Chapter 6

Application of Diallel Crosses in Tournament Designs

6.1 INTRODUCTION

Among the four types of diallels discussed by Griffing (1956), system IV is the most commonly used in plant breeding. Suppose there are v lines and let us consider a cross of the form $i \times j$ with $i < j = 1, \ldots, v$. With all possible $n_c = v(v-1)/2$ crosses. This is sometimes referred to as the modified or half-diallel.We shall refer to it as a complete diallel cross (CDC). The common practice with diallel cross experiment is to evaluate the crosses in completely randomized designs or randomized complete block designs as environment designs, e.g.Kempthorne and Curnow(1961). Due to limitation of homogeneous experimental units in a block to accommodate all the chosen crosses, the estimate of genetic parameters would not be precise enough if a complete block design was adopted for large number of crosses. To overcome this problem, many researchers used balanced incomplete block (BIB) designs, partially balanced incomplete block (PBIB) designs with two associate classes etc. by treating the crosses as treatments. These designs have interesting optimality properties when making inferences on a complete set of orthonormalised treatment contrasts. However, in diallel cross experiments the interest of the experimenter is in making comparisons among general combining ability (gca) effects of lines and not crosses and therefore, using these designs as mating designs may result into poor precision of the comparison among lines. Further, the analysis of a diallel cross experiment in incomplete block depends on the incidence of lines rather than the incidence of the crosses as treatments with in a block. It is therefore apparent that special techniques are required to obtain good designs for diallel crosses experiments.

If there is heterogeneity in one direction in the experimental material then the adoption of a randomized complete block design with crosses as treatments, would result in a large error variance even with moderate number of inbred lines. In order to control the error, one would therefore look for suitable incomplete block design for diallel crosses. Das and Giri (1986) and Ceranka and Mejza (1988) made the use of available incomplete block design for instance a BIB design, for the diallel experiment treating the treatments of BIB design as crosses. Ghosh and Divecha (1997) proposed another approach in which they started with an incomplete block design for the usual treatment-block structure, treating the treatments as a lines and maiking all possible pairwise crosses among the lines within the block.

Several authors such as Gupta and Kageyama (1994), Dey and Midha (1996), Mukherjee (1997), Das et al. (1998), Parsad et al. (1999) and Sharma (2004) addressed the problem of finding optimal designs by using nested incomplete block designs (NBIB), triangular PBIB designs, nested balanced block (NBB) designs, GD PBIB designs and circular designs, etc. Gupta and Kageyama (1994) and Sharma (2004) reported optimal designs in which every cross is replicated once but their designs differ in their parametric values with proposed designs. Das et al. (1998) and Parsad et al. (1999) reported optimal designs for single as well more replications. Dey and Midha (1996) reported optimal and efficient designs in which the crosses are replicated in the range $3 \le r \le 10$. These designs also differ in parametric values of our proposed designs.

Das and Dey (2004) has derived the conditions for a block design to be orthogonal in the sense that contrasts among the general combining ability effects, after eliminating the block effects, are estimated free from the specific combining ability effects. Conditions are also derived for such a design to be universally optimal. Some remarks are made on the existence of universally optimal designs.

In this chapter we have solved the problem of tournament design using the techniques of complete diallel crosses (CDC). Here, CDC is obtained using the following methods.

1. Latin Square Design (LSD) after deleting one column where the size of LSD must be odd.

2. LSD after deleting one row which is repeated in crossing plan so is to make the plan CDC when size of LSD is even.

Since each player (or team) will play with other player (or team) so we will have the score of both the teams. Further on the basis of the scores of the player (or team) we can obtain the performance of each player (or team) from the analysis of CDC plan. To estimate the effect of the *i*th player (or team) we use the expression

$$g_i = \frac{1}{\theta} Q_i$$

Where, g_i is the performance of *i*th team, θ is a non-zero eigen value of C- matrix of CDC plan and Q_i is the adjusted total score total.

Next, the variance of two players (or teams) performance effect can easily be obtained from

$$V(\hat{g}_i - \hat{g}_j) = \frac{2}{\theta}\sigma^2$$

Where, σ^2 is the Error Mean Square.

6.2 MATHEMATICAL MODEL

The statistical model for genetic yield from a partial diallel crosses plan of type IV, in the sense of Griffing's complete diallel crosses type IV (1956), following Hinkelmann (1975), is given by

$$y_{ijk} = \mu + g_i + g_j + \beta_k + e_{ijk} \quad (i \neq j = 1,2,...,v; k = 1,2,...,b), \tag{6.1}$$

where μ is the general mean effect, g_i, denotes the general combining ability effect of the *i*th line, β_k is the *k*th block effect, and e_{ijk} is the environmental effect in the *ijk*th observation, distributed normally with mean zero and variance σ^2.

