Chapter 5

Construction of Partially Balanced Residual Treatment Effects Designs for Comparing Test Treatment with a Control

5.1 INTRODUCTION

In residual treatment effects designs, each experimental unit receives some or all of the treatments in an appropriate sequence over a number of successive periods and the effect of the treatment continues beyond the period of its application. These designs have been discussed in the literature under various names, viz., cross-over designs, change-over designs, and repeated measurement designs. Residual treatment effects designs have application in many research areas including agriculture sciences, dairy husbandry, bioassay procedure, medical applications, psychological experiments, and industrial settings.

The most commonly used residual treatment effects designs are balanced residual effects designs of Williams (1949). Patterson (1952) developed some balanced residual treatment effects designs for which the number of periods is less than the number of treatment effects designs. Good sources are Hedayat and Afsarinejad (1975), Jones and Kenward (1989), Afsarinejad (1990), Ratkwasky et al. (1993), Stufken (1996) and Aggarwal and Jha (2001).

In many experimental situations, interest is focused on the simultaneous comparison of several test treatments to a control treatment rather than on all pairwise comparisons. Pigeon (1984) and Pigeon and Raghavarao (1987) introduced control balanced residual treatment effects designs for the situation where one treatment is a control or standard and is to be compared with the v test treatments, and they have also given methods of construction of control balanced residual treatment effects designs and have estimated their efficiencies. Aggarwal and Jha (2004) has developed some new families of control balanced residual treatment effects designs and which are Schur-optimal. Majumdar (1988) studied the optimality aspects of these designs. Some recent references include Ting (2002) and Hedayat and Yang (2005, 2006).

Giovagnoli and Wynn (1985) studied the Schur-otimality criteria in the context of comparing treatments with a control in a continuous block design set up. Schur-otimality criteria is a weaker notion than the universal optimality of Kiefer (1975), it still is a wide enough class which includes A-, D-, E-, optimality criteria as members. For more information on the theory and applications of Schur-optimality, see Marshall and Olkin (1979), Giovagnoli and Wynn (1981), Shah and Sinha (1989), and Butler (2008). Aggarwal and Jha (2009) develop some new families of control balanced residual treatment effects designs, and which are Schur-optimal.

In literature we found that all the work till date has been done on balanced residual treatment effects designs for comparing test treatments with control. In this chapter we tried to introduce a new concept which is called a partially balanced residual treatment effects designs for comparing test treatments with control. Other then definition we have also tried to gave some parametric relations. In this chapter we also gave the methods to construct the control partially balanced residual treatment effects designs. This chapter is divided into three sections. The first section consists of the definition of control partially balanced residual treatment effects designs and parametric relations. The second section deals with the mathematical model and the third sections deals with the methods of construction. This section is further divided into three sub section based on the singular, semi-regular and regular GD designs and each sub section is further categories into two on the bases of plot size i.e. whether k^* is odd or even.

5.2 PRELIMINARY

We have proposed the following definition.

Definition: Let $v+1$ treatments be represented as x, 0, 1, 2,…., $v-1$ such that control treatment has a symbol x and then v test treatments have symbols 0, 1, 2,…., $v-1$. A control partially balanced residual treatment effects designs (CPBRTED) based on m-associate class PBIB design is an arrangement of $v+1$ treatments in p periods ($p \leq v+1$) and N units such that:

(i) No treatment occurs more than once in an experimental units

(ii) The control treatment occurs t_0 times in each period and each test treatment occurs t_1 times in each period.

(iii) The control treatment occurs with each test treatment in λ_0 units and every pair of test treatment (θ, Φ) occurs together in λ_i columns if θ and Φ are the ith associates for all i =1, 2.

(iv) Deleting the last period, the control treatment occurs with each test treatment in μ_0 units and every pair of treatments (*θ, Φ*) occurs together in μ_i columns if *θ* and *Φ* are the *i*th associates for all $i = 1, 2$.

(v) The ordered treatment pairs (*x, i*) and (*i, x*) (i =0, 1, 2,…, *v* − 1) occurs in successive periods in successive periods in γ_0 units and the every ordered pair of treatment (*θ, Φ*) occurs together in successive periods in γ_i columns if *θ* and *Φ* are the *i*th associates for all $i = 1, 2$.

