CHAPTER 1

INTRODUCTION
The importance and development of classical hydrodynamics at the hands of mathematicians and scientists like Newton, Bernoulli and Euler is quite well known. In fact, it has successfully explained many a fluid phenomena. Nevertheless, there remained certain problems involving, in particular, the flows in the vicinity of solid bodies, that could not be explained. This resulted in incorporating the effects of viscous friction which eventually lead to the derivation of Navier-Stokes equations. These highly coupled nonlinear partial differential equations presented a very high degree of mathematical complexity in solving them. In fact, it was for very few special cases where the exact solutions could be obtained. To overcome this difficulty, Prandtl (1904) gave an idea that could eliminate most of the mathematical complexities from the Navier-Stokes equations and yet retain its essence. This concept led to the development of a theory termed as “Boundary Layer Theory“ and has been responsible for the modern day flights with special reference to space shuttle program.

The Navier-Stokes equations of motion of a viscous fluid along with the energy equals are, mathematically speaking, very complex. Exact solutions to these equations are known for only a few cases, most of which have very specialised and often impractical boundary conditions. The complicated nature of these equations dose not, however, eliminate the need for answers to problem of practical importance. For this reason an engineer must be content with the compromise of accepting either
an approximate solution to these fundamental equations, an exact solution to simplified or approximate versions of the equations, or sometimes, an approximate solution of approximate equations. This does not destroy the value of the exact fundamental equations, for they are necessary in order to understand the physical implications of making any approximation or simplification.

Since the Navier-Stokes equations express a balance among inertia forces, viscous forces, pressure forces and body forces, one such a simplification may come from neglecting certain of the forces has being small in comparison to others when the conditions of the flow and the relative magnitude of the terms will permit. A very important case of this is found in instances in which one decides that all the viscous forces are negligible with respect to the inertia and pressure forces. If this is so, one can assume that the fluid viscosity is negligibly small and the Navier-Stokes equations reduce to the simpler Euler equations. This simplification has many applications, the general theories of acrodynamics being an example.

However, when one considers the flow of the fluid past a solid surface the assumption of a vanishingly small viscosity may lead to results that are not verified in experiment. If the basic assumption is made that the fluid particles adjacent to the surface adhere to it and have zero velocity, there must exists velocity gradients in the fluid motion since the velocity must change from zero at the surface to some finite value at points removed from the surface. Immediately in the vicinity of the surface
the velocity gradient may be so large that, even if the fluid viscosity is small, the product of the velocity gradient and the viscosity may not be negligible. The extent of the region near the wall in which the viscous stress may not be negligible, even in fluids of small viscosity, will depend on the properties of the fluid, the shape of the wall, the velocity of the main stream of the fluid, etc.

Even in such a region it may be possible that not all the viscous stress or of a sizable nature. Nor may all the components of the inertia forces be significant. It is on these basis that Ludwig Prandtl in 1904, proposed his boundary layer theory. Prandtl's fundamental concept was that the motion of a fluid of small viscosity about a solid surface could be divided into two regions. One region, termed the boundary layer, near the body surface is defined as a region in which the velocity gradients are large enough so that the influence of viscosity cannot be neglected. The other region, termed the potential region (or potential core if low inside a body is considered) is defined as the region in which the influence of the presence of the body has died out enough so that the velocity gradients are so small that the fluid viscosity can be ignored.

In actual flows the influence of the body extends to all regions of the fluid, but this influence and the associated changes in the fluid velocity decrease rapidly with the distance from the body – particularly if the viscosity is small and the main fluid velocity is large (i.e., at large Reynolds's Numbers) since this decrease is continuous,
it is obviously not possible to define a precise law of the limit of the boundary layer and the beginning of the potential region. In practice the limit of the boundary layer, the boundary layer thickness, is taken to be the distance away from the surface at which the flow velocity has achieved some arbitrary percentage of the undisturbed, free stream flow - say 90 or 99%.

Heat is energy in transit due to a temperature difference. Heat transfer is the area of engineering that deals with the mechanisms responsible for transferring energy from one place to another when a temperature difference exist. In the study of heat transfer, both equilibrium and non-equilibrium processes are encountered. The science of heat transfer allows us to determine the time rate of energy transfer caused by non-equilibrium of temperatures.

