CHAPTER - III

VORTICITY IN ELASTICO-VISCOUS MHD FLOWS
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VORTICITY IN ELASTICO-VISCOUS MHD FLOWS

The classical theory of fluid dynamics was developed from studies of an imaginary ideal or 'perfect' fluid that is both incompressible and without viscosity or elasticity. Shearing motion in such fluid is completely frictionless, so that shear stresses are absent. Mathematical relations are available in abundance for "ideal" flow, but their application to real fluid motion was severally limited. This led to the development of "Dynamical theory" for the simplest class of real fluids, namely those possessing a linear relationship between shear stress and rate of shear, commonly described as "newtonian fluids".

An important method in the evolution of fluid dynamical theory is currently being developed. This is produced by the increasing importance in the processing industries and elsewhere of materials whose flow behaviour in shear can not be characterised by newtonian relationship. Examples of such non-newtonian materials are solutions and melts of high polymers, suspension of solids in liquids, emulsions and materials possessing both viscous and elastic properties. An understanding of the non-newtonian behaviour of the material is necessary so as to contributes substantially to the solution of a great variety of problems arising
in the various industries dealing with plastics and synthetic, fibres, petroleum, pharmaceuticals, cement, foods, paper, pulp paint, rubber, soap and detergents, cosmetics, biological fluids, solid rocket propellants, fermentation processes, oil field operations, ore processing and printing etc. Besides this, non-newtonian problems arise in the flow of blood, in the development of nuclear reactors using Thorium and Uranium slurries. Further it has applications in the form as the infection of non-newtonian fluids in to the boundary layer on immersed bodies such as bearings, ships hulls and submarines for reduced drag, addition of magnesium or boron particles to jet engine fuels for greater thrust, the propulsion of rockets by electrostatic acceleration of electrically charged colloidal particles etc.

These fluids are studied by different approaches the physical chemists are primarily interested in relating the flow in physical, chemical and molecular properties of the fluid while rheologists are concerned with describing the fluid behaviour in terms of some mathematical model. The conventional classifications of fluid behaviour is very wide but here we will very briefly highlight the elastico-viscous fluids.

Elastico-Viscous fluids are those materials which when sheared exhibit a certain degree of elasticity in addition to viscosity
visco-elastic fluids flow when subjected to stress but a part of their deformation in gradually recovered upon the removal of the stress. Distortion or deformation of the elastic elements from equilibrium form during shear results in the storage of elastic strain energy while some energy is lost due to viscous dissipation. This stored energy causes elastic recovery on the removal of the shear stresses. This is a consequence of interfacial tension forces restoring the deformed elements to their original form at rest. Thus, in this fluid, unlike enelastic viscous fluid, strain (however small it may be) cannot be ignored for it is responsible for the recovery of the fluid to original state and for the possible reverse flow that follows the removal of the stress. A number of workers like Dewitt [3], Lodge [14], Noll [18], Oldroyed [19,20], Pao [22], Walters [29,30,31,32], Green and Rivlin [8] have developed the constitutive earuation of visco-elastic fluids.

As in the ordinary flow, the vorticity will also be associated with the flow of elastico-viscous flow vorticity, which is responsible for energy dissipation in the fluid flow will be more important in this type of flow. Undoubtedly when the flow of elastico-viscous conducting fluid is a MHD flow, the vorticity considerations will become more complicated but of immense practical use.

In this chapter I have given two research papers namely :-
1. A note on the vorticity of MHD flow of two immiscible viscoelastic Rivin Ericksen fluids under gravity between two porous parallel non-conducting plates inclined at a certain angle with the horizontal.

2. A note on the flow of two immiscible conducting viscoelastic Rivlin-Ericksen fluid between two parallel plates in the presence of transverse uniform magnetic field.
PAPER - I

A NOTE ON THE VORTICITY OF MHD FLOW OF TWO-IMMISCIBLE VISCO-ELASTIC RIVLIN ERICKSEN FLUIDS UNDER GRAVITY BETWEEN TWO POROUS PARALLEL NON-CONDUCTING PLATES INCLINED AT A CERTAIN ANGLE WITH THE HORIZONTAL.

It has been observed that the study of flow of fluids through porous media is playing an important role in the recovery of crude oil effectively from the pores of the reservoir rocks by displacement with immiscible water and to form polymetric adhesive joints between the solids. Kapur and Shukla [12] have studied the problems of flow of immiscible fluids under constant or time dependent pressure gradient. Lahiri and Ganguly [13] investigated the unsteady flow of two immiscible visco-elastic Maxwell conducting liquids between two parallel plates under a uniform magnetic field when the pla upper plate is given a transient and oscillating velocity.

Gupta and Singh [7], Sengupta and Ray[24] have studied the unsteady flow of two immiscible visco-elastic Rivlin-Ericksen fluids between two parallel porous plates under a time dependent pressure gradient. Also Mittal et al [15,16,17] have studied the vorticity of some flows which give an insight in the study of loss of energy in the flow.
FORMULATION OF THE PROBLEM

To get the result let us first consider the case of n-immiscible fluids.