(vi) For every pair of treatments (*θ, Φ*) the number of columns in which *θ* occurs with *Φ* in the last period is same as the number of units in which *Φ* occurs with *θ* in the last period.

The parameters of a CPBRTED are *v, p, N,* t_o, t_1, λ_0, λ_i, μ_0, μ_i, ν_0, ν_i ($i = 1, 2, 3\ldots, m$) and satisfy the following parametric relations :

a) $N = t_0 + vt_1$

b) $(p-1)t_0 = vt_1$

c) $p(p-1)t_0 = v\lambda_0$

d) $p(p-1)(vt_1 - t_0) \geq v(\lambda_1 + (v-2)\lambda_2)$

e) $(p-1)(p-2)t_0 = v\mu_0$

f) $(p-1)(p-2)(vt_1 - t_0) \geq v(\mu_1 + (v-2)\mu_2)$

g) $(p-1)t_0 = 2v\gamma_0$

h) $(p-1)(vt_1 - t_0) \geq v(\gamma_1 + (v-2)\gamma_2)$

5.3 MATHEMATICAL MODEL

Let us consider a residual treatment effects design of first order in which *v* treatments are compared with a control, which is a standard treatment, using *N* experimental units and the experiment lasts for *p* periods and let *D(v+1, N, p)* denote the class of all such residual treatment effects designs. Following Stufken (1996), the linear model for the design is

$$y_{ij} = \mu + \alpha_i + \beta_j + \tau_{d(i,j)} + \rho_{d(i-1,j)} + e_{ij} \tag{5.3}$$

where y_{ij} ($1 \le i \le p$; $1 \le j \le N$) is the response observed in the *i*th period on the *j*th unit, μ the general mean, α_i the *i*th period effect, β_j the *j*th unit effect, $\tau_{d(i,j)}$ the direct effect of the treatment $d(i, j)$, $\rho_{d(i-1,j)}$ the first-order residual effect of the treatment $d(i, j)$ with $\rho_{d(0,j)} = 0$ for all *j*, e_{ij} are the random errors assumed to be normally and independently distributed with mean zero and variance σ^2. All the parameters in (5.3.1) are assumed to be fixed.

5.4 METHODS OF CONSTRUCTION

We discuss below methods of construction of control partially balanced residual treatment effects designs.

5.4.1 CPBRTED using Singular GD designs.

In this section using the singular GD designs, we obtained CPBRTED, when *k* is either even or odd.

5.4.1.1 Singular GD design when k* is odd

Let us consider a singular GD design with parameters v^*, r^*, k^*, b^*, λ_1^*, λ_2^*, m^*, n^*. We add a control treatment x at first place to each block of singular GD design. Then corresponding to each block of singular GD (with the control treatment x added) we obtain a Latin square of order $(k^* + 1) \times (k^* + 1)$ as follows:

1. Consider the treatments of the given block with the control treatment x added at the first place.

2. For the second row take the second element of the first row and the remaining element for the second row is the sequence of the second element of the first row.

3. For the third row take the last element of the first row and the remaining are obtained as described in step 2.

4. Now for the fourth row we take the third element of the first row and for fifth row second last element of the first row and so on.

Following the same procedure for each block of singular GD design and place the entire Latin squares side by side, CPBRTED is obtained with parameters

$$v = v^*, N = b^*(k^* + 1), p = (k^* + 1), t_0 = b^*, t_1 = r^*, \lambda_0 = \lambda_1^*(k^* + 1) = \lambda_1, \lambda_2 = \lambda_2^*(k^* + 1)$$
$$\mu_0 = \lambda_1^*(k^* - 1) = \mu_1, \mu_2 = \lambda_2^*(k^* - 1), \gamma_0 = 2\lambda_1^*, \gamma_1 = \lambda_1^*, \gamma_2 = \lambda_2^*$$

Example 5.4.1.1 Consider S82 with parameters $v^*= 12,\ b^* = 4,\ r^* = 3,\ k^* = 9,\ \lambda_1^* = 3,\ \lambda_2^* = 2,\ m^* = 4,\ n^* = 3$ from Clatworthy (1973). We consider the first block of the design with control treatment at first place. Obtain the Latin square of order $(k^*+1)\times(k^*+1)$ from each block using *5.4.1.1*. Now putting all Latin squares side by side, we get CPBRTED. The blocks the designs are as follows.