The subject of heat transfer has a great impact on all energy problems, covering spectrum ranging from the routine task of heating or cooling buildings to the sophisticated problems associate with nuclear power generation. In the case of climate control for a building, heat balances must be made that equate heat addition from lights, electric motors, fossil-fuel, fired engines, people, and solar energy entering windows with heat losses through walls, cracks and doors. The heat transfer problems associated with a nuclear power plant are much more complex. A carefully controlled nuclear reaction must be monitored to release the correct amount of heat to turn water into steam to drive a stream turbine. The stream leaving the turbine is then
condensed in a heat exchanger (condenser) requiring the use of cooling water, which is normally extracted from a river or a lake and which, when returns to its origin, may result in thermal pollution. There is extensive demand up to on heat transfer analyses to determine its rates during the stream generation and condensation processes and to estimate the quantity of heat that may be effectively dispersed into a given body of water without altering its biological balance.

The practicing engineer encounters numerous heat transfer problems during his daily activities. As examples, the chemical engineer must concern himself with heat transfer rates in various chemical processing operations, the electrical engineer must design his electrical motors so that they do not overheat, and he must worry about proper sizing of electrical transmission to prevent excess loss of power during transmission due to Joulean dissipation; the civil and structural engineer must be careful to prevent the creation of thermal stresses in concrete structures since heat is generated during the curing (drying) of concrete resulting in differential expansion of the structural components; metallurgical and ceramic engineers must control temperature accurately during heat treatment of various metals and ceramics to achieve the desired properties of the heat-treated material; the biomedical engineer is often interested in the effects of temperature level on leaving organisms; and the mechanical engineer is concerned with heat transfer rates when designing the heating systems for buildings, developing new power plants, improving their modynamic cycle efficiencies, and working on thermal pollution problems. In addition, modern
processes such as xerography demand extensive knowledge of heat conduction, convection and radiation. Consequently, we see that the engineering science of heat transfer has broad applications in technology and is not limited in scope to one or two isolated areas.

Although heat transfer has been studied for a number of years, its popularity in engineering curricula was brought about as a result of the space effort. Many problems related to generating the power necessary to put man in space and of shielding the space capsule up on re-entry into the earth’s atmosphere made it necessary to embark upon a through study of the mechanisms of heat transfer.

With the energy shortage currently upon us, it becomes even more important to study heat transfer so that we can utilize our energy reserves more efficiently. By improved methods of energy transport, by new designs that decrease heat losses, by more efficient generation of power and by improved usage of power, we can draw up on our limited energy resources in an economical manner.

The importance of a thorough knowledge of science of heat transfer and the necessity of being able to analyze, quantitatively, problems involving a transfer of heat have become increasingly important as modern technology has become more and more complex. In almost every phase of scientific and engineering work, processes involving the exchange of energy through a flow of heat are encountered. Recent technological implications have given rise to increased interest in mixed convection
problems in vertical channels. The physical situations involve with both buoyancy aided and opposed cases for laminar and turbulent flows.

Free and forced convection flows in vertical ducts have been investigated extensively. The majority of the recent studies have dealt with the circular tube geometry but increasing attention is being focused on the parallel plate duct. This configuration is relevant to solar energy collection, as in the conventional flat plate collector and Trombe wall, and in the cooling of modern electronic systems. In the later application, electronic components are mounted on circuit cards, an array of which is positioned vertically in a cabinet forming vertical flat channels through which cards are passed.

Mechanical and chemical engineers are particularly concerned with problems of heat transfer. Modern power generation involves the production of work from either a combustible fuel or a nuclear reaction. This energy is converted into useful work by means of boilers, turbines, condensers, air-heaters, water pre-heaters, pumps etc. All these pieces of apparatus involve a transfer of heat by one means or another, as does almost every piece of apparatus found in a chemical process industry or a petroleum refinery. Certainly designing the familiar internal combustion engines, gas turbines and jet engines requires a complete understanding of heat transfer for thorough analysis of the combustion and cooling processes.
The importance of heat transfer in the production of comfort cooling or fort heating is readily-apparent. This influences the design of building structure of all kinds. Existing literature for the parallel plate vertical channel deals mostly with the limiting cases of free and forced convection: little information is available for mixed convection consider the situation in which the channel walls are cooled by forced flow in the upward direction at a prescribed coolant flow rate at the duct entrance. Assume that the wall heating is sufficiently intense that free convection effects are significant. Such a mixed convection problem has not being fully treated in the literature.