The general form of the equations for visco-elastic incompressible conducting fluids of Rivlin-Ericksen model flowing through porous media in presence of a transverse magnetic field are given by

\[
\frac{\partial u_j}{\partial t} = -\frac{\partial p}{\partial x} \cdot \rho_j \cdot \left( \alpha_j + \beta_j \frac{\partial}{\partial t} \right) \frac{\partial^2 u_j}{\partial y^2} - \frac{\alpha_j}{k_j} u_j + g \sin \theta_0 + \frac{\sigma \beta_3^2 u_j}{\rho_j}
\]

\[
0 = -\frac{1}{\rho_j} \frac{\partial p}{\partial y} + g \cos \theta_0
\]

...(3.1)

...(3.2)

The boundary conditions are

\[ u_j = 0 \text{ at } y = 0 \text{ and } n_n = 0 \text{ at } y = n \]

.....(3.3)

As \( u_j \) is a function of \( y \) and \( t \) only, the expression not involving \( y \) on the right hand side of the equation (3.1) is a function of \( t \) alone and we can write

\[
p = g \rho_j \times \sin \theta_0 - \nabla \rho_j c
\]

...(3.4)

Were \( c \) is, in general, a function of time. Equation (3.1) then
reduces to

\[
\frac{\partial u_j}{\partial t} = c + \left( \alpha_j + \beta_j \frac{\partial}{\partial t} \right) \frac{\partial^2 u_j}{\partial y^2} - \frac{\alpha_j}{k_j} u_j + \frac{\sigma_j \beta_0^2}{\rho_j} u_j \quad \ldots \quad (3.5)
\]

**SOLUTION OF THE PROBLEM**

For the problem, we consider \( C = C_0 e^{-mt} \) where \( C_0 \) and \( m(>0) \) are constants. The equation (3.5) then reduced to

\[
\frac{\partial^2 u_j}{\partial t^2} = C_0 e^{-mt} + \left( \alpha_j + \beta_j \frac{\partial}{\partial t} \right) \frac{\partial^2 u_j}{\partial y^2} - \frac{\alpha_j}{k_j} u_j + \frac{\sigma_j \beta_0^2}{\rho_j} u_j \quad \ldots \quad (3.6)
\]

In considering the flow of n-visco-elastic incompressible and immiscible fluids each occupying a height ‘1’ between two parallel and stationary non-conducting plates inclined at an angle \( \theta_0 \) with the horizontal and in presence of a transverse magnetic field we introduce the following notation:-

- \( B_0 \Rightarrow \) Magnetic Induction Vector

- \( \mu_j \Rightarrow \) Coefficient of viscosity of the fluids

- \( \rho_j \Rightarrow \) Density of the fluids

- \( K_j \Rightarrow \) Permeability of the fluids
\[ \alpha_j \Rightarrow \text{Kinematic viscosity of the fluids} \]

\[ \sigma_j \Rightarrow \text{Conductivity of the fluids} \]

\[ \beta_j \Rightarrow \text{Kinematic visco-elasticity of the fluids} \]

\[ j \Rightarrow 1, 2, \ldots, n. \]

Let us consider a cartesian co-ordinate system with X-axis along the lower plate and parallel to the direction of flow, while Y-axis is chosen perpendicular to it and in upward sense. The magnetic field is along Y-axis.

Assuming \( u_j = u_j(x, y, t), v_j = 0, w_j = 0 \), the equation of continuity \( \frac{\partial u_j}{\partial x} = 0 \) leading to \( u_j = u_j(y, t) \).

Adopting a similar mathematical formulation, Sengupta and Ray have studied the hydromagnetic flow of n-immiscible visco-elastic Rivlin-Ericksen fluids between to porous parallel non conducting plates inclined at a certain angle with the horizontal under the action of time varying pressure gradient.

Let us take the solution of equation (3.6) as

\[ u_j = u^{(i)}(y)e^{-\alpha t}. \]

... (3.7)

Thus equation (3.6) becomes
\[ \frac{\partial^2 u^{(j)}}{\partial y^2} - \frac{\left( a_j/k_i + \alpha_j S_j^2 - m \right)}{\alpha_j - \beta_j m} u^{(j)} + \frac{C_0}{(\alpha_j - \beta_j m)} = 0 \]

where

\[ S_j^2 = \frac{\sigma_j B_0^2}{\mu_j} \]

or

\[ \frac{\partial^2 u^{(j)}}{\partial y^2} - a_j^2 u^{(j)} + b_j^2 = 0 \] ... (3.8)

Where

\[ a_j^2 = \frac{a_j/k_i + \alpha_j S_j^2 - m}{\alpha_j - \beta_j m}, \quad b_j^2 = \frac{C_0}{(\alpha_j - \beta_j m)} \] ... (3.9)

Neglecting the surface tension at the interface, the vorticity at the interfaces can be written as :

\[ u_j = A_re^{-mt} \text{ on } y = rl \] ... (3.10)

\[ j = r, (r + 1) \text{ for } r = 1, 2, \ldots, (n - 1) \]

where \( A_r \) are constants.

From (3.7) and (3.10) we can write

\[ u^{(r)} = u^{(r+1)} = A_r \] ....(3.11)

on \( y = rl \) for \( r = 1, 2, \ldots, (n - 1) \)

solution of equation (3.8) is

\[ u^{(j)} = B_1^{(j)} e^{a_j y} + B_2^{(j)} e^{-a_j y} + \frac{b_j^2}{a_j^2} \] ... (3.12)
The Constants of Integration $B_1^{(i)}$ and $B_2^{(i)}$ have been calculated from (3.3) and (3.11) and are given by

$$B_1^{(i)} = \frac{1}{2 \sinh a_i l} \left[ A_1 - \frac{b_i^2}{a_i^2} \left( 1 - e^{a_i l} \right) \right] \quad \ldots \quad (3.13)$$

$$B_2^{(i)} = \frac{1}{2 \sinh a_i l} \left[ A_1 + \frac{b_i^2}{a_i^2} \left( e^{a_i l} - 1 \right) \right] \quad \ldots \quad (3.14)$$

