mean diameter of abrasive grain (mm), \( f \) is cyclic frequency of vibration (cycles/sec); \( K \) is a constant of proportionality and is given by

\[ K = \frac{2}{3} \left( \pi K_1 \right)^{1/4} K_2^{3/4} \]  

(2.2)

Material removal rate due to particle impacting on work material is given by:

\[ V_2 = 8.6 K_1 \left[ \frac{\rho_a}{H_w} \right]^{3/4} f^{5/2} A^{3/2} C d_m \]  

(2.3)

where, average diameter of abrasive grain is given by \( d_1 = K_2 d_m^2 \); and \( \rho_a \) is density of work material (kg/mm\(^3\)). For a given area of tool face, the number of active abrasive grains (N) is given by:

\[ N = K_1 \frac{C}{d_m^2} \]

G.E. Miller [75] also proposed analytical model for material removal with the following assumptions:

(i) Abrasive particles are of cubical size,
(ii) Plastic deformation is directly proportional to the stress,
(iii) Plastic flow stress equals Burger vector times Shear modulus,
(iv) Cross sectional area of cut does not change during machining,
(v) Viscosity effects in water slurry are negligible.

According to above assumptions, the material removal rate is given by:

\[ V = K \left[ \frac{A d_m fFP_{atm}}{b_w G_w q_w RV_{wg} \rho_a} \right] \left( \frac{C}{C + 1} \right) \]  

(2.4)
But depth of cut per (Dc) unit time is given by:

\[
D_c = K \left[ \frac{Ad_m fp P_{atm}}{b_w G_w q_w RV_{wg} \rho_a} \right] \left( \frac{C}{C + 1} \right) \tag{2.5}
\]

where, K is a constant of proportionality; \(G_w\) is shear modulus (MPa or N/mm\(^2\)); \(q_w\) is work hardening capacity (MPa or N/mm\(^2\)); \(R\) is tool radius (mm); \(b_w\) is Burger vector (mm); \(V_{wg}\) volume of tool-work gap (mm\(^3\)) and \(P_{atm}\) is atmospheric pressure (MPa or N/mm\(^2\)).

Developments in application of ultrasonic machining processes have resulted in several numbers of researchers to carry out investigations in the field of mechanism on conventional and hybrid of ultrasonic machining, called rotary ultrasonic machining. Some researchers have focused on development of mathematical and empirical relations to estimate material removal rate in conventional ultrasonic machining processes.

E.V.Nair and A.Ghosh [25] developed a model based on distribution of strain energy with elastic wave propagation for determining the volume of material removed per second as:

\[
V = V_f N_f
\]

where, \(V_f\) is volume fractured by single grit per cycle, and number of active grains in the working gap (N) is given by:

\[
N = \frac{3}{2} C \left[ \frac{D^2 \left( d_m + A \sin \theta_a \right)}{d_m^3 \left( \rho_y \rho_r + C \right)} \right]
\]

\(\theta_a = \theta < \) at which vibrating tool makes contact with abrasive (at \(t = \theta_a / 2\pi f\)); \(D\) is cutting tool diameter (mm). The model explained the experimental observation about the existence of an optimum value of abrasive grain diameter for maximum MRR and also quantity of MRR could be predicted through purely theoretical analysis unlike previous theories.

Rajurkar et al [8, 16] carried out dynamic analysis of the ultrasonic machining on ceramic materials and observed the presence of two phenomena that contribute to material removal; the deformation at the point of impact, and the brittle structure below the impact zone. Based on dynamic impact test, it was observed that the material removal in the USM process appears to be a function of impact velocity, which is determined by the frequency...
and amplitude of the vibrating tool. Material at the impact zone was removed by fragmentation and by chipping microfracture due to the high compressive stresses developed in the machined area. According to the authors, based on the combined effects of impact indentation and fracture phenomenon by assuming work material is semi-infinite solid, axis of the moving grit is perpendicular to the free surface during the machining process and speed of abrasive grit is same as that of the vibrating cutting tool; Volumetric MRR in mm³/s is given by:

\[ V = K_1 (1000)^{-0.8} r_{\text{m}} \rho_m^{4/5} f^{8/5} A^{8/5} \]  \hspace{1cm} (2.7)

where, \( K_1 = \frac{c_3 A_r K_s^2}{6} \) and \( K_s = \left[ \frac{5\pi^3(1 - \nu_w)}{2\mu_w} \cos^2(2\pi ft - \Phi) \right]^{2/5} \)

