CHAPTER II

POLARIZATION

INDUCED PHASE
2.1 Introduction

The results of the studies presented in different chapters of this dissertation are based on the utilization of polarization induced phase difference between different zones of the apertures of the imaging systems. It is therefore imperative that a brief review of the concept of polarization-induced phase (PIP) be presented before we enter into the subject matter of the present dissertation.

It is well known that the phase difference between two different pairs of orthogonal components of an elliptically polarized light is different in general. This implies that if an elliptically polarized beam passes through a linear polarizer, the transmitted beam will have an associated phase that depends on the state of polarization of the original beam and also on the orientation of the linear polarizer. This provides a simple but useful technique for introducing a phase in a beam of light or a phase difference between two beams derived from an original beam. The phase introduced by utilizing this method is termed as polarization phase.

In what follows an analytical expression for the phase of the component of an elliptically polarised light transmitted through a linear polarizer is derived.

2.2 Theory

In order to calculate the complex amplitude of the beam passing through a polariser which receives an elliptically polarized beam as its input, we make use of Jones matrix formalism. An elliptically polarized beam propagating along z-axis may be represented by a column matrix, known as Jones vector, whose components represent the two orthogonal transverse components $E_x$ and $E_y$ of the wave.
Ignoring the time harmonic term, the Jones vector for an elliptically polarised light is given by,

\[ \mathbf{e}_i = \begin{pmatrix} a \\ be^{i\alpha} \end{pmatrix} \]  \hspace{1cm} (2.1)

where \( a \) and \( b \) are the amplitudes of the \( x \) and \( y \) components of the beam respectively and \( \Delta \) is the phase difference between these two components. It can be easily shown that the vibrational pattern of the beam is determined by \( a, b \) and \( \Delta \).

The azimuth \( \Psi \) (Fig. 2.1) of the vibrational ellipse is given by the following relation [90],

\[ \tan 2\psi = \frac{2ab}{a^2 - b^2 \cos \Delta} \]  \hspace{1cm} (2.2)

Fig. 2.1 Geometry of the vibration ellipse
When a beam of light represented by $\mathbf{e}$ is sent through a linear polariser whose transmission axis makes an angle $\alpha$ with the chosen x-axis, the beam transmitted through the polariser is given by

$$E_0 = P(\alpha)\mathbf{e},$$

(2.3)

where $P(\alpha)$ is the characteristic Jones matrix of the polariser having the following form:

$$P(\alpha) = \begin{vmatrix} \cos^2 \alpha & \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha & \sin^2 \alpha \end{vmatrix}$$

(2.4)

The output from the polariser can therefore be written as,

$$E_0 = \left[a \cos \alpha + b \sin \alpha e^{i\Delta} + i(c \sin \alpha + d \cos \alpha \sin \Delta)\right] \begin{vmatrix} \cos \alpha \\ \sin \alpha \end{vmatrix}$$

$$= R e^{i\Delta'} \begin{vmatrix} \cos \alpha \\ \sin \alpha \end{vmatrix}$$

(2.5)

where,

$$R = \left[a^2 \cos^2 \alpha + b^2 \sin^2 \alpha + ab \sin 2\alpha \cos \Delta\right]^{1/2}$$

(2.6)

and

$$\Delta' = \tan^{-1} \left[\frac{b \sin \alpha \sin \Delta}{a \cos \alpha + b \sin \alpha \cos \Delta}\right]$$

(2.7)

The above three equations reveal the following facts:

(a) The beam of light emerging from the polarizer is linearly polarized with the electric vector making an angle $\alpha$ with the x-axis.

(b) The transmitted beam has an amplitude given by $R$ which is dependent on $\Delta$ and $\alpha$. 

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The phase associated with the transmitted beam is dependent on the angles $\alpha$ and $\Delta$.

Equation 2.7 further shows that as long as the light waves incident on the polariser is elliptically polarized, (i.e., $a>0$, $b>0$, and $0<\Delta<2\pi$), the polarization phase introduced is non-zero and can be varied by changing $\alpha$ or $\Delta$ or the magnitudes of ‘$a’$ and ‘$b’$.