$$B_1^{(r)} = \frac{\left( A_r - A_r \cdot e^{a_r l} \right) - b_r^2 / a_r^2}{e^{a_r l} - e^{a_{r-2} l}} \quad \ldots \quad (3.15)$$

$$B_2^{(r)} = \frac{\left( A_r - A_r \cdot e^{a_r l} \right) - b_r^2 / a_r^2}{e^{-a_r l} - e^{-a_{r-2} l}} \quad \ldots \quad (3.16)$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$

$$B_1^{(n)} = \frac{A_{n-1} e^{a_{n-1} l} + b_n^2 / a_n^2 \left( 1 - e^{-a_n l} \right)}{e^{a_n l (n-2) l} - e^{a_n, n l}} \quad \ldots \quad (3.17)$$

$$B_2^{(n)} = \frac{A_{n-1} e^{a_{n-1} l} + b_n^2 / a_n^2 \left( 1 - e^{-a_n l} \right)}{e^{-a_n l (n-2) l} - e^{a_n, n l}} \quad \ldots \quad (3.18)$$

Hence with the help of equation (3.7) and (3.12), the complete velocity distribution is given by

$$u_j = \left( B_1^{(i)} e^{a_j y} + B_2^{(i)} e^{-a_j y} + \frac{b_j^2}{a_j^2} \right) e^{-mt} \quad \ldots \quad (3.19)$$

In order to determine the constants $A_1, \ldots, A_r$, we shall consider the continuity of shear at the interfaces given by
\[
\left( a_r + \beta_r \frac{\partial}{\partial t} \right) \frac{\partial u_r}{\partial y} = \left( a_{r+1} + \beta_{r+1} \frac{\partial}{\partial t} \right) \frac{\partial u_{r+1}}{\partial y} . \quad \text{... (3.20)}
\]

Substituting (3.19) in (3.20), we get

\[
(\alpha_r - m\beta_{r+1})a_r \begin{bmatrix} B_1^{(r)}e^{a_r y} - B_2^{(r)}e^{-a_r y} \end{bmatrix} = (\alpha_{r+1} - m\beta_{r+1})a_{r+1} \begin{bmatrix} B_1^{(r+1)}e^{a_{r+1} y} - B_2^{(r+1)}e^{-a_{r+1} y} \end{bmatrix}
\]

\[A_r\] can be calculated by putting the values of \(B_1^{(r)}\) and \(B_2^{(r)}\).

**PARTICULAR CASE**

**TWO LIQUIDS**

In case, when there are only two immiscible liquids, the velocity components are given by

\[
u_j = \begin{bmatrix} B_1^{(j)}e^{a_j y} + B_2^{(j)}e^{-a_j y} + \frac{b_j^2}{a_j^2} \end{bmatrix} e^{-mt} . \quad \text{... (3.21)}
\]

For \(j=1\), the lower liquid occupying the region \(0 \leq y \leq 1\), and for \(j=2\), the upper liquid occupying the region \(1 \leq y \leq 2\), \(B_1^{(1)}, B_2^{(1)}, B_1^{(2)}, B_2^{(2)}\) are given by (3.13), (3.14), (3.17), (3.18)

\[
B_1^{(1)} = \frac{1}{2 \sinh a_1 l} \left[ A_1 - \frac{b_1^2}{a_1^2} \left( 1 - e^{-a_1 l} \right) \right] \quad \text{... (3.22)}
\]

\[
B_2^{(1)} = -\frac{1}{2 \sinh a_1 l} \left[ A_1 - \frac{b_1^2}{a_1^2} \left( e^{a_1 l} - 1 \right) \right] \quad \text{... (3.23)}
\]

\[
B_1^{(2)} = \frac{A_1 e^{a_2 l} + \frac{b_2^2}{a_2^2} \left( 1 - e^{-a_2 l} \right)}{1 - e^{2a_2 l}} \quad \text{... (3.24)}
\]
\[ B_2^{(2)} = \frac{A_1 e^{a_1 \lambda} + b_1^2}{a_1^2 (1 - e^{a_1 \lambda})} \left( 1 - e^{-2a_1 \lambda} \right) \] ... (3.25)

Velocity at the common interface is given by

\[ u = A_1 e^{-mt} \] ... (3.26)

where \( A_1 \) can easily be calculated from

\[
(a_1 - m\beta_1)a_1 \left[ B_1^{(1)} e^{a_1 \lambda} - B_2^{(1)} e^{-a_1 \lambda} \right] = (a_2 - m\beta_2)a_2 \left[ B_1^{(2)} e^{2a_2 \lambda} - B_2^{(2)} e^{-2a_2 \lambda} \right]
\]

on substituting the values of \( B_1^{(1)}, B_1^{(2)}, B_2^{(1)}, B_2^{(2)} \) the value of \( A_1 \) is

\[
A_1 = \frac{(a_1 - m\beta_1)^{b_1^2/a_1^2} (1 - \text{Cosech } a_1 \lambda) - (a_2 - m\beta_2)^{b_2^2/a_2^2} (1 - \text{Cosech } a_2 \lambda)}{(a_1 - m\beta_1)a_1 + a_2 (a_2 - m\beta_2) \text{Cosech } a_2 \lambda}
\]

then the interface velocity can be written as

\[
u = \frac{(a_1 - m\beta_1)^{b_1^2/a_1^2} (1 - \text{Cosech } a_1 \lambda) - (a_2 - m\beta_2)^{b_2^2/a_2^2} (1 - \text{Cosech } a_2 \lambda)}{(a_1 - m\beta_1)a_1 + (a_2 - m\beta_2)a_2 \text{Cosech } a_2 \lambda} e^{-mt}
\] .. (3.27)

or \( u = u_0 e^{-mt} \) ... (3.28)