Convection is the term applied to the energy transfer process which is observed to occur in fluids mainly because of the transport of energy by means of the motion of the fluid itself. The processes of conduction of energy by molecular interchange is, of course, still present, but high energy (or hot) portions of the fluid are brought into contact with the lower energy regions (cooler regions) by the virtue of the fact that there are gross displacement of the fluid particles.

Surface and body forces are encountered in the study of continuum fluid mechanics. Surface forces act on the boundaries of medium through direct contact. Forces developed without physical contact and distributed over the volume of the fluid, are termed body forces. Gravitational and electromagnetic forces are examples of body forces.
Fluids in which shear stress is directly proportional to rate of deformation are Newtonian fluids. The term Non-Newtonian is used to classify all fluids in which shear stress is not directly proportional to shear rate.

Flows in which variations in density are negligible are termed incompressible; when density variations within a flow are not negligible, the flow is called Compressible.

Compressible flows occur frequently in engineering applications. Common examples include compressed air systems used to power shop tools and dental drills, transmission of gases in pipelines at high pressure, and pneumatic or fluidic control and sensing systems. Compressibility effects are very important in the design of modern high-speed aircraft and missiles, power plants, fans and compressors.

Flows completely bounded by solid surfaces are called internal or duct flows. Flows over bodies immersed in an unbounded fluid are termed external flows. Both internal and external flows may be laminar or turbulent, compressible or incompressible.

The internal flow of liquids in which the duct does not flow full-where there is a free surface subject to a constant pressure is termed open-channel flow. Common example of open channel flow include flow in rivers, irrigation ditches, and aqueducts.
The density of a fluid is the ratio of the mass of the fluid in a fluid element to its volume.

\[
\rho = \frac{\text{mass}}{\text{volume}} \quad \text{units Kg/m}^3
\]

Viscosity:

It is property of the fluid, which offers the resistance to the movement of one layer over the adjacent layer.

\[
\tau = \mu \frac{dv}{dx}
\]

where \( \mu = \) viscosity coefficient

\[
\frac{dv}{dx} = \text{velocity gradient}
\]

\( \tau = \) Shear stress

Absolute (or) Dynamic viscosity:

It is defined as the ratio of shear stress per unit angular deformation. It is denoted by \( \mu : \)

\[
\mu = \frac{t}{\frac{dv}{dx}}
\]
Kinetic viscosity:

It is defined as a ratio of dynamic viscosity to density. It is denoted by $v$

$$ v = \frac{\mu}{\rho} $$

Reynold’s Number:

It is measure of relative magnitude of the inertia force to the viscous force occurring in the flow.

$$ Re = \frac{\rho L V}{\mu} $$

where

$\rho$ = density of fluid,

$L$ = dimensional parameter,

$V$ = velocity of the fluid,

$\mu$ = dynamic viscosity.

Prandtl Number:

It is the ratio of kinetic viscosity to thermal diffusivity

$$ Pr = \frac{\mu C_p}{k} $$
where \( C_p \): Special heat on constant pressure

\[ K = \text{Thermal conductivity of thread} \]

\( \mu = \text{Dynamic Viscosity} \)

**Grashof Number:**

It is defined as the ratio of product of inertia force and buoyancy force to the square of viscous force.

\[ G = \frac{g\beta \Delta T L^3}{\gamma} \]

where \( g = \text{acceleration due to gravity} \),

\( \beta = \text{thermal expansion coefficient} \),

\( \Delta T = \text{temperature difference} \),

\( L = \text{dimensional parameter} \),

\( \gamma = \text{kinematic viscosity} \).