If the incident beam is polarized linearly (i.e., if $\Delta = 0, \pm \pi$), the output beam from the polariser undergoes no phase change, although in general, the amplitude $R$ now assumes the form,

$$R = a \cos \alpha + b \sin \alpha \quad (2.8)$$

and is still dependent on $\alpha$.

It is interesting to note that, if the incident beam on the polariser is circularly polarised, (i.e., if $a = b, \Delta = (2n + 1)\pi / 2, n=1,2...$), the amplitude $R$ of the transmitted beam becomes independent of the orientation of the analyser, but the polarisation-induced phase varies continuously with the orientation of the analyser, i.e.,

$$\Delta' = \pm \alpha \quad (2.9)$$

This can be readily shown by substituting $a = b$ and $\Delta = (2n + 1)\pi / 2$ in Eq. 2.7. The positive sign corresponds to left circular polarization and the negative sign corresponds to right circular polarization of the input beam. The above relation shows that when circularly polarized light waves intercept a linear polarizer, the phase associated with the output beam is numerically equal to the angle the polarizer makes with the X-axis.
In the analysis so far we have considered the absolute phase change suffered by a polarised beam on passing through a linear polariser. If two different linear components of elliptically polarised light are selected (by two linear polarisers whose transmission axes do not coincide), they will have a phase difference depending upon the direction of vibration of the components and upon the state of polarization of the original beam. This offers an interesting possibility of introducing any pre-specified phase difference between two interfering beams. It is this phase difference, rather than the absolute phase associated with a single transmitted beam, that is more important for application purposes.

Let us consider two linearly polarized components of an elliptically polarized light which are obtained in the output side of two polarizers having their transmission axes oriented at angles $\alpha$ and $\beta$ to the x-axis. In this case the beam obtained in the output side of the polarizer $P(\alpha)$ will have an overall phase $\Delta_1$, given by the following expression, identical to Eq. 2.7

$$\Delta_1' = \tan^{-1} \left[ \frac{b \sin \alpha \sin \Delta}{(a \cos \alpha + b \sin \alpha \cos \Delta)} \right]$$

Similarly the phase associated with the light passing through the polarizer $P(\beta)$ is given by,

$$\Delta_2' = \tan^{-1} \left[ \frac{b \sin \beta \sin \Delta}{(a \cos \beta + b \sin \beta \cos \Delta)} \right]$$

The input beam, in both the above cases is given by the Jones vector $e$, of Eq. 2.1

The phase difference between these two linearly polarized components of the elliptically polarized beam considered, is given by,

$$\Delta_p = \Delta_1' - \Delta_2'$$

$$= \tan^{-1} \frac{ab \sin \Delta \sin (\alpha - \beta)}{a^2 \cos \alpha \cos \beta + b^2 \sin \alpha \sin \beta + ab \sin (\alpha + \beta) \cos \Delta}$$

This polarization induced phase difference may be of extreme practical importance in areas such as optical imagery and interferometry. The most interesting and useful feature of the phenomenon of polarization induced phase variation is the
fact that the phase introduced in a linearly polarized component of an elliptically polarized beam can be continuously varied simply by changing the relative orientations of some polarizing devices. It may be noted that, in general, both the amplitude transmission ratio and the phase difference vary with the variation of the polarization parameters of the input beam and the orientation of the polarizer. This is however not desirable in many applications. We often need to introduce only a phase difference without changing the amplitude ratio. Fortunately, for a certain state of polarization of the input beam, the phase alone can be varied without changing the amplitude of the transmitted beam. Again, if the input beam is linearly polarized, the output beam varies only in amplitude. Thus, the phase and amplitude of a beam can be either independently or jointly varied in a continuous manner by taking benefit of the polarization properties of light waves. The polarization-induced phase discussed above has been demonstrated by interferometric experiments reported in the reference [91].