CHAPTER - 2

MATHEMATICAL FORMULATION
Consider the developing magnetohydrodynamic laminar free convection flow between vertical flat plates. The distance between the plates is taken as ‘2b’. Rectangular co-ordinate system is used. The fluid is taken to have a uniform vertical upward stream-wise velocity distribution at the channel entrance. Since the temperatures of the two walls may be different, resulting in an asymmetric heating. A uniform transverse magnetic field of strength $H_0$ is applied perpendicular to the walls (i.e., in the y-direction).

![Diagram of flow configuration](image-url)

**Fig. 2.1.1. Configuration of flow**
The governing equations are:

\[ \rho \frac{Dq}{Dt} = -\nabla p + \mu \nabla^2 q + \mu \epsilon \mathbf{J} \times \mathbf{H} - \rho \mathbf{g} \] \hspace{1cm} (2.1.1)

\[ \nabla \cdot q = 0 \] \hspace{1cm} (2.1.2)

\[ \rho C_p [q \cdot \nabla] T = k \nabla^2 T + \phi \] \hspace{1cm} (2.1.3)

Equation of state is

\[ \rho = \rho_0 [1 - \beta (T - T_0)] \]

where \( \phi \) \hspace{1cm} Dissipation function

\[ = \mu \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]^2 \]

\[ + 2 \mu \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] \]

The Ohm's law is \( \mathbf{J} = \sigma \mathbf{E} + \mu \epsilon \mathbf{q} \times \mathbf{H} \)

Where \( \mathbf{q} = (u, v, 0) \) and \( \mathbf{H} = (0, H_0, 0) \)

The plates of maintained at a uniform temperature \( T_1 \) which exceeds ambient temperature \( T_0 \). Fluid rises between them by natural convection and is assumed to enter the channel and is assumed to enter the channel at \( T_0 \) with a flat velocity profile.
For moderate differences between $T_1$ and $T_n$, the flow is governed by the so-called "incompressible natural convention equations" expressing conservation of mass, momentum and energy.

The following assumptions are made in obtaining the governing equations of motion.

1. Flow is steady, laminar, viscous, incompressible and developed.
2. All the physical properties of the fluid are assumed to be constant.
3. The energy dissipation is neglected [79].
4. It is assumed that the electric field $\vec{E}$ and the induced magnetic field are neglected.

Using the above assumptions the governing equations become

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{-------------------(2.1.4)}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \frac{dp}{dx} - g - \frac{\sigma_{\mu} R^2 H_0^2 u}{\rho} \quad \text{-------(2.1.5)}
\]

\[
\frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \quad \text{-------------------(2.1.6)}
\]

It is customary to express the body force in terms of a buoyancy force. Define

\[
p^b = p - p_0 \quad \text{-------------------(2.1.7)}
\]
where, $P_0$ is the pressure which would obtain at a particular elevation in the channel if the temperature is uniform at $T_0$ throughout the fluid flow.

Noting that

$$\frac{dp_0}{dx} = -P_0 g \quad \text{------------------(2.1.8)}$$

Equation (2.1.5) can be written as,

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \frac{dp}{dx} + g \beta (T - T_0) - \frac{\mu}{\rho} \frac{\partial^2 u}{\partial x^2} \quad \text{------------------(2.1.9)}$$

where $\beta$ is the exansivity of the fluid defined by

$$\beta = - \frac{1}{\rho} \left[ \frac{\partial P}{\partial T} \right]_p \quad \text{------------------(2.1.10)}$$

The boundary conditions are

At $x = 0$, $0 \leq y \leq b$; $u = U$, $v = 0$, $T = 0$, $P = 0$

At $x > 0$, $y = 0$; $\frac{\partial u}{\partial y} = 0$, $v = 0$, $\frac{\partial T}{\partial y} = 0 \quad \text{------------------(2.1.11)}$

At $x > 0$, $y = b$; $u = 0$, $v = 0$, $T = T_i$
The following non-dimensional variables are used to present the basic equations and boundary conditions in dimensionless form

\[
U = \frac{b^2 u}{l v G_r}, \quad \nu = \frac{b v}{v}
\]
\[
X = \frac{x}{l G_r}, \quad Y = \frac{y}{b}
\]
\[
\theta = \frac{(T - T_0)}{(T_i - T_0)} \quad P = \frac{p^2 b^4}{\rho l^2 v^2 Gr^3}
\]
\[
M^2 = \frac{\alpha \mu b^2 H a^2 b^2}{\rho v}
\]

where Gr is a Grashoff number, defined by

\[
G_r = \frac{\rho (T_i - T_0) b^4}{l v^2}
\]