x	1	5	9	2	6	10	3	7	11	*x*	3	7	11	4	8	12	1	5	9
1	5	9	2	6	10	3	7	11	*x*	3	7	11	4	8	12	1	5	9	*x*
11	*X*	1	5	9	2	6	10	3	7	9	*x*	3	7	11	4	8	12	1	5
5	9	2	6	10	3	7	11	*x*	1	7	11	4	8	12	1	5	9	*x*	3
7	11	*x*	1	5	9	2	6	10	3	5	9	*x*	3	7	11	4	8	12	1
9	2	6	10	3	7	11	*x*	1	5	11	4	8	12	1	5	9	*x*	3	7
3	7	11	*x*	1	5	9	2	6	10	1	5	9	*x*	3	7	11	4	8	12
2	6	10	3	7	11	*x*	1	5	9	4	8	12	1	5	9	*x*	3	7	11
10	3	7	11	*x*	1	5	9	2	6	12	1	5	9	*X*	3	7	11	4	8
6	10	3	7	11	*x*	1	5	9	2	8	12	1	5	9	*X*	3	7	11	4

x	2	6	10	3	7	11	4	8	12	*x*	4	8	12	1	5	9	2	6	10
2	6	10	3	7	11	4	8	12	*x*	4	8	12	1	5	9	2	6	10	*x*
12	*X*	2	6	10	3	7	11	4	8	10	*x*	4	8	12	1	5	9	2	6
6	10	3	7	11	4	8	12	*x*	2	8	12	1	5	9	2	6	10	*x*	4
8	12	*X*	2	6	10	3	7	11	4	6	10	*x*	4	8	12	1	5	9	2
10	3	7	11	4	8	12	*x*	2	6	12	1	5	9	2	6	10	*x*	4	8
4	8	12	*X*	2	6	10	3	7	11	2	6	10	*x*	4	8	12	1	5	9
3	7	11	4	8	12	*x*	2	6	10	1	5	9	2	6	10	*x*	4	8	12
11	4	8	12	*x*	2	6	10	3	7	9	2	6	10	*x*	4	8	12	1	5
7	11	4	8	12	*x*	2	6	10	3	5	9	2	6	10	*X*	4	8	12	1

$v = 12,\ N = 40,\ p = 10,\ t_0 = 4,\ t_1 = 3,\ \lambda_0 = 30 = \lambda_1,\ \lambda_2 = 20,\ \mu_0 = 24 = \mu_1,\ \mu_2 = 16,\ \gamma_0 = 6,\ \gamma_1 = 3,\ \gamma_2 = 2$

5.4.1.2 Singular GD design when k* is even

Let us consider a singular GD design with parameters v^*, r^*, k^*, b^*, λ_1^*, λ_2^*, m^*, n^*. We add a control treatment x at first place to each block of singular GD design. Then to corresponding each block of singular GD (with the control treatment x added) we obtain a mutually orthogonal Latin square of order $(k^*+1)\times(k^*+1)$. We first obtain the Latin square for each block of singular GD design with control treatment x added at first place of each block. For (k^*+1) size, there will be k^* MOLS for each block with the control treatment. Construct the k^* MOLS from each block of the singular group divisible design and then put all mutually orthogonal Latin squares side by side, this design is a CPBRTED with parameters.

$$v = v^*, N = b^*k^*(k^*+1), p = (k^*+1), t_0 = v^*r^*, t_1 = k^*r^*, \lambda_0 = k^*r^*(k^*+1) = \lambda_1,$$
$$\lambda_2 = \lambda_2^* k^*(k^*+1), \mu_0 = \lambda_1^* k^*(k^*-1) = \mu_1, \mu_2 = \lambda_2^* k^*(k^*-1), \gamma_0 = 2k^*\lambda_1^*, \gamma_1 = k^*\lambda_1^*,$$
$$\gamma_2 = k^*\lambda_2^*$$

Example 5.4.1.2 Consider S2 with parameters $v^* = 6 = b^*$, $r^* = 4 = k^*$, $\lambda_1^* = 4$, $\lambda_2^* = 2$, $m^* = 3$, $n^* = 2$ from Clatworthy (1973). We consider the first block of the design with control treatment at first place. Obtain the MOLS of order $(k^*+1)\times(k^*+1)$ from each block using *5.4.1.2* and then putting all MOLS side by side, we get CPBRTED