The *C*- matrix of the model (*6.1*) is defined as

$$C_d = G_d - k^{-1}N_d N_d' \tag{6.2}$$

Or $\quad C_d = \theta(I_v - \frac{1}{v}E_{vv})$ (6.3)

where, θ is the eigen value with multiplicity *v-1*.

Where,

$$G_d = \begin{bmatrix} S_{d_i} & d_{ii'} \\ & S_{d_i} \end{bmatrix} \quad (6.4)$$

S_{d_i} is the number of times each line occurred.

$d_{ii'}$ is the number of times each cross occurs in CDC plan

N_d is the $p \times b$ incidence matrix of lines versus blocks and

k^{-1} is the size of the crosses

6.3 METHODS OF CONSTRUCTION

In this section the analysis and construction of tournament design and performance of the *i*th team has been obtained.

6.3.1 Complete Diallel Crosses using LSD after deleting one column where the size of LSD is odd.

Let '*v*' be the odd number of players (or teams) that participate in a tournament. There are $\frac{(v-1)}{2}$ courts (or tables) available for the matches the players (or teams) play several rounds of the game to decide the winner. We want to arrange the tournament schedule in the following way:

(1) There are 'v' rounds of the game and in each round one player (or team) will not play.

(2) Every player (or team) plays in (v-1) rounds.

(3) Every player (or team) plays opposite to every other player (or team) exactly once.

(4) In each round (v-1) players (or teams) will play.

(5) Total number of matches will be $\frac{v(v-1)}{2}$ these $\frac{v(v-1)}{2}$ games are arranged in 'v' rounds and there will be $\frac{(v-1)}{2}$ games in each round.

The solution is based on complete diallel crosses. Let us consider a v an odd size LSD then delete the last column from the design. Now using CDC plan we will obtain ${}^{v}C_{2}$ crosses. We denote number of players (or teams) by s and the cross denotes the number of matches. If i and j denote the ith and jth player (or team) then $i \times j$ is the match between them and i < j and each player plays against the other exactly once and each player (or team) plays (v-1) rounds of game. There will v rounds of game and each round will have $\frac{(v-1)}{2}$ games.

6.3.1.1 GENERALIZATION

$$C_d = (v-2)I_v + E_{vv} - \frac{(I_v + (v-2)E_{vv})}{\frac{(v-1)}{2}}$$

$$= \frac{(v-1)(v-2)I_v + (v-2)E_{vv} - 2I_v - 2(v-2)E_{vv}}{(v-1)}$$

$$= \frac{v(v-3)}{(v-1)} I_v - \frac{(v-3)}{(v-1)} E_{vv}$$

$$= \frac{v(v-3)}{(v-1)} \left(I_v - \frac{1}{v} E_{vv}\right)$$

$\theta = \frac{v(v-3)}{(v-1)}$ is the eigen value with multiplicity $(v-1)$.

Example 6.3.1 Let us consider 5 players (or teams), with these players (or teams) we form a LSD

1	2	3	4	5
2	3	4	5	1
3	4	5	1	2
4	5	1	2	3
5	1	2	3	4

Deleting the last column of the above design. Now using CDC plan we obtain 10 crosses which implies that there will be 10 matches in all.

1 × 2	3 × 4
2 × 3	4 × 5
1 × 4	3 × 5
1 × 5	2 × 4
1 × 3	2 × 5

There will be 5 rounds and in each round 4 players (or teams) will play at a time and there will be 2 matches in each round. To estimate the effect of each player (or team) and also to estimate the performance of 2 players (or teams) we need to calculate *C*-matrix.

$$C_d = \left. \left(\begin{bmatrix} 4 & 1 & 1 & 1 & 1 \\ 1 & 4 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 3 & 3 & 3 & 3 \\ 3 & 4 & 3 & 3 & 3 \\ 3 & 3 & 4 & 3 & 3 \\ 3 & 3 & 3 & 4 & 3 \\ 3 & 3 & 3 & 3 & 4 \end{bmatrix} \right) \middle/ 2 \right.$$

$$= \left. \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix} \middle/ 2 \right.$$

$$= \left. \left\{ \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \right\} \middle/ 2 \right.$$

$$= \frac{(5I_v - E_5)}{2}$$

$$= \frac{5}{2}(I_v - \frac{1}{5}E_5)$$

$\theta = \frac{5}{2}$ is the eigen value with multiplicity 4.

The effect of each player (or team) will be

$$g_i = \frac{2}{5}Q_i$$

The performance effect of 2 players (or teams) will be

$$V(\hat{g}_i - \hat{g}_j) = \frac{4}{5}\sigma^2$$

Hence, the efficiency of the tournament design is given by

$$E = \frac{\frac{2}{4}\sigma^2}{\frac{4}{5}\sigma^2} = \frac{10}{16} = 0.625$$

6.3.2 Complete Diallel Crosses using LSD after deleting one row which is repeated in crossing plan where the size of LSD is even.