Equation (2.1.4), (2.1.6) and (2.1.9) can be given in dimensionless form as follows:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad \text{-------------------(2.1.14)}
\]
\[
U \frac{\partial U}{\partial X} + \nu \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} - \frac{dP}{dX} + \theta - M^2 U \quad \text{-------------------(2.1.15)}
\]
\[
U \frac{\partial \theta}{\partial X} + \nu \frac{\partial \theta}{\partial Y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial Y^2} \quad \text{-------------------(2.1.16)}
\]
where the prandtl number, defined by

$$Pr = \frac{\mu c_p}{\kappa}$$

----------(2.1.17)

Then the boundary conditions in equations (2.1.14), (2.1.15), and (2.1.16) are:

for $X = 0, 0 \leq Y \leq 1; U = 1, \nu = 0, \theta = 0, \psi = 0$

for $X > 0, Y > 0; \frac{\partial U}{\partial Y} = 0, \nu = 0, \frac{\partial \theta}{\partial Y} = 0$

----------(2.1.18)

for $X > 0, Y = 1; U = 0, \nu = 0, \theta = 1$

where

$$r_T = \frac{(T_1 - T_0)}{(T_1 - T_0)}$$

----------(2.1.19)

To obtain a solution of the convection problem formulated above, an additional equation expressing the global conservation of mass at any cross section in the channel is also required.

This equation is taken as

$$\int_0^1 UdY = 1$$

----------(2.1.20)

The system of non-linear equations (2.1.14) to (2.1.16) are solved by a numerical method based on finite difference approximation. An implicit difference technique is employed where by the differential equations are transformed into a set of simultaneous linear algebraic equations.
Problem-II

Consider the laminar free and forced (mixed or combined) convection between two vertical plates. The distance between the plates is taken to be \( b \) in figure. (2.2.1) Rectangular co-ordinate system is used and the fluid having a uniform upward stream-wise velocity distribution at the channel entrance. The walls are heated at uniform wall temperature, but the temperature of the two walls may be different, resulting in an asymmetric heating situations. A uniform transverse magnetic field of strength \( H_0 \) is applied perpendicular to the walls (i.e., in the \( y \)-direction).

![Flow Configuration Diagram]

**Fig. 2.2.1: Flow configuration**
The governing equations for the steady viscous flow are:

\[ \rho \frac{Dq}{Dt} = -\nabla P + \mu \nabla^2 q + \mu_e \vec{J} \times \vec{H} - \vec{g} \]  
\[ \nabla \cdot q = 0 \]  
\[ \rho C_p \left[ \vec{q} \cdot \nabla \right] T = k \nabla^2 T + \phi \]

Where \( \phi = \text{Dissipation function} \)

\[ \phi = \mu \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]^2 + 2 \mu \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] \]

The Ohm's law is \( \vec{J} = \sigma \left[ \vec{E} + \mu_e \vec{q} \times \vec{H} \right] \)

Where \( \vec{q} = (u, v, 0) \) and \( \vec{H} = (0, H_p, 0) \)

The following assumptions are made:

1. Flow is steady, laminar, viscous, incompressible and developed.

2. The flow is assumed to be two-dimensional steady and the fluid properties are constant except for the variation of density in the buoyancy term of momentum equation.

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3. A uniform magnetic field $H_0$ is applied perpendicular to the walls (i.e., in the $Y$ direction)

4. It is assumed that the electric field $E$ and the induced magnetic field are neglected [79,95]

Using the above assumptions the governing equations become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} \text{(2.2.4)}

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \frac{dp}{dx} - \frac{1}{\rho} \frac{\sigma \mu_0^2 H_0^2 u}{\rho}$$  \hspace{1cm} \text{(2.2.5)}

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left[ \frac{\partial u}{\partial y} \right]^2$$  \hspace{1cm} \text{(2.2.6)}

The boundary conditions are:

At $x = 0$, $0 \leq y \leq b$: $u = u_0$, $v = 0$, $T = 0$, $P_1 = 0$

At $x > 0$, $y = 0$: $u = 0$, $v = 0$, $T = T_1$  \hspace{1cm} \text{(2.2.7)}

At $x > 0$, $y = b$: $u = 0$, $v = 0$, $T = T_2$
The following non-dimensional variables used are:

$$U = \frac{u}{u_n} \quad V = \frac{v}{v_n}$$

$$X = \frac{x}{bR_e} \quad Y = \frac{y}{b}$$

$$\theta = \frac{(T - T_0)}{(T_e - T_0)}$$

$$P = \frac{P_1}{\rho u_0^2}$$

$$P_r = \frac{\mu C_\nu}{K} \quad R_r = \frac{u_0 b}{v}$$

$$M^2 = \frac{\sigma u_0^2 H_e b^2}{\rho v}$$

Equations (2.2.4), (2.2.5) and (2.2.6) are placed in dimensionless form yielding,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad \text{(2.2.9)}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} \frac{dP}{dX} - M^2 U \quad \text{(2.2.10)}$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{p_r} \frac{\partial^2 \theta}{\partial Y^2} + F \left[ \frac{\partial U}{\partial Y} \right]^2 \quad \text{(2.2.11)}$$
Then the Boundary Conditions will be,

At $X = 0$, $0 \leq Y \leq 1$ : \hspace{1em} U = 1, \hspace{0.5em} V = 0, \hspace{0.5em} \theta = 0, \hspace{0.5em} P = 0$

At $X > 0$, $Y = 0$ : \hspace{1em} U = 0, \hspace{0.5em} V = 0, \hspace{0.5em} \theta = r_T \hspace{1em} \text{----------(2.2.12)}$

At $X > 0$, $Y = 1$ : \hspace{1em} U = 0, \hspace{0.5em} V = 0, \hspace{0.5em} \theta = 1$

To obtain a solution of the convection problem formulated above, the additional equation

$$
\int U dY = 1 \hspace{2em} \text{----------(2.2.13)}
$$

expressing the global conservation of mass at any cross-section in the channel is also required. The system of non-linear equations (2.2.9) to (2.2.11) are solved by a numerical method based on finite difference approximation. An implicit difference technique is employed where by the differential equations are transformed into a set of simultaneous linear algebraic equations.
The motion of a laminar conducting fluid between two parallel plates is considered. The channel has a height of 2a and width of d. A rectangular co-ordinate system is used with the origin at the centre of the channel at the left-hand edge. The variable y increases in an upward direction and the variable z increases perpendicularly to both x and y so as to form a right hand co-ordinate system. The velocity and temperature profiles develop towards the direction of increasing x. A uniform magnetic field of strength $H_0$ is applied in the positive y-direction.

The governing equations are:

$$\frac{Dp}{Dt} + \rho \nabla \cdot \vec{q} = 0$$

$$\rho \frac{DT}{Dt} = -\nabla p + \mu \nabla^2 \vec{q} + \mu_e J \times \vec{B} + \vec{F}$$

Where $\vec{F}$ is the body force per unit volume vector $(X,Y,Z)$

$$\rho C_p \frac{DT}{Dt} = k \nabla^2 T + \mu \phi$$

Where $\phi$ = Dissipation function

$$\left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]^2 + 2 \left[ \frac{\partial u}{\partial x} \right]^2 + \left[ \frac{\partial v}{\partial y} \right]^2$$
The Ohm's law is 
\[ \mathbf{I} = \sigma \left[ \mathbf{E} + \mathbf{\mu}_e \mathbf{q} \times \mathbf{H} \right] \]

Where \( \mathbf{q} = (u, v, 0) \) and \( \mathbf{H} = (0, H_e, 0) \)

The following assumptions are made

1. Flow is steady, laminar, viscous, incompressible and developed.
2. It is assumed that electric field \( \mathbf{E} \) and induced magnetic field are neglected [79,95].
3. All the physical properties of the fluid are assumed to be constant.
4. It is assumed that body force is neglected.