X	1	4	5	2	x	1	4	5	2	x	1	4	5	2	x	1	4	5	2
1	4	2	X	5	4	2	5	1	x	5	x	1	2	4	2	5	x	4	1
4	2	5	1	x	5	x	1	2	4	2	5	x	4	1	1	4	2	X	5

5	X	1	2	4	2	5	x	4	1	1	4	2	x	5	4	2	5	1	x
2	5	x	4	1	1	4	2	x	5	4	2	5	1	X	5	X	1	2	4

X	2	5	6	3	x	2	5	6	3	x	2	5	6	3	x	2	5	6	3
2	5	3	X	6	5	3	6	2	x	6	x	2	3	5	3	6	x	5	2
5	3	6	2	x	6	x	2	3	5	3	6	x	5	2	2	5	3	x	6
6	X	2	3	5	3	6	x	5	2	2	5	3	x	6	5	3	6	2	x
3	6	x	5	2	2	5	3	x	6	5	3	6	2	X	6	X	2	3	5

X	3	6	4	1	x	3	6	4	1	x	3	6	4	1	x	3	6	4	1
3	6	1	X	4	6	1	4	3	x	4	x	3	1	6	1	4	x	6	3
6	1	4	3	x	4	x	3	1	6	1	4	x	6	3	3	6	1	x	4
4	X	3	1	6	1	4	x	6	3	3	6	1	x	4	6	1	4	3	x
1	4	x	6	3	3	6	1	x	4	6	1	4	3	X	4	X	3	1	6

X	4	1	2	5	x	4	1	2	5	x	4	1	2	5	x	4	1	2	5
4	1	5	x	2	1	5	2	4	x	2	x	4	5	1	5	2	x	1	4
1	5	2	4	x	2	x	4	5	1	5	2	x	1	4	4	1	5	x	2
2	X	4	5	1	5	2	x	1	4	4	1	5	x	2	1	5	2	4	x
5	2	x	1	4	4	1	5	x	2	1	5	2	4	x	2	X	4	5	1

x	5	2	3	6	x	5	2	3	6	x	5	2	3	6	x	5	2	3	6
5	2	6	x	3	2	6	3	5	x	3	x	5	6	2	6	3	x	2	5
2	6	3	5	x	3	x	5	6	2	6	3	x	2	5	5	2	6	x	3
3	X	5	6	2	6	3	x	2	5	5	2	6	x	3	2	6	3	5	X
6	3	x	2	5	5	2	6	x	3	2	6	3	5	x	3	X	5	6	2

X	6	3	1	4	x	6	3	1	4	x	6	3	1	4	x	6	3	1	4
6	3	4	x	1	3	4	1	6	x	1	x	6	4	3	4	1	x	3	6
3	4	1	6	x	1	x	6	4	3	4	1	x	3	6	6	3	4	x	1
1	X	6	4	3	4	1	x	3	6	6	3	4	x	1	3	4	1	6	X
4	1	x	3	6	6	3	4	x	1	3	4	1	6	x	1	X	6	4	3

$v = 6,\ N = 120,\ p = 5,\ t_0 = 24\ ,\ t_1 = 16,\ \lambda_0 = 80 = \lambda_1,\ \lambda_2 = 40,\ \mu_0 = 48 = \mu_1,\ \mu_2 = 24,\ \gamma_0 = 32,\ \gamma_1 = 16,\ \gamma_2 = 8$

5.4.2 CPBRTED using Semi-regular GD designs

In this section using the semi-regular GD designs and on the basis of we obtained CPBRTED, when k^* is either even or odd.