Let ‘v’ be the even number of players (or teams) that participate in a tournament. There are $\frac{v}{2}$ courts (or tables) available for the matches the players (or teams) play several rounds of the game to decide the winner. We want to arrange the tournament schedule in the following way:

(1) There are (v-1) rounds of the game and in each round one player (or team) will not play.

(2) Every player (or team) plays in (v-1) rounds.

(3) Every player (or team) plays opposite to every other player (or team) exactly once.

(4) In each round v players (or teams) will play.

(5) Total number of matches will be $\frac{v(v-1)}{2}$ these $\frac{v(v-1)}{2}$ games are arranged in (v - 1) rounds and there will be $\frac{v}{2}$ games in each round.

The solution is based on complete diallel crosses. Let us consider a v an even size LSD then delete the repeated row from the design. Now using CDC plan we will obtain ${}^{v}C_2$ crosses. We denote number of players (or teams) by s and the cross denotes the number of matches. If i and j denote the ith and jth player (or team) then

$i \times j$ is the match between them and i < j and each player plays against the other exactly once and each player (or team) plays v rounds of game. There will (v - 1) rounds of game and each round will have $\frac{v}{2}$ games.

6.3.2.1 GENERALIZATION

$$C_d = (v-2)I_v + E_{vv} - \frac{(v-1)E_{vv}}{\frac{v}{2}}$$

$$= (v-2)I_v + (1 - \frac{2(v-1)}{v})E_{vv}$$

$$= (v-2)I_v + \frac{(v-2)}{v}E_{vv}$$

$$= (v-2)(I_v + \frac{1}{v}E_{vv})$$

$\theta = (v-2)$is the eigen value with multiplicity $(v-1)$.

Example 6.3.2 Let us consider there are 6 players (or teams) participating in a tournament. With these 6 players (or teams) we obtain LSD.

1	2	3	4	5	6
2	3	4	5	6	1
3	4	5	6	1	2
4	5	6	1	2	3
5	6	1	2	3	4
6	1	2	3	4	5

Using CDC plan we obtain 18 crosses.

$$
\begin{array}{ccc}
1\times 2 & 3\times 4 & 5\times 6 \\
2\times 3 & 4\times 5 & 1\times 6 \\
3\times 5 & 2\times 6 & 1\times 4 \\
2\times 4 & 1\times 5 & 3\times 6 \\
1\times 3 & 4\times 6 & 2\times 5 \\
1\times 2 & 3\times 4 & 5\times 6
\end{array}
$$

Since the last row is being repeated therefore we delete the last row and hence we are left with 15 crosses which imply there will be 15 matches.

$$
\begin{array}{ccc}
1\times 2 & 3\times 4 & 5\times 6 \\
2\times 3 & 4\times 5 & 1\times 6 \\
3\times 5 & 2\times 6 & 1\times 4 \\
2\times 4 & 1\times 5 & 3\times 6 \\
1\times 3 & 4\times 6 & 2\times 5
\end{array}
$$

There will be 5 rounds and in each round 4 players (or teams) will play at a time and there will be 2 matches in each round. To estimate the effect of each player (or team) and also to estimate the performance of 2 players (or teams) we need to calculate C-matrix.

$$
C_d = \left. \begin{bmatrix} 5 & 1 & 1 & 1 & 1 & 1 \\ 1 & 5 & 1 & 1 & 1 & 1 \\ 1 & 1 & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 & 1 \\ 1 & 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 1 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 \end{bmatrix} \middle/ 3 \right.
$$

$$
= \left. \begin{bmatrix} 10 & -2 & -2 & -2 & -2 & -2 \\ -2 & 10 & -2 & -2 & -2 & -2 \\ -2 & -2 & 10 & -2 & -2 & -2 \\ -2 & -2 & -2 & 10 & 2 & -2 \\ -2 & -2 & -2 & -2 & 10 & -2 \\ -2 & -2 & -2 & 2 & -2 & 10 \end{bmatrix} \middle/ 3 \right.
$$

$$= \frac{12}{3}(I_6 - \frac{1}{6}E_6)$$

Hence $\theta = 4$ is the Eigen value with multiplicity 5

The effect of each player (or team) will be

$$g_i = \frac{1}{4}Q_i$$

The performance effect of 2 players (or teams) will be

$$V(\hat{g}_i - \hat{g}_j) = \frac{1}{2}\sigma^2$$

Hence, the efficiency of the tournament design is given by

$$E = \frac{\frac{2}{5}\sigma^2}{\frac{1}{2}\sigma^2} = \frac{4}{5} = 0.8$$