Here, \( c_3 \) is gap coefficient,
\[ \Phi = \text{phase lag used in the equation of amplitude}, \]
\[ \varphi = \text{phase lag used in the equation of amplitude i.e. } A(t) = A \sin(2\pi f t - \varphi) \]

The above model could also be useful for predicting analytically an optimum grain size for a specified MRR.

Lee and Chan [9] proposed a model based on brittle fracture by assuming pre-existing flaws in the workpiece material for the initiation of median or lateral cracks. For the size of the median or radial crack \( C \), relation with critical load \( P \) is given by \( C^m = kP \); with \( m \) being assumed equal to 1.5. Cutting tool was assumed as a slender column. The expression for determining volumetric MRR is given as:

\[ \text{MRR} = \frac{2\pi k_1^2 f}{3N} (aW + bU)^2 \]  \hspace{1cm} (2.8)

where, \( a = \frac{T}{\Delta t} \); \( b = \frac{U_i E_1}{L_1} \left[ 1 + \frac{T}{\pi \Delta t} \cos(\omega t_a + \Phi) \right] \)

Above equation also can be simplified to:

\[ \text{MRR} = \frac{k^1}{N} (aW + bU)^2 \]  \hspace{1cm} (2.9)
where, \( \varphi \) = Phase used in the equation of amplitude i.e. \( U(t) = U_0 \sin(\omega t + \varphi) \), \( t \) is shocking time, \( k_1 \) is a constant as mentioned in the assumptions, \( a \) and \( b \) are coefficients determined by the machining conditions, \( N \) is the number of the effective abrasive particles in the machining area; \( W \) is the static load; \( E \) is elastic modulus of tool and \( U \) is the amplitude of the tool tip. Any increase in tool tip vibration amplitude, static load and abrasive particle size causes increase in MRR and surface roughness on the workpiece.

Zhang et al [10] also proposed a model based on fracture mechanics and failure for estimation of the material removal rate as:

\[
MRR = K \cdot f \cdot d^2 \cdot S^{-1} \cdot C_1^{-3/2} \cdot H_v^{-1/2} \left( W + \frac{2 \pi f AM}{\Delta T} \right)^2
\]  

(2.10)

The material removed by one grain in one indentation becomes:

\[
V = \pi C_L^2 C_h
\]

where, \( C_L \) is lateral crack, and \( C_h \) is depth of crack / indentation, \( K \) is a proportional constant, \( f \) is the frequency of the tool tip vibration, \( d \) is the mean diameter of the abrasive grains, \( S \) is the area of the bottom face of the tool, \( C \) is the concentration of abrasive in the slurry, \( K_{IC} \) is the fracture toughness, \( H_v \) is the Vickers diamond hardness of the workpiece, \( W \) is the static load, \( A \) is the amplitude of the tool tip, \( M \) is the equivalent quality of the vibrating system and \( \Delta t \) is the time of the tool tip impacting to the workpiece through abrasive grains. An increase in terms of the amplitude of the tool tip, the static load applied and the size of the abrasive grain will result in an increase of material removal rate in ultrasonic machining. Decrease of the grain size and amplitude, increase of the static load will result improve the finished surface.

Lee and Deng [23] reported the effects of whisker orientation on the material removal rates and mechanisms in ultrasonic machining of Al\(_2\)O\(_3\)/SiC\(_w\) ceramic composites. It was observed that material removal rate and machined surface roughness in ultrasonic machining of the ceramic composites varied according to the orientation of the hot pressing direction. The relation for estimating the volume of material removed is given by:

\[
MRR = K \sum N \pi E^{5/4} K_{IC}^{-1} H_v^{-2} P^{7/4}
\]  

(2.11)
where,  $E$ and $H_v$ are Young’s modulus and Vickers hardness of the material, and $P$ is the mean vertical impact force acting from one particle to material during machining, $N$ is number of impact particles, $K_{IC}$ is fracture toughness of work material and $K$ is a constant.