Using the above assumptions the governing equations become:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

\[ \rho \left[ \mu \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} \right] = \frac{\partial^2 u}{\partial y^2} - \mu \frac{\partial p}{\partial x} - \sigma \mu_e^2 J H_e^2 u \]

\[ \rho C_v \left[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2} + \mu \left[ \frac{\partial u}{\partial y} \right]^2 \]
A set of dimensionless variables may now be introduced as follows:

\[ Y = \frac{y}{a} \quad V = \frac{U a}{\nu} \quad X = \frac{x \mu}{\rho u_0 a^2} \]

\[ \Pr = \frac{\mu C_p}{k} \quad F_i = \frac{\mu_i}{C_p(T_i - T_i)} \quad P = \frac{P}{\rho u_0^2} \]  \hspace{1cm} \text{-------------------(2.3.7)}

\[ 0 = \frac{(T - T_h)}{(T_i - T_h)} \quad M_i^2 = \frac{\sigma \mu_i^2 \rho_i^2}{\mu} \quad U = \frac{u}{u_0} \]

The determining equations are then:

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \]  \hspace{1cm} \text{-------------------(2.3.8)}

\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} - \frac{\partial P}{\partial X} - M_i^2 U \]  \hspace{1cm} \text{-------------------(2.3.9)}

\[ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{P} \frac{\partial^2 \theta}{\partial Y^2} + E_i \left[ \frac{\partial U}{\partial Y} \right]^2 \]  \hspace{1cm} \text{-------------------(2.3.10)}
Problem - IV

The motion of a laminar conducting fluid between the two insulating cylinders, which are concentric, is considered. The cylindrical co-ordinate system \((r, \theta, z)\) is used. The origin of the co-ordinate system is located at the extreme left of the channel along the center line of the cylinders. \(z\) is the co-ordinate which increases in the down stream direction, \(r\) is the radial co-ordinate and \(\theta\) is the angular co-ordinate and is perpendicular to the \((r, z)\) plane. Note that the origin of the co-ordinate system is not placed in the center of the region in which the fluid passes. A uniform magnetic field of strength \(H_0\) is applied in the radial direction. The inner radius of the channel is \(r_i\) and the outer radius is \(r_o\). The channel is of length \(L\), which must be long compared with the entry length.

Governing equations are

\[
\frac{Dp}{Dt} = -\frac{\partial p}{\partial t} + q \cdot V_p = 0 \tag{2.4.1}
\]

\[
\rho \frac{D\bar{q}}{Dt} = -\nabla p + \mu \nabla^2 \bar{q} + \mu_0 j \times H \tag{2.4.2}
\]

\[
\rho C_p \frac{DT}{Dt} = k \nabla^2 T + \phi \tag{2.4.3}
\]

Where \(\phi\) = Dissipation function

\[
\phi = \mu \left[ \frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \right]^2 + 2 \mu \left[ \frac{\partial V_r}{\partial r} \right]^2 + \left[ \frac{\partial V_z}{\partial z} \right]^2
\]
The Ohm's law is \( \vec{J} = \sigma \left[ \vec{E} + \mu_\perp \vec{q} \times \vec{H} \right] \)

Where \( \vec{q} = (q_x, q_y, q_z) \) and \( \vec{H} = (0, H_y, 0) \)

The following assumptions are made:

1. Flow is steady, laminar, viscous, incompressible and fully developed with no free charge present.

2. There are no applied (external) magnetic fields other than in the \( z \)-direction.

3. No motion in the \( \theta \) direction at any time.

4. Electric field \( \vec{E} \) and induced magnetic field are neglected \([79, 95]\).

5. All the physical properties of the fluid are assumed to be constant.

Using the above assumptions, the governing equations in cylindrical coordinates become:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r u_r \right) + \frac{\partial u_z}{\partial z} = 0 \\
\rho \left[ u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} \right] = -\frac{\partial p}{\partial z} \\
+ \mu \left[ \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} \right] - \frac{\sigma \mu_\perp H_0^2}{r^2} u_z
\]

\[\text{(2.4.4)}\]

\[\text{(2.4.5)}\]
A set of non-dimensional variables are now introduced as follows:

\[
\begin{align*}
Z &= \frac{v}{r_i \mu_i \rho} \\
v &= \frac{\rho_{i} \epsilon_{i}}{\mu} \\
R &= \frac{r}{r_{i}} \\
p &= \frac{p}{\rho \mu_{0}^{2}} \\
P_{r} &= \frac{\mu C_{p}}{k} \\
U &= \frac{u_{z}}{u_{0}} \\
T &= \frac{T - T_{w}}{T_{w} - T_{w_{0}}} \\
E_{k} &= \frac{u_{0}^{2}}{C_{p} (T_{w} - T_{w_{0}})} \\
M^2 &= \frac{\sigma \mu_{0}^{2} H_{w_{0}}^{2}}{\mu} 
\end{align*}
\]  