5.4.2.1 Semi-regular GD design when k^* is odd

Let us consider a semi-regular GD design with parameters $v^*, r^*, k^*, b^*, \lambda_1^*, \lambda_2^*, m^*, n^*$. We add a control treatment x at first place to each block of semi-regular GD design. Then corresponding to each block of semi-regular GD (with the control treatment x added) we obtain a Latin square of order $(k^*+1)\times(k^*+1)$. Putting all Latin square side by side we get CPBRTED with parameters

$$v = v^*, N = b^*(k^*+1), p = (k^*+1), t_0 = b^*, t_1 = r^*, \lambda_0 = r^*(k^*+1), \lambda_1 = 0, \lambda_2 = \lambda_2^*(k^*+1),$$
$$\mu_0 = r^*(k^*-1), \mu_1 = 0, \mu_2 = \lambda_2^*(k^*-1), \gamma_0 = 2r^*, \gamma_1 = 0,, \gamma_2 = \lambda_2^*$$

Example 5.4.2.1 Consider SR18 with parameters $v^* = 6, b^* = 4, r^* = 2, k^* = 3, \lambda_1^* = 0, \lambda_2^* = 1, m^* = 3, n^* = 2$ from Clatworthy (1973). We consider the first block of the design with control treatment at first place. Obtain the Latin Square of order $(k^*+1)\times(k^*+1)$ from each block using *5.4.2.1*. Now putting all Latin square side by side, we get CPBRTED.

x	1	2	3	*x*	1	5	6	*x*	2	4	6	*X*	3	4	5
1	2	3	*X*	1	5	6	*x*	2	4	6	*x*	3	4	5	*x*
3	*X*	1	2	6	*x*	1	5	6	*x*	2	4	5	*X*	3	4
2	3	*x*	1	5	6	*x*	1	4	6	*x*	2	4	5	*x*	3

$v = 6,\ N = 16,\ p = 4,\ t_0 = 4,\ t_1 = 2,\ \lambda_0 = 8,\ \lambda_1 = 0,\ \lambda_2 = 4,\ \mu_0 = 4,\ \mu_1 = 0,\ \mu_2 = 2,\ \gamma_0 = 4,\ \gamma_1 = 0,\ \gamma_2 = 1$

5.4.2.2 Semi-regular GD design when k^* is even

Let us consider a semi-regular GD design with parameters $v^*,\ r^*,\ k^*,\ b^*,\ \lambda_1^*, \lambda_2^*, m^*, n^*$. We add a control treatment *x* at first place to each block of semi-regular GD design. Then to corresponding each block of semi-regular GD (with the control treatment *x* added) we obtain a MOLS of order $(k^* + 1) \times (k^* + 1)$. We first obtain the Latin square for each block of singular GD design with control treatment *x* added at first place of each block. For size $(k^* + 1)$, there will be k^* MOLS for each block with the control treatment. Construct the k^* MOLS from each block of the semi-regular GD design and then put all MOLS side by side, this design is a CPBRTED with parameters.

$$v = v^*, N = b^* k^* (k^* + 1), p = (k^* + 1), t_0 = v^* r^*, t_1 = k^* r^*, \lambda_0 = k^* r^* (k^* + 1), \lambda_1 = \lambda_1^* k^* (k^* + 1),$$
$$\lambda_2 = \lambda_2^* k^* (k^* + 1), \mu_0 = r^* k^* (k^* - 1), \mu_1 = \lambda_1^* k^* (k^* - 1), \mu_2 = \lambda_2^* k^* (k^* - 1), \gamma_0 = 2k^* r^*,$$
$$\gamma_1 = k^* \lambda_1^*, \gamma_2 = k^* \lambda_2^*$$

Example 5.4.2.2 Consider SR36 with parameters $v^* = 8,\ b^* = 8,\ r^* = 4,\ k^* = 4,\ \lambda_1^* = 0,\ \lambda_2^* = 2,\ m^* = 4,\ n^* = 2$ from Clatworthy (1973). We consider the first block of the design with control treatment at first place. Obtain the MOLS of order $(k^* + 1) \times (k^* + 1)$ from each block using *5.4.2.2.* Now put all MOLS side by side, we get CPBRTED

x	1	2	4	3	*x*	1	2	4	3	*x*	1	2	4	3	*x*	1	2	4	3
1	2	3	*x*	4	2	3	4	1	*x*	4	*x*	1	3	2	3	4	*x*	2	1
2	3	4	1	*x*	4	*x*	1	3	2	3	4	*x*	2	1	1	2	3	*x*	4