From equations (3.21), (3.22), (3.23), (3.24), (3.25) and (3.28), the velocity in the lower liquid, in the region \( 0 \leq y \leq 1 \) is

\[
u_1 = \left[ u_0 \frac{\text{Sinh } a_1 y - b_1^2 \left( \text{Sinh } a_1 y - \text{Sinh } a_1 (y - 1) \right)}{\text{Sinh } a_1 \lambda - \frac{b_1^2}{a_1^2} \left( \frac{\text{Sinh } a_1 y - \text{Sinh } a_1 (y - 1)}{\text{Sinh } a_1 \lambda} - 1 \right)} \right] e^{-mt} \] ... (3.29)

or
\[ u_2 = \frac{u_1}{u_0} = \left[ \frac{\text{Sinh} a_1 y - \frac{b_1^2}{a_1} u_0 \{\text{Sinh} a_1 y - \text{Sinh} a_1 (y-1) - \text{Sinh} a_1\}}{\text{Sinh} a_1} \right] e^{-mt} \]

...(3.30)

Similarly, from equations (3.21),(3.22),(3.23),(3.24),(3.25) and (3.28), the velocity in the upper liquid, in the region \( 1 \leq y \leq 2 \) is

\[ u_2 = \left[ \frac{u_0}{\text{Sinh} a_2} \left( \frac{\text{Sinh} a_2 (2-y) + \text{Sinh} a_2 (y-1)}{\text{Sinh} a_2} \right) \right] e^{-mt} \]

...(3.31)

or

\[ \frac{u_2}{u_0} = \left[ \frac{\text{Sinh} a_2 (2-y) - \frac{b_2^2}{a_2^2} u_0 \{\text{Sinh} a_2 (y-1) + \text{Sinh} a_2 (2-y)\}}{\text{Sinh} a_2} \right] e^{-mt} \]

...(3.32)

From equation (3.30) we have

\[ \zeta_1 = \frac{1}{\text{Sinh} a_1} \left[ a_1 \text{Cosh} a_1 y - \frac{b_1^2}{a_1 u_0} \{\text{Cosh} a_1 y - \text{Cosh} a_1 (y-1)\} \right] \]

...(3.33)

By giving the different values to \( y, a_1, b_1 \) and taking \( u_0 = 1 \) we get different values of \( \zeta_1 \) as shown in table (3.1) and the \( \zeta_1 - y \) graphs are drawn as shown in the figure (3.1). From equation (3.32) we have
\[
\zeta_2 = \frac{1}{\sinh a_2} \left[ -a_2 \cosh a_2 (2 - y) - \frac{b_2^2}{a_2 u_0} \{ \cosh a_2 (y - 1) - \cosh a_2 (2 - y) \} \right]
\]

... (3.34)

By giving the different values to \( y, a_2, b_2 \) and taking \( u_0 = 1 \), we get the different values of \( \zeta_2 \) as shown in table (3.2) and \( \zeta_2 - y \) graphs are drawn as shown in the figure (3.2).
<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$Y$</th>
<th>0</th>
<th>.11</th>
<th>.21</th>
<th>.31</th>
<th>.41</th>
<th>.51</th>
<th>.61</th>
<th>.71</th>
<th>.81</th>
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<td>$a_i = \frac{1}{i}$</td>
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<td></td>
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<td>$\frac{2}{4}$</td>
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<td>29.33</td>
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<td>14.89</td>
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<tr>
<td></td>
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<td>0.55</td>
<td>-1.90</td>
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<td></td>
<td>$a_i = \frac{5}{i}$</td>
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<td>11.90</td>
<td>7.04</td>
<td>3.99</td>
<td>1.95</td>
<td>0.41</td>
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### TABLE - 3.2

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<td>0</td>
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<td>a₁ = \frac{1}{1}</td>
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<td>a₁ = \frac{4}{1}</td>
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<tr>
<td>a₁ = \frac{5}{1}</td>
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<td>a₁ = \frac{6}{1}</td>
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<tr>
<td>\frac{q}{b} = .1</td>
</tr>
<tr>
<td>\frac{q}{b} = .2</td>
</tr>
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</tr>
<tr>
<td>\frac{q}{b} = .4</td>
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<td>\frac{q}{b} = .5</td>
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</table>

| \frac{q}{b} = .6 | | | | | | | | |

| \frac{q}{b} = .7 | | | | | | | | |

| \frac{q}{b} = .8 | | | | | | | | |

| \frac{q}{b} = .9 | | | | | | | | |

| \frac{q}{b} = 1 | | | | | | | | |
Fig. 3.1
$\zeta_2$ Vs $\gamma'$ Curve

- $a_2 = \frac{5}{1}, \quad \frac{a_2}{b_2} = 0.5$
- $a_2 = \frac{4}{1}, \quad \frac{a_2}{b_2} = 0.4$
- $a_2 = \frac{3}{1}, \quad \frac{a_2}{b_2} = 0.3$
- $a_2 = \frac{2}{1}, \quad \frac{a_2}{b_2} = 0.2$
- $a_2 = \frac{1}{1}, \quad \frac{a_2}{b_2} = 0.1$

Fig. 3.2
DISCUSSION

From table (3.1) and table (3.2) and fig. (3.1) and fig. (3.2) it is clear that:

(i) The vorticity of both the liquids at the interface is maximum and it decreases as we move away the Interface.

(ii) The rate of decrease of vorticity in first liquid is more predominant than the second. In first liquid, we get region of irrotationality in about midway the interface and the porous plate by while in the second liquid although the vorticity decreases but the region of irrotationality may never be expected.