**Nusselt Number:**

It is the ratio of heat transfer rate \( q \) to the rate at which heat would be conducted within the fluid under a temperature gradient.

\[ Nu = \frac{hL}{k} \]
where $h = \text{heat transfer coefficient}$

$k = \text{thermal conductivity}$

$L = \text{dimensional parameter}.$

**Reynolds Number:**

It is a measure of heating effect due to the work done on the fluid by shearing stresses.

$$Ek = \frac{U^3}{C_p\Delta T}$$

where $U = \text{overall heat transfer coefficient,}$

$C_p = \text{specific heat at constant pressure}$

$\Delta T = \text{temperature difference}.$

**Steady flow:**

Fluid flow is said to steady if at any point in the flowing fluid various characteristics such as velocity, pressure, density, temperature etc., which describe the behavior of fluid in motion do not change with the time.
Unsteady flow:

Fluid flow is said to be unsteady if at any point in the flowing fluid, any one or all the characteristics which describe the behaviour of the fluid in motion change with time.

Uniform flow:

When the velocity of fluid flow, does not change both in magnitude and direction from point to point in the flowing fluid for any given instant of time, the flow is said to be uniform flow, otherwise, it is said to be non-uniform flow.

Laminar flow:

A flow is said to be Laminar when the various fluid particles move in layers with one layer of fluid, sliding smoothly over an adjacent layer.

Boundary Layer:

The small region in the immediate vicinity of the boundary surface in which the velocity of flowing fluid increases gradually from zero at the boundary surface to the velocity of the main stream is called the Boundary Layer.

Boundary Layer Theory:

Prandl's Concept of Boundary Layer

The concept of boundary layer had its origin at the hands of Ludwig prandl (1904) and has turned out to be one of the corner stones of modern fluid mechanics.
According to Prandtl, for flows with low viscosity (high Reynolds number), the viscous effects are confined to a thin layer, called boundary layer, along the surface. The velocity of the fluid varied over this layer from a relatively high value out in the flow to zero on the surface (no-slip condition). Outside this layer the fluid behaves like a perfect fluid. Though the viscosity is small, the shearing stress quite significant. As a direct consequence of the above assumptions, the Navier-Stokes equations are greatly simplified and the flow inside the boundary layer is governed by much simpler set of equations, called boundary layer equations. Though the entire flow was divided into two regions, there existed no sharp boundary between the two. The transition from the viscous flow inside the boundary layer to inviscid flow outside it, is a gradual one. The pressure gradient across the boundary layer is negligible and determined by the outside potential flow.

Convection:

Heat convection is due to the capacity of moving matter to carry heat energy such as transporting load from one place to another. Convection is possible only in presence of a fluid medium. When a fluid flows inside a duct or outside a body, with a temperature difference exist between them, then heat transfer take place between solid and the fluid. This is due to the motion of fluid relative to the surface of the solid and the fluid. This is due to the motion of fluid relative to the surface of the
solid. This type of heat transfer is called convection. There are two types of convective heat transfers.

If the fluid motion is set up by buoyancy effects resulting from the density variation caused by temperature difference in the fluid, the heat transfer is said to be “Free or Natural Convection”. If the fluid motion is artificially created by means of an external agency like a blower or fan, the heat transfer is termed as “Forced Convection”.

Free Convection Boundary Layers

In many cases, the fluid motions and transport that affect our immediate surroundings are induced by buoyancy. Starting from the air circulation around our bodies to the oceanic circulation, such flows are found. In such cases, the buoyancy force owe its origin to the density difference caused by inhomogeneities in temperature, differences in the concentration of chemical species, Changes in the material phase etc., in the gravitational field. Of course, the buoyancy force can be created by other fields such as centrifugal or coriolis forces. In situations of this type the flow and the temperature fields are coupled and have to be considered simultaneously. However, the velocities are relatively small. In recent years the free convection flows have drawn much attention. Though Prandtl’s boundary layer theory was originally developed for flows with high Reynolds number, interestingly
this concept can be effectively employed to describe free convection flow over rigid surfaces, where the flow is weak.

Methods of Solution

Though the boundary layer equations are simplified version of Navier-Stokes equations are simplified version of Navier-Stokes equations yet they are quite complex in the sense that they are coupled, non-linear partial differential equations. Consequently, considerable mathematical difficulties are associated with them. There are only few special cases, where the analytical solutions are possible, if the discussions are restricted to steady, incompressible and constant property flows and hence of little practical use. Consequently, numerous accurate numerical methods have been developed for solving the complete set of boundary layer equations. With the advent of large digital computers, exact and efficient numerical methods are being widely used and more complex problems have been and are being attempted.