The determining equations are then:

\[
\begin{align*}
\frac{1}{R} \frac{\partial (VR)}{\partial R} + \frac{\partial U}{\partial Z} &= 0 \\
V \frac{\partial U}{\partial R} + U \frac{\partial U}{\partial Z} &= -\frac{\partial P}{\partial Z} + \frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} - \frac{M^2 U}{R^2} \\
V \frac{\partial T}{\partial R} + U \frac{\partial T}{\partial Z} &= \frac{1}{Pr} \left[ \frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} \right] + E_{k} \left[ \frac{\partial U}{\partial R} \right]^2 
\end{align*}
\]

It should be noted at this point that it is assumed the input values of velocity and temperature are assumed to be constant every where to the left of the channel.
Problem V

Consider magnetohydrodynamic free convection flow along a vertical wall in a porous medium. The two-dimensional co-ordinate system is used. The magnetic field is applied perpendicular to the rectangular channel.

The governing equations for the steady, viscous in compressible flow of an electrically conduction fluid for the Brinkman-extended Darcy model are:

\[ \rho \frac{D\tilde{q}}{Dt} = -\nabla p + \mu \nabla^2 \tilde{q} + \mu_0 \tilde{J} \times \tilde{H} + \rho g - \frac{\mu q}{k_1} \]  \hspace{1cm} (2.5.1)

The Ohm's law \( \tilde{J} = \sigma \left[ \tilde{E} + \mu_0 \tilde{q} \times \tilde{H} \right] \)  \hspace{1cm} (2.5.2)

\[ \nabla \cdot \tilde{q} = 0 \]  \hspace{1cm} (2.5.3)

\[ \nabla \cdot \tilde{H} = 0 \]  \hspace{1cm} (2.5.4)

\[ \nabla \times \tilde{E} = 0 \]  \hspace{1cm} (2.5.5)

\[ \rho C_p \left[ \tilde{q} \cdot \nabla \right] T = k \nabla^2 T + \phi \]  \hspace{1cm} (2.5.6)
where $\phi = \text{Dissipation function}$

$$= \mu \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]^2 + 2 \mu \left[ \frac{\partial u}{\partial x} \right]^2 + \left[ \frac{\partial v}{\partial y} \right]^2 \right]$$

Where $q = (u, v, 0)$ and $H = (0, H_0, 0)$

The following assumptions are made:

1. The flow is only in $x$ and $y$ directions.
2. The electric field $E$, and induced magnetic field are neglected [79,95].
3. The energy dissipation is neglected.
4. Pressure term will be neglected.
5. The Boussinesq approximations has been used for the buoyancy term and constant properties have been assumed.

Introducing the above assumptions, the equations (2.5.1) to (2.5.6) for the Brinkman extended Darcy model are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{(2.5.7)}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta (T - T_{\infty}) \frac{nu}{k_i} + \frac{\mu H_0^2 u}{\rho} \quad \text{(2.5.8)}$$
\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \]  \hfill (2.5.9)

The Boundary conditions are:

\begin{align*}
\text{At} \quad x = 0 & \quad : u = 0, \ T = T_w \\
\text{At} \quad y = 0 & \quad : u = 0, \ v = 0, \ T = T_w \\
\text{At} \quad y \to \infty & \quad : u = 0, \ T = T_\infty
\end{align*}

The equations (2.5.7) to (2.5.9) and boundary conditions (2.5.10) are put in dimensionless form by using the following transformations:

\[ U = \frac{v u}{(L^2 g \beta (T_w - T_\infty))} \quad K = \frac{k_1}{L^2} \quad Y \equiv \frac{v}{L} \]

\[ \psi = \frac{v L}{v} \quad X = \frac{\psi^2 x}{(g \beta (T_w - T_\infty)) L^4} \]  \hfill (2.5.11)

\[ 0 = \frac{\psi}{(T - T_\infty)} \left( \frac{T - T_\infty}{T_w - T_\infty} \right) \]
The non-dimensional form of the governing equations are:

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \]  

\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} - \frac{U}{\kappa} + \Theta - M^2 U \]  

\[ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{P} \frac{\partial^2 \theta}{\partial Y^2} \]  

Where

\[ M^2 = \frac{\sigma \mu \epsilon^2 H \theta^3 L^2}{\mu} \]

The boundary conditions are

\[ U(X,0) = U(X,\infty) = U(0,Y) = V(X,0) = 0 \]

\[ \theta(X,0) = 1, \theta(X,\infty) = \theta(0,Y) = 0 \]