(iii) In my opinion, for this to happen the porosity of non-conducting plates is solely responsible. It is clear from the data that the change in porosity has marked the fact on the flow of the two liquids.
A NOTE ON THE FLOW OF TWO IMMISCIBLE CONDUCTING VISCO-ELASTIC RIVLIN-ERICKSEN FLUID BETWEEN TWO PARALLEL PLATES IN THE PRESENCE OF TRANSVERSE UNIFORM MAGNETIC FIELD.

FORMULATION OF THE PROBLEM

Let us consider that an unlimited mass of conducting visco-elastic Rivlin-Ericksen fluid is separated by two parallel plates of finite depth “2d”. The fluid was initially at rest and both the fluids separated by the plates were set in motion by the action of time dependent pressure gradient. A transverse uniform magnetic field of strength $B_0$ has been applied perpendicular to the direction of flow. The effects due to perturbation of the field and due to induced magnetic field have been neglected.

We choose the origin on the fixed lower plate, y-axis in the direction of flow and x-axis perpendicular to the base pointing upward ‘u’ denotes the vorticity of the unsteady uni-directional flow.

The equation of continuity gives

$$\frac{\partial u}{\partial y} = 0$$

Which shows that $u$ is a function of $x$ and $t$ only.
Assuming the motion to be slow, the equation of motion can be written as

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \left( \alpha + \beta \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial x^2} - \frac{\sigma B_0^2}{\rho} u \quad \cdots (3.35)
\]

(Neglecting the term of cross viscosity) where

\[
\alpha = \frac{\phi_1}{\rho} = \text{Kinematical coefficient of viscosity},
\]

\[
\beta = \frac{\phi_2}{\rho} = \text{Kinematical coefficient of visco-elasticity}
\]

\[
\rho = \text{densities of the fluids being constant.}
\]

**SOLUTION OF THE PROBLEM**

Let us suppose that the upper plate is moving parallel to itself with a transient vorticity \( ve^{-mt} \). A transient pressure gradient \( p_0 e^{-mt} \) is applied to the two fluids.

Putting \( u = ve^{-mt} \) and \( \frac{\partial p}{\partial y} = p_0 e^{-mt} \), the equation of motion (3.35) becomes

\[
\frac{d^2 v}{dx^2} - \frac{\sigma B_0^2 - m\rho}{\rho(\alpha - m\beta)} v = \frac{p_0}{\rho(\alpha - m\beta)}
\]

or

\[
\frac{d^2 v}{dx^2} - a^2 v = b \quad \cdots (3.36)
\]

where

\[
a^2 = \frac{\sigma B_0^2 - m\rho}{\rho(\alpha - m\beta)} \\
b = \frac{p_0}{\rho(\alpha - m\beta)}
\]

\cdots (3.37)
The solution of (3.36) is

\[ v = C_1 \cosh ax + C_2 \sinh ax - \frac{b}{a^2} \quad \text{... (3.38)} \]

where \( C_1, C_2 \) are constants.

The boundary conditions are

\[
\begin{align*}
v_1 &= 0 \text{ when } x = -d \quad \text{when } t \geq 0 \\
v_2 &= u_0 \text{ when } x = 0 \quad \text{... (3.39)}
\end{align*}
\]

\[
\begin{align*}
v_1 &= u_0 \text{ when } x = 0 \quad \text{when } t \geq 0 \\
v_2 &= u \text{ when } x = d \quad \text{... (3.40)}
\end{align*}
\]

Now, we will discuss the flow of both the fluids under the boundary conditions (3.39) and (3.40).

[A] We first consider the flow for the first fluid under the boundary condition (3.39).

Putting the values of (3.39) in (3.38), we have

\[
v = \left( u_0 + \frac{b}{a^2} \right) \cosh ax + \left( u_0 + \frac{b}{a^2} \right) \coth ad - \frac{b}{a^2} \coth ad \sinh ax - \frac{b}{a^2} \quad \text{... (3.41)}
\]

and

\[
u = \left[ \left( u_0 + \frac{b}{a^2} \right) \cosh ax + \left( u_0 + \frac{b}{a^2} \right) \coth ad \right. \\
\left. - \frac{b}{a^2} \coth ad \right] \sinh ax - \frac{b}{a^2} e^{-mt} \quad \text{... (3.42)}
\]
From (3.42) we can find the vorticity as

\[
\therefore \xi_1 = \left[ \left( \frac{a^2 u_0 + b}{a} \right) \text{Sinh} ax + \left( \frac{a^2 u_0 + b}{a} \right) \text{Coth} ad - \frac{b}{a} \frac{\text{Cos Sec} ad}{\text{Cosh} ax} \right] \text{Cosh} ax e^{-mt}
\]

... (3.43)

Now, by giving different values to \(a, d, t, m, x\) and Taking \(u = u_0 = b = 1\), we get the different values of \(\xi_1\) which are shown in tables (3.3), (3.4), (3.5) and (3.6) and \(\xi_1 - x\) graphs are drawn as shown in figures (3.3), (3.4), (3.5) and (3.6) respectively.

[B] We now consider the flow for the second fluid under the boudary condition (3.40).