The most widely used numerical method for solving boundary value problems is the method of finite differences. In this class, usually a rectangular grid is placed on the flow field across and the flow quantities are to be calculated only at the nodes of the grid. The system be calculated only at the nodes of the grid. The system of partial differential equations is converted into a system of algebraic equations by approximating each derivative by an appropriate difference quotient. The boundary conditions are incorporated to ensure the consistency of the algebraic equation.
Consequently, the object is to see that the algebraic equations are solved efficiently and accurately. The finite difference method can be implicit or explicit. Implicit schemes are usually unconditionally stable. A discussion on the stability, convergence properties and relative merits of different finite-difference schemes can be found in Issaeson and Keller (1966), Keller (1968) and Ames (1969). Sometimes difficulties are faced when one of the boundaries is at infinity. As the finite difference method requires finite number of grid points, we overcome this difficulty by following any of the two ways. First one is to take a large and finite interval and assume it to be infinite. If the further increase in the interval size brings no change in the computed solution, the solution is taken to be the final one. The second way is to transform the infinite interval to a finite one (see Sills (1969)). Several finite difference methods have been used to solve the parabolic, boundary layer equations and notable among them being the methods due to Flugge - Lotz and Blotner (1962), Marvin and Sheaffer (1969), Crank-Nicolson (1947) and the implicit box-scheme due to Keller (Keller (1970), Keller and Cebecci (1971) and Cebecci and Bradshaw (1984)).

Downstream solution to differential equations, which describe the system, Bodoia and Osterle [19] solved the entry problem by means of a finite-difference procedure. Hartmann[45] determined the fully developed velocity profile for a conducting fluid, which has the same conditions for Poiseuille flow, imposed on its motion, except for the addition of magnetic field applied across the channel.
Nagarajan [64] obtained the velocity and temperature profiles in the entrance region of a plane channel for laminar flow between two parallel plates.

Fluid transport systems composed of pipes or ducts have many practical applications in chemical plants and oil refineries, city water supply system and biofluid mechanism which one seem to be maze of pipes. Power plants contain many pipes and ducts for transporting fluid involved in the energy-conservation process. Uflyand [101] and Cheknarar [27] analyzed the time transient case of this problem, which implies a development time, rather than a development length. The entry problem for this case is solved by a technique similar to that of Shobst, et. Al.[87], which was used in the solution of the entry problem for a plane channel.

Flow and heat transfer with in porous media are of great practical interest. Applications include chemical reactors, thermal storage systems, thermal insulation, petroleum reservoirs, nuclear waste, etc. A considerable number of publications are now available for the problems of free convection in porous media.

Although many solutions are available for fully developed flows, only a few works deal with the development problem. Schlichting [85], following the original Blasius [17] technique for a flat plate, obtained the velocity profile for the entrance region by means of a marching process between an upstream and a downstream solution to the differential equations, which describe the system. Bodoia and Osterle [19] solved the entry problem by means of a finite-difference procedure.
Hartmann [45] determined the fully developed velocity profile for a conducting fluid, which has the same conditions for poiseuille flow, imposed on its motion, except for the addition of magnetic field applied across the channel. Nagarajan [64] obtained the velocity and temperature profiles in the entrance region of a plane channel for laminar flow between two parallel plates.

Inghan [47] presented the analysis for study free convective flow past a semi-infinite vertical flat plates at large Grashoff number. Arthur et.al [6] studied the developing natural convection in an asymmetrically heat open ended vertical channels. Dash and Das [34] presented a study of the free convective flow of a viscous incompressible liquid past a hot vertical plate wall with transverse sinusoidal suction in the presence of heated source.

Acharya and Pady [1] discussed the free convective flow of a viscous fluid past a vertical plate with constant porosity assuming the temperature to the spanwise cosinusoidal. Sahoo and Sahoo [82] obtained an exact analysis of the effect of natural convection and mass transfer of the flow past a uniformly accelerate permeable vertical plate in the presence of uniform magnetic field.

Wilks[111] studied the MHD free convection about a semi-infinite vertical plate in a strong cross-field. Wilks and Hunt [112] studied numerically the natural convection flow about a heated semi-infinite horizontal flat plate in the presence of a strong cross magnetic field. Fully developed MHD free convection flow of a viscous
electrically conducting fluid in a vertical parallel plates was studied by Hughes and Young [46]. Yang and Yu [117] considered the free and forced convection MHD flow between two vertical plates.