### 2.3 MODEL OF MATERIAL REMOVAL MECHANISM IN USM

In the present model, material removal would be assumed to take place mainly due to impact of abrasives, between tool and workpiece, followed by indentation and brittle fracture of the workpiece. The model does not consider the deformation of the tool. The following assumptions have been proposed for determining the mechanism of material removal during conventional ultrasonic machining:

(i) All abrasive particles are identical in shape and size,

(ii) The work material removal rate is proportional to the volume of work material per impact,

(iii) Local spherical bulges have identical shape and diameter,

(iv) All impacts are identical.

The abrasive particles are characterized by the mean grit diameter, $d_g$. The relation between diameters of the local spherical bulges ($d$) and that of abrasive grit diameter is $d = \mu d_g^2$. During indentation by the abrasive grit onto the workpiece and the surface of the tool, the local spherical bulges contact the surfaces and the effect of impact on the workpiece is characterized by $d$.

The indentation due to direct impact of the abrasive particle on work material is exhibited in Fig.2.2. As the indentation on the workpiece proceeds, the contact zone between the abrasive grit and workpiece is established and increases proportionately with duration of machining. The contact zone created by indentation, is circular in nature and is characterized by its diameter ‘2x’. At full indentation, the indentation depth in the workpiece is characterized by $\Delta x$. Due to direct impact of abrasive particles on the brittle work material, brittle fracture takes place leading to hemi-spherical fracture of diameter ‘2x’ at the point of contact between the grit and the workpiece [76]. Thus material removal per abrasive particle ($P_a$) is given as

$$P_a = \frac{2}{3} \pi x^3$$  \hspace{1cm} (2.12)
Now considering the geometry of indentation as shown Fig.2.2

\[ AB^2 = AC^2 + BC^2 \]

\[
\left( \frac{d}{2} \right)^2 = \left( \frac{d}{2} - \Delta x \right)^2 + x^2
\]

\[ x^2 = d\Delta x \text{ neglecting } \Delta x^2 \text{ as } \Delta x << d \]

Therefore, material removal per abrasive particle (2.12) becomes

\[ P_a = \frac{2}{3} \pi (d\Delta x)^{3/2} \quad (2.13) \]

If at any moment of time, there are an average ‘n’ of grit particles and the tool is vibrating at a frequency ‘f’, then material removal rate (MRR) can be expressed as

\[ MRR = P_a.n.f \]

\[ = \frac{2}{3} \pi (d\Delta x)^{3/2} nf \quad (2.14) \]

Now, considering the tool and the workpiece are pressed each other through contact points causing indentations on work material (\(\Delta x\)) and tool material (\(\Delta t\)), both of them would deform or wear out. And then the indentation process starts and finally completes with an indentation of \(\Delta x\) and \(\Delta t\) on the work and tool respectively. The tool vibrates in harmonic motion. The interaction of tool and workpiece with abrasive grains may occur during first quarter of each cycle. The cycle time is \(T\). Out of this quarter; some part is used to engage the tool with abrasive grains. Thus the time of indentation \(\tau\) can be roughly estimated as:

\[ \frac{\Delta}{a_o} = \frac{\tau}{T/4} \]

\[ \tau = \frac{T\Delta}{4a_o} = \frac{T(\Delta x + \Delta t)}{4a_o} \quad (2.15) \]

During machining, the impulse of force on the tool and work would be balanced. Thus assuming \(F_{max}\) as the maximum indentation force per abrasive, the total impulse on the tool can be expressed as:
In the USM, the tool is usually fed with an average force $F$. Thus

$$F = n.f \cdot \frac{1}{2} F_{\text{max}} \cdot \tau$$ \hspace{1cm} (2.16)