Putting the values of (3.40) in (3.38), we have

\[
v = \left( u_0 + \frac{b}{a^2} \right) \text{Cosh} ax + \left( u + \frac{b}{a^2} \right) \text{Cosech} ad
\]

\[- \left( u_0 + \frac{b}{a^2} \right) \text{Coth} ad \right] \text{Sinh} ax - \frac{b}{a^2} \]

... (3.44)

and, hence, it becomes

\[
u = \left[ \left( u_0 + \frac{b}{a^2} \right) \text{Cosh} ax + \left( u + \frac{b}{a^2} \right) \text{Cosech} ad
\]

\[- \left( u_0 + \frac{b}{a^2} \right) \text{Coth} ad \right] \text{Sinh} ax - \frac{b}{a^2} \right] e^{-mt} \]

... (3.45)

From equation (3.45) we can find the vorticity of the second fluid as
\[ \zeta_2 = \left[ \left( \frac{a^2 u_0 + b}{a} \right) \sinh ax + \left( \frac{a^2 u + b}{a} \right) \cosh ad \right] - \left( \frac{a^2 u_0 + b}{a} \right) \coth ad \cosh ax \right] e^{-mt} \quad \ldots (3.46) \]

Now, by giving the different values of \( a, d, m, t, x \) and taking \( u_0 = u = b = 1 \), we get the different values of \( \zeta_2 \) which are shown in tables (3.7), (3.8), (3.9), (3.10) and then \( \zeta_2 - x \) graphs are drawn as shown in figures (3.7), (3.8), (3.9) and (3.10) respectively.
TABLE 3.3

\[
a = 5, d = 5, u = 1, u_0 = 1, b = 1
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m = 0)</td>
<td>2.2902</td>
<td>3.8852</td>
<td>6.4719</td>
<td>10.7106</td>
<td>17.6883</td>
<td>29.1696</td>
<td>48.1016</td>
</tr>
<tr>
<td>(m = 1)</td>
<td>0.8425</td>
<td>1.4292</td>
<td>2.3809</td>
<td>3.9402</td>
<td>6.5053</td>
<td>10.7309</td>
<td>17.6956</td>
</tr>
<tr>
<td>(m = 1)</td>
<td>0.3099</td>
<td>0.5258</td>
<td>0.8758</td>
<td>1.4495</td>
<td>2.3931</td>
<td>3.9476</td>
<td>6.5098</td>
</tr>
<tr>
<td>(m = 2)</td>
<td>1.1140</td>
<td>0.1934</td>
<td>0.3222</td>
<td>0.5332</td>
<td>0.8804</td>
<td>1.4522</td>
<td>2.3948</td>
</tr>
<tr>
<td>(m = 3)</td>
<td>0.0419</td>
<td>0.1185</td>
<td>0.1961</td>
<td>0.3238</td>
<td>0.5342</td>
<td>0.8810</td>
<td>1.3264</td>
</tr>
<tr>
<td>(m = 4)</td>
<td>0.0154</td>
<td>0.0261</td>
<td>0.0436</td>
<td>0.0721</td>
<td>0.1191</td>
<td>0.1965</td>
<td>0.3241</td>
</tr>
<tr>
<td>(m = 5)</td>
<td>0.0056</td>
<td>0.0096</td>
<td>0.0160</td>
<td>0.0265</td>
<td>0.0438</td>
<td>0.0723</td>
<td>0.1192</td>
</tr>
</tbody>
</table>

TABLE - 3.4

\[ a = 0.5, \ d = 1, \ u = 1, \ u_0 = 1, \ b = 1 \]

<table>
<thead>
<tr>
<th>( \varsigma )</th>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 0 )</td>
<td>( t = 0 )</td>
<td>1.5718</td>
<td>3.0751</td>
<td>5.3634</td>
<td>9.0208</td>
<td>14.9807</td>
<td>24.7644</td>
<td>40.8694</td>
</tr>
<tr>
<td>( m = 1 )</td>
<td>( t = 1 )</td>
<td>0.5782</td>
<td>1.1312</td>
<td>1.9731</td>
<td>3.3185</td>
<td>5.5110</td>
<td>9.1103</td>
<td>15.0350</td>
</tr>
<tr>
<td>( m = 1 )</td>
<td>( t = 2 )</td>
<td>0.2127</td>
<td>0.4161</td>
<td>0.7258</td>
<td>1.2208</td>
<td>2.0274</td>
<td>3.3515</td>
<td>5.5310</td>
</tr>
<tr>
<td>( m = 1 )</td>
<td>( t = 3 )</td>
<td>0.0782</td>
<td>0.1513</td>
<td>0.2670</td>
<td>0.4491</td>
<td>0.7458</td>
<td>1.2329</td>
<td>2.0347</td>
</tr>
<tr>
<td>( m = 1 )</td>
<td>( t = 4 )</td>
<td>0.0287</td>
<td>0.0563</td>
<td>0.0982</td>
<td>0.1652</td>
<td>0.2743</td>
<td>0.4535</td>
<td>0.7485</td>
</tr>
<tr>
<td>( m = 1 )</td>
<td>( t = 5 )</td>
<td>0.0105</td>
<td>0.0207</td>
<td>0.0361</td>
<td>0.0607</td>
<td>0.1009</td>
<td>0.1668</td>
<td>0.2753</td>
</tr>
<tr>
<td>( m = 1 )</td>
<td>( t = 6 )</td>
<td>0.0038</td>
<td>0.0076</td>
<td>0.0132</td>
<td>0.0223</td>
<td>0.0371</td>
<td>0.0613</td>
<td>0.1013</td>
</tr>
</tbody>
</table>
TABLE - 3.5

\[
a = 1, \ d = .5, \ u = 1, \ u_0 = 1, \ b = 1
\]