A numerical and experimental investigation of the developing laminar free convection heat transfer flow in vertical parallel plate channels with asymmetric heating was considered and they obtained a closed form fully developed solutions by Aung et al. [8].

The channel walls were maintained either at uniform heat fluxes or uniform wall temperatures. Bodea and Osterle [20] examined the effect of free convection on fully developed flow with symmetric wall heating at constant temperature between vertical plates. Anand et al [4] presented the numerical results for the effect of wall conduction on forced convection between asymmetrically heated vertical plates to uniform wall temperatures.

Vajravelu [106] examined the free convection heat transfer effects on the flow of a viscous fluid confined between two long parallel vertical plates moving with equal velocities but in opposite directions and when the plates are kept at uniform temperature. Natural convection effects on the flow of a viscous fluid in an open ended vertical channel were examined by Sparrow et al [94].

Yao [118] present an analysis of combined convection in a channel with symmetric uniform temperature and uniform flux heating. The solutions, valid in the
developing flow region, reveal fundamental information different length scales that distinguish various convective mechanisms traversed by the fluid before reaching the fully developed state.

An analysis of free convection flow and heat transfer about a flat plate parallel to the direction of the generating body force was made by Ostrach [66]. Jayaprakash Narayana and Uberoi [50] studied the laminar combined forced and free convection heat transfer from a vertical thin needle in a variable external stream. The effect of combined free and forced convection heat transfer in channels was studied by Tao [99]. Zeldin and Schmidt [122] made a numerical and experimental study of developing mixed convection flow in a vertical tube.

The introduction of the boundary layer approximation in the porous media literature has intensified research efforts on the analytical studies of convective heat transfer about heated surfaces in porous media Cheng [29]. One of the most basic problems in natural convection heat transfer is buoyant boundary layer flow along a heated vertical wall adjacent to a fluid-saturated porous medium. This basic problems was analyzed for the first time by Cheng and Minkowycz[30] who relies on the theoretical frame work provided by boundary layer theory. The same phenomenon has been studied in related configurations such as the flow on the out side of a vertical cylindrical surface imbedded a porous medium Minkowycz, and Cheng,[62 ], or the flow along the inner wall of cylindrical well filled with porous material Bejan., [13 ].

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The time-dependent boundary layer flow triggered by the sudden imposition of a temperature difference between a wall and fluid-saturated medium was studied by Ingham et al. [48].

Among the practical considerations that stimulate the continuing interest in this flow is the engineering of efficient thermal insulation systems. This is why the vertical boundary layer concept has been used consistently in the analytical treatment of natural convection in an enclosed space filled with porous medium. The boundary layer treatment of enclosed porous layers begins with Weber's paper [110] and continued with the works of Simpkins and Blythe [89, 90], Blythe et al. [18], Bejan [12] and Bergholz [16].

The Darcy model (which assumes proportionally between the velocity and pressure gradient has been extensively used to investigate a number of interesting fluid and heat transfer problems associated with heated bodies embedded in fluid-saturated porous media (e.g., Wooding [113]; Cheng [29]; Cheng and Minkowycz [30]). The model, however, is valid only for slow flows through porous media with low permeability. Muskat [63] added a velocity square term (known as the Forchheimer term) to account for the porous inertia effect on the pressure drop, while Brinkman [21] introduced a viscous diffusion term to consider the boundary frictional drag on impermeable walls.
Kimura and Bejan [53] have analyzed natural convection in a stably heated corner where Darcy flow is driven by competing thermal gradients. Puniyatma Sing et al. [74] have analyzed the free convection boundary layer flow along a vertical surface in a porous medium by employing a generalized equation of Darcy's law in which the convection term is taken into account. The temperature of the wall and the permeability of the porous media is constant. Recently, the problems related to double-diffusive flow which combine heat and mass transfer have been addressed by Bejan and Khair [14] and Kumari et al. [55]. Prasad et al. [73] have experimentally studied convection in a vertical porous annulus comprising various combinations of ball sizes and fluids.