4	*X*	1	3	2	3	4	*x*	2	1	1	2	3	*X*	4	2	3	4	1	*X*
3	4	*x*	2	1	1	2	3	*x*	4	2	3	4	1	*x*	4	*x*	1	3	2

x	5	6	8	7	*x*	5	6	8	7	*x*	5	6	8	7	*x*	5	6	8	7
5	6	7	*x*	8	6	7	8	5	*x*	8	*x*	5	7	6	7	8	*x*	6	5
6	7	8	5	*x*	8	*x*	5	7	6	7	8	*x*	6	5	5	6	7	*x*	8
8	*X*	5	7	6	7	8	*x*	6	5	5	6	7	*x*	8	6	7	8	5	*X*
7	8	*x*	6	5	5	6	7	*x*	8	6	7	8	5	*x*	8	*x*	5	7	6

x	2	7	1	8	*x*	2	7	1	8	*x*	2	7	1	8	*x*	2	7	1	8
2	7	8	*x*	1	7	8	1	2	*x*	1	*x*	2	8	7	8	1	*x*	7	2
7	8	1	2	*x*	1	*x*	2	8	7	8	1	*x*	7	2	2	7	8	*x*	1
1	*X*	2	8	7	8	1	*x*	7	2	2	7	8	*x*	1	7	8	1	2	*X*
8	1	*x*	7	2	2	7	8	*x*	1	7	8	1	2	*x*	1	*x*	2	8	7

x	6	3	5	4	*x*	6	3	5	4	*x*	6	3	5	4	*x*	6	3	5	4
6	3	4	*x*	5	3	4	5	6	*x*	5	*x*	6	4	3	4	5	*x*	3	6
3	4	5	6	*x*	5	*x*	6	4	3	4	5	*x*	3	6	6	3	4	*x*	5
5	*X*	6	4	3	4	5	*x*	3	6	6	3	4	*x*	5	3	4	5	6	*X*
4	5	*x*	3	6	6	3	4	*x*	5	3	4	5	6	*x*	5	*x*	6	4	3

x	3	8	6	1	*x*	3	8	6	1	*x*	3	8	6	1	*x*	3	8	6	1
3	8	1	*x*	6	8	1	6	3	*x*	6	*x*	3	1	8	1	6	*x*	8	3
8	1	6	3	*x*	6	*x*	3	1	8	1	6	*x*	8	3	3	8	1	*x*	6
6	*X*	3	1	8	1	6	*x*	8	3	3	8	1	*x*	6	8	1	6	3	*X*
1	6	*x*	8	3	3	8	1	*x*	6	8	1	6	3	*x*	6	*x*	3	1	8

x	7	4	2	5	*x*	7	4	2	5	*x*	7	4	2	5	*x*	7	4	2	5
7	4	5	*x*	2	4	5	2	7	*x*	2	*x*	7	5	4	5	2	*x*	4	7
4	5	2	7	*x*	2	*x*	7	5	4	5	2	*x*	4	7	7	4	5	*x*	2
2	*X*	7	5	4	5	2	*x*	4	7	7	4	5	*x*	2	4	5	2	7	*X*
5	2	*x*	4	7	7	4	5	*x*	2	4	5	2	7	*x*	2	*x*	7	5	4

x	4	1	7	6	*x*	4	1	7	6	*x*	4	1	7	6	*x*	4	1	7	6
4	1	6	*x*	7	1	6	7	4	*x*	7	*x*	4	6	1	6	7	*x*	1	4
1	6	7	4	*x*	7	*x*	4	6	1	6	7	*x*	1	4	4	1	6	*x*	7
7	*X*	4	6	1	6	7	*x*	1	4	4	1	6	*x*	7	1	6	7	4	*X*

6	7	x	1	4	4	1	6	x	7	1	6	7	4	x	7	x	4	6	1
x	8	5	3	2	x	8	5	3	2	x	8	5	3	2	x	8	5	3	2
8	5	2	x	3	5	2	3	8	x	3	x	8	2	5	2	3	x	5	8
5	2	3	8	x	3	x	8	2	5	2	3	x	5	8	8	5	2	x	3
3	X	8	2	5	2	3	x	5	8	8	5	2	x	3	5	2	3	8	X
2	3	x	5	8	8	5	2	x	3	5	2	3	8	x	3	x	8	2	5

$v = 8,\ N = 160,\ p = 5,\ t_0 = 32\ ,\ t_1 = 16,\ \lambda_0 = 80,\ \lambda_1 = 0,\ \lambda_2 = 40,\ \mu_0 = 48,\ \mu_1 = 0,\ \mu_2 = 24,\ \gamma_0 = 32,\ \gamma_1 = 0,\ \gamma_2 = 8$

5.4.3 CPBRTED using Regular GD designs.

In this section using the regular GD designs and on the basis of we obtained CPBRTED, when k^* is either even or odd.