Again, if the flow strength of the work is taken $\sigma_w$, then

$$F_{\text{max}} = \sigma_w \pi x^2$$ \hspace{1cm} (2.17)

Therefore,

$$F = \frac{1}{2} \sigma_w \pi x^2 \cdot n f$$

$$F = \frac{1}{2} n f \sigma_w \pi x^2 \frac{T(\Delta x + \Delta t)}{4a_0}$$ \hspace{1cm} (2.18)

If ‘$A$’ is total area of the tool facing the workpiece, then volume of abrasive slurry of one grain thickness is $A d_g$.

If ‘$n$’ is the number of grits then the total volume of $n$ grits is $\frac{\pi d_g^3}{6} n$

Thus, the concentration of abrasive grits in the slurry ($C$) is related as follows:

$$n \frac{\pi d_g^3}{6} = A d_g C$$

where, $C = n \frac{\pi d_g^2}{6 A}$ \hspace{1cm} (2.19)

Therefore, the number of grits in the abrasive slurry is estimated as:

$$n = \frac{6 A C}{\pi d_g^2}$$ \hspace{1cm} (2.20)

Now, assuming the indentation to be inversely proportional to the flow strength then,

$$\frac{\Delta t}{\Delta x} = \frac{\sigma_w}{\sigma_t} = \lambda$$ \hspace{1cm} (2.21)

Thus, combining above equations, ‘$F$’ can be written as
\[ F = \frac{1}{2} \frac{6AC}{\pi d_g^2} f \pi \sigma_x^2 T \Delta x (1 + \lambda) \]

or, \[ F = \frac{3AC}{d_g^2} (fT) \pi \frac{w}{4a_o} \mu d_g^2 \Delta \lambda_x (1 + \lambda) \]

or, \[ F = \frac{3AC}{d_g^2} (fT) \pi \frac{w}{4a_o} \mu d_g^2 \Delta \lambda_x (1 + \lambda) \]

\[ \Delta \lambda_x = \frac{4a_o F}{3 \mu d AC \sigma_w (1 + \lambda)} (fT) \]

Now, the material removal rate (MRR) can be expressed as

\[ MRR = P_o n f \]

= \[ \frac{2}{3} \pi \sigma^3 n f \]

= \[ \frac{2}{3} \pi \frac{6CA}{\pi d_g^2} f \cdot \Delta \lambda_x \]

= \[ 4 \pi \frac{cA}{\pi d_g^2} \cdot f \cdot (d \Delta \lambda)^{3/2} \]

= \[ \frac{4cA}{d_g^2} \cdot f \cdot (\mu d_g^2 \Delta \lambda)^{3/2} \]

= \[ 4 c A d_g \mu^{3/2} \cdot f \left( \frac{4 F a_o}{3 \mu A c \sigma_w (1 + \lambda)} \right)^{3/4} \]

(2.24)

From above expressions with usual notations, it can be written as

\[ MRR \propto \left( cA \right)^{1/4} \left( \mu a_o F \right)^{3/4} d_g f \]

\[ \left( \sigma_w (1 + \lambda) \right)^{3/4} \]
\[ \propto (cA)^{1/4} \left\{ \frac{\mu a_o F}{\sigma_w (1 + \lambda)} \right\}^{3/4} d_g f \]

\[ \propto Ad_g f \frac{c^{1/4} (\rho a_o)^{3/4}}{\sigma_w^{3/4} (1 + \lambda)^{3/4}} \mu^{3/4} \]

where \( \sigma_w \) is strength of work material, \( A \) is total surface area, \( \lambda \) is ratio of indentation of the tool and work material, \( a \) is amplitude of the tool tip.