Problems involving vertical porous layers bounded by parallel wall have been extensively studied and a detailed review has been written by Cheng [29]. Cheng and Minkowycz and their associates [30,51,62] have studied the free convection boundary layer flows by using Darcy's law as the momentum equation. Plumb and Huenefeld [69] have analyzed the buoyancy induced boundary layer adjacent to a vertical heated surface using a non-Darcy flow model. Darcy's law is considered to be valid for low speed flows, whereas the speed in the filter is not always small and the convective force may be important. Yamamoto, and Iwamura, [116].

Soundalgekar [92] has derived an exact solution for the flow past an impulsively started infinite vertical plate in its own plane known as the Stoke's
problem for the vertical plate. Recently Pop and Soundalgekar [71], Raptis [75] and Georgantopoulos et al. [40] have studied the free convection flow past an accelerated vertical infinite plate. The effect of mass transform on the flow past a uniformly accelerated vertical plate which is either at uniform temperature or its supplied heat at constant rate is studied by Soundalgekar [93].

Treating a fluid-saturated porous medium as a continuum, Vafai and Tien [104] integrated the momentum equation for a fluid over a local control volume, and derived a volume averaged momentum equation. This generalized momentum equation for non-Darcy flows reveals the importance of the convective inertia term for highly porous materials. Subsequently, Chen et al. [28] evoked the boundary layer approximations and solved the generalized equation to investigate free convection from a vertical flat plate in highly porous medium, while the corresponding forced convection problems was treated by Kaviany [52], who employed the finite difference calculation procedure developed by Cebeci and Bradshaw [24]. Kaviany also employed the Karman-Pohlhansen integral relation and obtained useful asymptotic expressions for the local Nusselt number. He, however, dropped the Forchheimer term in his finite difference calculations.

Laminar free convection through a channel in the absence of porous material has been studied extensively by many authors like Sparrow et al. [96], Ostrach [66], but the same in the presence of a porous medium has not been given much attention.
The fully developed free convection in a viscous fluid flow through a vertical stratum was investigated considering the Brinkman model [21].

Das and Kalita [33] discussed the two-dimensional steady MHD flow of a viscous, electrically conducting fluid past an infinite vertical porous plate. The induced magnetic field and the viscous and the joulean dissipations were considered. Krishna Prasad and Chandra [54] studied the low Reynold's number flow of a viscous incompressible fluid in channels of slowly varying cross section with permeable boundaries.

Ahmadi and Manvi [3] have derived a general equation of motion and applied the results obtained to some basic flow problems. Gulab Ram and Mishra [43] and Varshney [108] applied these equations to study the MHD flow of conduction fluid through porous media. Effects of free convection and mass transform flow through a porous medium are studied by Raptis et al.[77], Raptis [78] and Raptis et al.[76]. Pillai [68] studied the hydromagnetic boundary layer analysis for natural convection in vertical porous enclosures.

Tong and Subrahmanian [100] studied the effect of the no slip boundary condition on flows in two dimensional rectangular enclosures with sufficiently high to allow exhibition of boundary layer characteristics. By using modified Oseen method [41,110] the boundary layer equations were solved which were derived from

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the Brikman's extended model [21] were solved by comparing the results to those obtained from the Darcy model [47].

Sinha Roy et al. [91] studied the finite difference solution of the problem of viscous flow and heat transfer between two porous rotating disks. Yadav [115] studied the unsteady flow of viscous liquid between two co-axial porous circular cylinders. Sanyal and Bhattacharyya [83] studied free and forced convection flow of a conducting fluid through a vertical annulus.

Benjamin [15] was probably the first to consider the problem of a flow over a wavy wall. His analysis is based on the assumption that the basic flow is parallel. The method of conformal transformation has been used by Lyne [58] to investigate the steady streaming generated by an oscillatory viscous flow over a wavy wall under the assumption that the amplitude of the wave is smaller than the stokes layer thickness. Lassen and Gangwani [57] studied the flow stability under boundary layer approximation. Shankar and Sinha [86] have made detailed study of the Rayleigh problem for a wavy wall.

Capani [23] found the steady, laminar, incompressible flow over a periodic wavy surface with prescribed surface velocity distribution from the solution of the two dimensional Navier-Stokes equations. Patidar and Purohit [67] studied free convection flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls.