5.4.3.1 Regular GD design when k^* is odd

Let us consider a regular GD design with parameters $v^*,\ r^*,\ k^*,\ b^*,\ \lambda_1^*,\ \lambda_2^*, m^*,\ n^*$. We add a control treatment x at first place to each block of regular GD design. Then corresponding to each block of regular GD (with the control treatment x added) we obtain a Latin square of order $(k^*+1)\times(k^*+1)$. Putting all Latin square side by side we get CPBRTED with parameters

$$v = v^*, N = b^*(k^*+1), p = (k^*+1), t_0 = b^*, t_1 = r^*, \lambda_0 = r^*(k^*+1), \lambda_1 = \lambda_1^*(k^*+1),$$
$$\lambda_2 = \lambda_2^*(k^*+1), \mu_0 = r^*(k^*-1), \mu_1 = \lambda_1^*(k^*-1), \mu_2 = \lambda_2^*(k^*-1), \gamma_0 = 2r^*, \gamma_1 = \lambda_1^*,$$
$$\gamma_2 = \lambda_2^*$$

Example 5.4.3.1 Consider R42 with parameters $v^* = 6$, $b^* = 6$, $r^* = 3$, $k^* = 3$, $\lambda_1^* = 2$, $\lambda_2^* = 1$, $m^* = 3$, $n^* = 2$ from Clatworthy (1973). We consider the first block of the design with control treatment at first place. Obtain the Latin square of order $(k^* + 1) \times (k^* + 1)$ for each block using *5.4.3.1*. Now putting all Latin squares side by side, we get CPBRTED

x	1	2	4	x	2	3	5	x	3	4	6	x	4	5	1	x	5	6	2
1	2	4	x	2	3	5	x	3	4	6	x	4	5	1	x	5	6	2	X
4	x	1	2	5	x	2	3	6	x	3	4	1	x	4	5	2	x	5	6
2	4	x	1	3	5	x	2	4	6	x	3	5	1	x	4	6	2	x	5

X	6	1	3
6	1	3	x
3	X	6	1
1	3	x	6

$v = 6,\ N = 24,\ p = 4,\ t_0 = 6,\ t_1 = 3,\ \lambda_0 = 12,\ \lambda_1 = 8,\ \lambda_2 = 4,\ \mu_0 = 6,$

$\mu_1 = 4,\ \mu_2 = 2,\ \gamma_0 = 6,\ \gamma_1 = 2,\ \gamma_2 = 1$

5.4.3.2 Regular GD design when k* is even

Let us consider a semi-regular GD design with parameters v^*, r^*, k^*, b^*, λ_1^*, λ_2^*, m^*, n^*. We add a control treatment x at first place to each block of semi-regular GD design. Then to corresponding each block of semi-regular GD (with the control treatment x added) we obtain a MOLS of order $(k^{*}+1)\times(k^{*}+1)$. We first obtain the Latin square for each block of singular GD design with control treatment x added at first place of each block. For ($k^{*}+1$) there will be k^{*} MOLS for each block with the control treatment. Construct the k^{*} MOLS from each block of the regular GD designs and then put all MOLS side by side, this design obtain is a CPBRTED with parameters.

$$v=v^{*}, N=b^{*}k^{*}(k^{*}+1), p=(k^{*}+1), t_0=v^{*}r^{*}, t_1=k^{*}r^{*}, \lambda_0=k^{*}r^{*}(k^{*}+1), \lambda_1=\lambda_1^{*}k^{*}(k^{*}+1),$$
$$\lambda_2=\lambda_2^{*}k^{*}(k^{*}+1), \mu_0=r^{*}k^{*}(k^{*}-1), \mu_1=\lambda_1^{*}k^{*}(k^{*}-1), \mu_2=\lambda_2^{*}k^{*}(k^{*}-1), \gamma_0=2k^{*}r^{*},$$
$$\gamma_1=k^{*}\lambda_1^{*}, \gamma_2=k^{*}\lambda_2^{*}$$