<table>
<thead>
<tr>
<th>$\zeta_t$</th>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>m = 0 t = 0</td>
<td>2.4088</td>
<td>6.0674</td>
<td>16.3163</td>
<td>44.2874</td>
<td>120.3918</td>
<td>327.1686</td>
<td>889.3330</td>
<td></td>
</tr>
<tr>
<td>M = 1 t = 1</td>
<td>0.8861</td>
<td>2.2321</td>
<td>6.0024</td>
<td>16.2924</td>
<td>44.2786</td>
<td>120.3586</td>
<td>327.1675</td>
<td></td>
</tr>
<tr>
<td>M = 1 t = 2</td>
<td>0.3260</td>
<td>0.8211</td>
<td>2.2081</td>
<td>5.9936</td>
<td>16.2892</td>
<td>44.27774</td>
<td>120.3580</td>
<td></td>
</tr>
<tr>
<td>M = 1 t = 3</td>
<td>0.1199</td>
<td>0.3020</td>
<td>0.8123</td>
<td>2.2049</td>
<td>5.9924</td>
<td>16.2887</td>
<td>44.2773</td>
<td></td>
</tr>
<tr>
<td>M = 1 t = 4</td>
<td>0.0411</td>
<td>0.1111</td>
<td>0.2988</td>
<td>0.8111</td>
<td>2.2045</td>
<td>5.9923</td>
<td>16.2887</td>
<td></td>
</tr>
<tr>
<td>M = 1 t = 5</td>
<td>0.0162</td>
<td>0.0408</td>
<td>0.1099</td>
<td>0.2984</td>
<td>0.8109</td>
<td>2.2044</td>
<td>5.9922</td>
<td></td>
</tr>
<tr>
<td>M = 1 t = 6</td>
<td>0.0059</td>
<td>0.0150</td>
<td>0.0404</td>
<td>0.1097</td>
<td>0.2983</td>
<td>0.8109</td>
<td>2.2044</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE - 3.6

\[ a = 1, \ d = 1, \ u = 1, \ u_0 = 1, \ b = 1 \]

<table>
<thead>
<tr>
<th>ζ_1 (x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>m = 0</td>
<td>1.7751</td>
<td>5.0896</td>
<td>13.9321</td>
<td>37.9110</td>
<td>103.0561</td>
<td>280.1403</td>
<td>761.5023</td>
</tr>
<tr>
<td>t = 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 1</td>
<td>0.6530</td>
<td>1.8723</td>
<td>5.1253</td>
<td>13.9466</td>
<td>37.9122</td>
<td>103.0578</td>
<td>280.1410</td>
</tr>
<tr>
<td>t = 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 1</td>
<td>0.2402</td>
<td>0.6888</td>
<td>1.8855</td>
<td>5.1307</td>
<td>13.9471</td>
<td>37.9128</td>
<td>103.0581</td>
</tr>
<tr>
<td>t = 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 1</td>
<td>0.0883</td>
<td>0.2533</td>
<td>0.6936</td>
<td>1.8874</td>
<td>5.1308</td>
<td>13.9473</td>
<td>37.9129</td>
</tr>
<tr>
<td>t = 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 1</td>
<td>0.0325</td>
<td>0.0932</td>
<td>0.2551</td>
<td>0.6943</td>
<td>1.8875</td>
<td>5.1309</td>
<td>13.9474</td>
</tr>
<tr>
<td>t = 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 1</td>
<td>0.0119</td>
<td>0.0342</td>
<td>0.0938</td>
<td>0.2554</td>
<td>0.6943</td>
<td>1.8875</td>
<td>5.1309</td>
</tr>
<tr>
<td>t = 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 1</td>
<td>0.0044</td>
<td>0.0126</td>
<td>0.0345</td>
<td>0.0939</td>
<td>0.2554</td>
<td>0.6943</td>
<td>1.8875</td>
</tr>
<tr>
<td>t = 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE - 3.7

$a = .5$, $d = .5$, $u = 1$, $u_0 = 1$, $b = 1$

<table>
<thead>
<tr>
<th>$\zeta_0$</th>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>m = 0</td>
<td>t = 0</td>
<td>-0.3108</td>
<td>0.9521</td>
<td>2.4582</td>
<td>4.5918</td>
<td>7.8975</td>
<td>13.2190</td>
<td>21.9148</td>
</tr>
<tr>
<td>m = 1</td>
<td>t = 1</td>
<td>-0.1143</td>
<td>0.3502</td>
<td>0.9043</td>
<td>1.6892</td>
<td>2.9053</td>
<td>4.8630</td>
<td>8.0620</td>
</tr>
<tr>
<td>m = 1</td>
<td>t = 2</td>
<td>-0.0420</td>
<td>0.1288</td>
<td>0.3326</td>
<td>0.3214</td>
<td>1.0688</td>
<td>1.7890</td>
<td>2.9658</td>
</tr>
<tr>
<td>m = 1</td>
<td>t = 3</td>
<td>-0.0154</td>
<td>0.0474</td>
<td>0.1223</td>
<td>0.2286</td>
<td>0.3931</td>
<td>0.6581</td>
<td>1.0910</td>
</tr>
<tr>
<td>m = 1</td>
<td>t = 4</td>
<td>-0.0056</td>
<td>0.0174</td>
<td>0.0450</td>
<td>0.0841</td>
<td>0.1446</td>
<td>0.2421</td>
<td>0.4013</td>
</tr>
<tr>
<td>m = 1</td>
<td>t = 5</td>
<td>-0.0020</td>
<td>0.0064</td>
<td>0.0165</td>
<td>0.0309</td>
<td>0.0532</td>
<td>0.0890</td>
<td>0.1476</td>
</tr>
<tr>
<td>m = 1</td>
<td>t = 6</td>
<td>-0.0007</td>
<td>0.0023</td>
<td>0.0060</td>
<td>0.0113</td>
<td>0.0195</td>
<td>0.0327</td>
<td>0.0543</td>
</tr>
</tbody>
</table>
TABLE - 3.8
a = .5, d = 1, u = 1, \( u_0 = 1 \), b = 1