Based on above assumptions, the expression for total material removal rate with total active number of abrasive particles (\( N \)) on the contact surface can be written as:

\[ MRR = 4ACNd_g \mu^{3/2} f \left( \frac{4Fa_o}{3\mu AC \sigma_w (1 + \lambda)} \right)^{3/4} \]  \hspace{1cm} (2.25)

### 2.4 BASIC INFLUENCES OF SOME PARAMETERS DURING USM PROCESS

Based on the literature survey, some of the process parameters, which usually govern the conventional ultrasonic machining processes, have been identified hereunder:

(a) **Abrasive Grit Size**

Abrasive grit size plays very important role for achieving higher quality of machined surface as well as higher material removal rate during machining of workpiece. Smaller size of abrasive grains, i.e. higher grit number, generally gives higher quality and accuracy of the machined Lower grit number, i.e. larger grit size results higher material removal rate. Material removal rate increases with increase in abrasive grit size but there is a limit in which material removal rate starts decreasing. However, surface roughness decreases with decrease in abrasive grit size.

(b) **Abrasive Materials**

The machining characteristics in ultrasonic machining operations are dependent on type of abrasive grains used. Selection of abrasive grits can be done based on physical properties of work materials to be machined. The following are common abrasive materials
used in stationary and rotary type of ultrasonic machining, i.e. Aluminum oxide (Al₂O₃), Silicon carbide (SiC), Boron carbide (B₄C), Boronsilicarbide and Diamond.

(c) **Slurry Concentration**

In higher slurry concentration, more abrasive particles are mixed with liquid or water medium generally results higher material removal rate. Moreover, the performance of machining process depends on the type of abrasives as well as slurry concentration.

(d) **Power Rating**

Power rating has some influence in material removal rate as well as profile accuracy on work surface. Higher rating of power supply gives to some extent, gradual increase in MRR and higher tool wear rate. Considering profile accuracy, lower to medium level power supply rating may be preferred.

(e) **Feed Force**

Material removal rate increases with increase in feed force as well as feed rate. At the same time, the profile accuracy of machined surface increases when tool feed rate increases up to middle range of the feed rate. Moreover, higher feed rate with same static force may cause higher tool wear. From analysis of plots, it can be concluded that feed rate with smaller grit size and lower slurry concentration can be used effectively for controlling the machining characteristics to achieve higher geometric accuracy.

(f) **Slurry Flow Rate**

Slurry is a mixture of abrasive grains with liquid medium. The purpose of slurry solution is for abrasion cutting of work surface as well as cooling system for the tool tip. Adequate amount of slurry solution must available during machining operations. Sufficient flow rate of abrasive slurry is required to avoid overheat of the tool which even could damage the machined surface as well as geometrical profile.
Precautions also must be observed that over large grain size of abrasives may cause lower MRR as well as lower rate of penetration of tool. Similarly, higher slurry concentration also may reduce the speed of the axial movement of the tool. Lower size of tool may reduce penetration rate of tool on workpiece but higher static force with small size of tool can cause bending of tool. An increase in amplitude or vibration frequency of tool gives higher MRR but only to some extent.

From the basic study of mechanism of USM and influences of various process parameters on different major machining criteria, it is clear that proper control of process parameters is highly required for obtaining the desire goal during ultrasonic machining applications. Type of grit size and grit material in USM process plays a key role during machining of conductive or non-conductive materials. Therefore, a special attention is highly needed to achieve better performance in machining operations. Increasing power supply increases amplitude and frequency of vibration which also material removal rate up to some extent during ultrasonic machining operations. Controlling of tool feed rate with adequate slurry flow rate is necessary for smooth performance of USM. As a whole, the machining performance of USM depends on type of grit size, grit materials, slurry concentration, power rating, frequency of vibration, vibration amplitude, tool feed force, slurry flow rate, work material, tool shapes, tool materials, etc. Moreover, based on size of grits, gap between the tool and work surface must be maintained as per requirement. To the best of author’s knowledge, very few works have been conducted till date for searching optimal parametric combinations on USM. Hence, an investigation on the influence of various USM parametric combinations such as grit size, slurry concentration, power rating, tool feed rate and slurry flow rate, etc. on different machining performance is essential for USM applications.
Fig. 2.1 Material removal mechanism of USM

Fig. 2.2 Illustrative diagram of interaction between grit and workpiece