Example 5.4.3.2 Consider R94 from Clatworthy (1973). We consider the first block of the design with control treatment at first place. Obtain the Latin square of order $(k^{*}+1)\times(k^{*}+1)$ for each block using *5.4.3.2* and then putting all Latin squares side by side. Hence we get CPBRTED

x	1	4	6	2	x	1	4	6	2	x	1	4	6	2	x	1	4	6	2
1	2	6	x	4	4	6	1	2	x	6	x	2	4	1	2	4	x	1	6
4	6	1	2	x	6	x	2	4	1	2	4	x	1	6	1	2	6	X	4
6	X	2	4	1	2	4	x	1	6	1	2	6	x	4	4	6	1	2	X
2	4	x	1	6	1	2	6	x	4	4	6	1	2	x	6	x	2	4	1

x	2	5	1	3	x	2	5	1	3	x	2	5	1	3	x	2	5	1	3
2	3	1	x	5	5	1	2	3	x	1	x	3	5	2	3	5	x	2	1
5	1	2	3	x	1	x	3	5	2	3	5	x	2	1	2	3	1	X	5

1	*X*	3	5	2	3	5	*x*	2	1	2	3	1	*x*	5	5	1	2	3	*X*
3	5	*x*	2	1	2	3	1	*x*	5	5	1	2	3	*x*	1	*x*	3	5	2

x	3	6	2	4	*x*	3	6	2	4	*x*	3	6	2	4	*x*	3	6	2	4
3	4	2	*x*	6	6	2	3	4	*x*	2	*x*	4	6	3	4	6	*x*	3	2
6	2	3	4	*x*	2	*x*	4	6	3	4	6	*x*	3	2	3	4	2	*X*	6
2	*X*	4	6	3	4	6	*x*	3	2	3	4	2	*x*	6	6	2	3	4	*x*
4	6	*x*	3	2	3	4	2	*x*	6	6	2	3	4	*x*	2	*x*	4	6	3

x	4	1	3	5	*x*	4	1	3	5	*x*	4	1	3	5	*x*	4	1	3	5
4	5	3	*x*	1	1	3	4	5	*x*	3	*x*	5	1	4	5	1	*x*	4	3
1	3	4	5	*x*	3	*x*	5	1	4	5	1	*x*	4	3	4	5	3	*X*	1
3	*X*	5	1	4	5	1	*x*	4	3	4	5	3	*x*	1	1	3	4	5	*x*
5	1	*x*	4	3	4	5	3	*x*	1	1	3	4	5	*x*	3	*x*	5	1	4

x	5	2	4	6	*x*	5	2	4	6	*x*	5	2	4	6	*x*	5	2	4	6
5	6	4	*x*	2	2	4	5	6	*x*	4	*x*	6	2	5	6	2	*x*	5	4
2	4	5	6	*x*	4	*x*	6	2	5	6	2	*x*	5	4	5	6	4	*X*	2
4	*X*	6	2	5	6	2	*x*	5	4	5	6	4	*x*	2	2	4	5	6	*x*
6	2	*x*	5	4	5	6	4	*x*	2	2	4	5	6	*x*	4	*x*	6	2	5

x	6	3	5	1	*x*	6	3	5	1	*x*	6	3	5	1	*x*	6	3	5	1
6	1	5	*x*	3	3	5	6	1	*x*	5	*x*	1	3	6	1	3	*x*	6	5
3	5	6	1	*x*	5	*x*	1	3	6	1	3	*x*	6	5	6	1	5	*X*	3
5	*X*	1	3	6	1	3	*x*	6	5	6	1	5	*x*	3	3	5	6	1	*x*

1	3	x	6	5	6	1	5	x	3	3	5	6	1	x	5	x	1	3	6

$v = 6$, $N = 120$, $p = 5$, $t_0 = 24$, $t_1 = 16$, $\lambda_0 = 80$, $\lambda_1 = 60$, $\lambda_2 = 40$, $\mu_0 = 48$, $\mu_1 = 36$, $\mu_2 = 24$, $\gamma_0 = 32$, $\gamma_1 = 12$, $\gamma_2 = 8$