The nature of natural convection under a downward facing heated horizontal surface in a vast fluid-saturated porous medium is numerically studied by Angirasa and Peterson. [5] Fathiia Moh. Al Samman et al. [38] studied transient free convection flow of a viscous dissipative fluid with mass transfer past a semi-infinite vertical plane. Jat and Jiankal [49] studied three-dimensional free convective MHD flow and heat transfer through a porous medium.

Frederick [39] studied natural convection heat transfer in a cubical enclosure with two active sectors on one vertical wall. Aydin [9] studied dependance of the natural convection over a vertical flat plate in the presence of the ribs. Harris et al. [44] studied free convection from a vertical plate in a porous media subjected to a sudden change in surface temperature.


Michael Vylynycky [60,61,59] studied forced convection heat transfer from a flat plate, mixed convection due to a finite horizontal flat plate embedded in a porous media and conjugate free convection due to a leaded vertical plate.

Lee and Kuo [56] studied laminar flow in annuli ducts with constant wall temperature. Pop and Na [70] studied Darcian mixed convection along slender vertical cylinders with variable surface heat flux embedded in a porous medium. Pop and Yan [72] studied forced convection flow past a circular cylinder and a sphere in a darcian fluid at large Peclet numbers.


Yih [119] studied free convection effect on MHD coupled heat and mass transfer of a moving permeable vertical surface. Campo [22] studied bounds for the optimal conditions of forced convective flows inside multiple channels whose plates are heated by a uniform flux.


Giri et al. [42] studied combined natural convection heat and mass transfer from vertical fin arrays. Cherepanov [31] studied two-dimensional convective heat transfer in a vertical channel.
NOMENCLATURE:

\( \beta \) : Thermal expansion co-efficient

\( \sigma \) : Electrical Conductivity

\( \theta \) : Dimensionless temperature difference

\( \mu \) : Dynamic viscosity

\( \rho \) : Mass Density

\( \alpha \) : The thermal diffusivity

\( \mu_c \) : Magnetic permeability

\( A \) : Surface area

\( b \) : Spacing between plates

\( C_p \) : Specific heat at constant pressure

\( E_k \) : Eckert number

\( g \) : acceleration due to gravity

\( Gr \) : Grashoff number

\( H_0 \) : Applied magnetic field

\( J \) : Current density vector

\( K \) : Thermal Conductivity

\( K \) : Permeability parameter

\( L \) : Characteristic length in \( X \)-direction

\( M \) : Magnetic parameter

\( Nu \) : Nusselt number
\[ P \quad : \quad \text{Dimensionless pressure difference} \]
\[ p \quad : \quad \text{Pressure} \]
\[ P^1 \quad : \quad \text{Static pressure} \]
\[ Pr \quad : \quad \text{Prandtl number} \]
\[ \bar{q} \quad : \quad \text{Velocity Vector} (u,v,0) \]
\[ Re \quad : \quad \text{Reynolds number} \]
\[ \text{r}_{w} \quad : \quad \text{Ratio of wall temperature difference} = \frac{[T_{1}-T_{0}]}{[T_{2}-T_{0}]} \]
\[ T \quad : \quad \text{Temperature} \]
\[ T_{F} \quad : \quad \text{The free – stream temperature} \]
\[ T_{in} \quad : \quad \text{Input temperature} \]
\[ T_{w} \quad : \quad \text{Wall temperature} \]
\[ U \quad : \quad \text{Dimensionless streamwise velocity} \]
\[ u \quad : \quad \text{Axial velocity} \]
\[ \text{uw} \quad : \quad \text{Uniform input velocity} \]
\[ v \quad : \quad \text{Transverse velocity} \]
\[ V \quad : \quad \text{Dimensionless transverse velocity} \]
\[ X \quad : \quad \text{Stream wise distance from channel entrance} \]
\[ X \quad : \quad \text{Dimensionless stream-wise distance from channel entrance} \]
\[ Y \quad : \quad \text{Dimensionless transverse co-ordinate} \]
\[ y \quad : \quad \text{Transverse co-ordinate} \]
\[ \nu \quad : \quad \text{Kinematic viscosity} \]
Subscripts

0 : Value at channel entrance (i.e., at $x=0$)
1 : Cold wall (i.e., value at $y=0$)
2 : Hot wall (i.e., value at $y=b$)
b : Bulk value
c : Value at center line