<table>
<thead>
<tr>
<th>( m )</th>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>-0.6122</td>
<td>0.6122</td>
<td>1.9935</td>
<td>3.8830</td>
<td>6.7639</td>
<td>11.3713</td>
<td>18.8812</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>-0.2252</td>
<td>0.2252</td>
<td>0.7334</td>
<td>1.4284</td>
<td>2.4883</td>
<td>4.1832</td>
<td>6.9460</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>-0.0828</td>
<td>0.0828</td>
<td>0.2698</td>
<td>0.5255</td>
<td>0.9153</td>
<td>1.5389</td>
<td>2.5553</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>-0.0304</td>
<td>0.0304</td>
<td>0.0992</td>
<td>0.1933</td>
<td>0.3367</td>
<td>0.5661</td>
<td>0.9400</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>-0.0112</td>
<td>0.0112</td>
<td>0.0365</td>
<td>0.0711</td>
<td>0.1238</td>
<td>0.2082</td>
<td>0.3458</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>-0.0041</td>
<td>0.0041</td>
<td>0.0134</td>
<td>0.0261</td>
<td>0.0455</td>
<td>0.0766</td>
<td>0.1272</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>-0.0015</td>
<td>0.0015</td>
<td>0.0049</td>
<td>0.0096</td>
<td>0.0167</td>
<td>0.0281</td>
<td>0.0468</td>
</tr>
</tbody>
</table>
\textbf{TABLE - 3.9}

\(a = 1, \ d = .5, \ u = 1, \ u_0 = 1, \ b = 1\)

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\zeta & x & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
m = 0 & t = 0 & -0.4898 & 1.5946 & 5.4109 & 15.1046 & 41.2042 & 112.0583 & 304.6259 \\
\hline
m = 1 & t = 1 & -0.1802 & 0.5866 & 1.9905 & 5.5566 & 15.1582 & 41.2239 & 112.0656 \\
\hline
m = 1 & t = 2 & -0.0662 & 0.2158 & 0.7322 & 2.0441 & 5.5763 & 15.1654 & 41.2266 \\
\hline
m = 1 & t = 3 & -0.0243 & 0.0793 & 0.2693 & 0.7520 & 2.0514 & 5.5790 & 15.1664 \\
\hline
m = 1 & t = 4 & -0.0089 & 0.0292 & 0.0991 & 0.2766 & 0.7546 & 2.0524 & 5.5794 \\
\hline
m = 1 & t = 5 & -0.0033 & 0.0107 & 0.0364 & 0.1017 & 0.2776 & 0.7550 & 2.0525 \\
\hline
m = 1 & t = 6 & -0.0012 & 0.0039 & 0.0134 & 0.0374 & 0.1021 & 0.2777 & 0.7550 \\
\hline
\end{array}
\]
TABLE - 3.10

\(a = 1, \ d = 1, \ u = 1, \ u_0 = 1, \ b = 1\)

<table>
<thead>
<tr>
<th>(m, t)</th>
<th>(x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>m = 0 t = 0</td>
<td></td>
<td>-0.9242</td>
<td>0.9242</td>
<td>3.7765</td>
<td>10.7308</td>
<td>29.3406</td>
<td>79.8190</td>
<td>216.9938</td>
</tr>
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<td>29.3637</td>
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<td>0.1250</td>
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<td>1.4522</td>
<td>3.9708</td>
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<td>29.3669</td>
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<td>0.0460</td>
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<td>0.0169</td>
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<td>0.0062</td>
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<td>0.1976</td>
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<td>0.0265</td>
<td>0.0727</td>
<td>0.1978</td>
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</table>
\[
a = .5, \quad d = .5, \quad u = 1, \quad u_0 = 1, \quad b = 1
\]
\( a = 0.5, \ d = 1, \ u = 1, \ u_0 = 1, \ b = 1 \)

Fig. 3.4
$a = 1, \ d = .5, \ u = 1, \ u_0 = 1, \ b = 1$

$m = 0, \ t = 0$
$m = 1, \ t = 1$
$m = 1, \ t = 2$
$m = 1, \ t = 3$
$m = 1, \ t = 4$
$m = 1, \ t = 5$
$m = 1, \ t = 6$

Fig. 3.5
$\zeta_1 \quad a = 1, \quad d = 1, \quad u = 1, \quad u_0 = 1, \quad b = 1$

$m = 0, \quad t = 0$ –––
$m = 1, \quad t = 1$ –––
$m = 1, \quad t = 2$ –––
$m = 1, \quad t = 3$ –––
$m = 1, \quad t = 4$ –––
$m = 1, \quad t = 5$ –––
$m = 1, \quad t = 6$ –––

Fig. 3.6
$a = .5, \; d = .5, \; u = 1, \; u_0 = 1, \; b = 1$

Fig. 3.7
\[ a = 1, \ d = .5, \ u = 1, \ u_0 = 1, \ b = 1 \]
$a = 1, \ d = 1, \ u = 1, \ u_o = 1, \ b = 1$

Fig. 3.10
DISCUSSION

From the tables (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9), and (3.10) and Figures (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9) and (3.10) we observe that

(1) The vorticity of both the fluids at the interface is minimum and it increases as the we move away the interface.

(2) The vorticity of both the fluids at the interface is minimum and the flow becomes irrotational near the interface.

(3) As ‘t’ increases the increase in vorticity becomes regular while on decrease in ‘t’ the increase in vorticity becomes very steep.

(4) In our opinion, the parallel plates and the transverse uniform magnetic field are responsible for such type of vorticity between the two plates.
BIBLIOGRAPHY


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