Problem-1

Writing equations (2.1.14), (2.1.15) and (2.1.16) in finite difference form and applying them to the j,k mesh point of a rectangular grid super imposed on the half-channel flow field (fig.3.1.2) there results.

\[
\frac{U(J + 1, k) + U(J + 1, k - 1) - 2U(j, k) - U(J, k - 1)}{2\Delta X} + \frac{V(J + 1, k) - V(J + 1, k - 1)}{\Delta Y} = 0
\]

----------------------------- (3.1.1)

\[
U(j, k) \frac{U(J + 1, k) - U(j, k)}{\Delta X} + V(j, k) \frac{U(J + 1, k + 1) - U(J + 1, k - 1)}{2\Delta Y}
\]

\[
= \frac{U(J + 1, k + 1) - 2U(j + 1, k) + U(j + 1, k - 1)}{(\Delta Y)^2} + \frac{P(j + 1) - P(j)}{\Delta X} + \Theta(j + 1, k) - \Theta(j + 1, k - 1)
\]

----------------------------- (3.1.2)

\[
U(j, k) \frac{\Theta(J + 1, k) - \Theta(j, k)}{\Delta X} + V(j, k) \frac{\Theta(J + 1, k + 1) - \Theta(j + 1, k - 1)}{2\Delta Y}
\]

\[
= \frac{1}{\rho_v} \frac{\Theta(J + 1, k + 1) - 2\Theta(j + 1, k) + \Theta(j + 1, k - 1)}{(\Delta Y)^2}
\]

----------------------------- (3.1.3)
Equation (2.1.20) can also be written in finite difference form by application of the Simpson's 1/3 rule yielding

\[
U(j + 1, 0) + 4[U(j + 1, 1) + U(j + 1, 3) + \cdots + U(j + 1, n)]
+ 2[U(j + 1, 2) + U(j + 1, 4) + \cdots + U(j + 1, n - 1)] = 3(n + 1) \quad \cdots (3.1.4)
\]

Equations (3.1.2) and (3.1.3) can be written as

\[
\beta_n U(j + 1, 0) + (\alpha_0 + \gamma_0) U(j + 1, 1) + \cdots + \Theta(j + 1, 0) + n p(j + 1) = \Phi_0
\]

\[
\alpha_1 U(j + 1, 0) + \beta_1 U(j + 1, 1) + \gamma_1(j + 1, 2) + \cdots + \Theta(j + 1, 0) + n p(j + 1) = \Phi_1
\]

\[
\alpha_N U(j + 1, N - 1) + \beta_N U(j + 1, N) + \cdots + \Theta(j + 1, 0) + n p(j + 1) = \Phi_N
\]

\[
\beta_n \Theta(j + 1, 0) + (\alpha_0 + \gamma_0) \Theta(j + 1, 1) + \cdots = \Phi_0
\]

\[
\alpha_1 \Theta(j + 1, 0) + \beta_1 \Theta(j + 1, 1) + \gamma_1 \Theta(j + 1, 2) = \Phi_1
\]

\[
\alpha_N \Theta(j + 1, N - 1) + \beta_N \Theta(j + 1, N) = \Phi_N - \gamma_N
\]
where

\[ \alpha_k = \frac{1}{(\Delta y)^2} + \frac{V(j, k)}{2 \Delta y} \]

\[ \beta_k = -\left[ \frac{2}{(\Delta y)^2} + M^2 + \frac{U(j, k)}{\Delta x} \right] \]

\[ \gamma_k = \frac{1}{(\Delta y)^2} - \frac{V(j, k)}{2 \Delta y} \]

\[ n = \frac{-1}{\Delta x} \]

\[ \Phi_k = -\left[ \frac{P(j) + U^2(j, k)}{\Delta x} \right] \]

\[ \overline{\alpha_k} = \frac{1}{P_r(\Delta y)^2} + \frac{V(j, k)}{2 \Delta y} \]

\[ \overline{\beta_k} = -\left[ \frac{2}{P_r(\Delta y)^2} + \frac{V(j, k)}{\Delta x} \right] \]

\[ \gamma_k = \frac{2}{P_r(\Delta y)^2} + \frac{V(j, k)}{2 \Delta y} \]

\[ \phi_k = -\left[ \frac{U(j, k) + U(j, k)}{\Delta x} \right] \]
The solution of the difference equations are obtained by first selecting some specific values for \( P \) and \( r_r \). The mean of marching procedure is used and the variables \( U, V, \theta \) and \( P \) for each row, beginning at row \( j+1 = 2 \) are obtained using the values at previous row \( j \). Thus by approaching equations to the points 1,2,3...n on row \( j \), \((2n+1)\) algebraic equations with \((2n+1)\) unknowns are obtained.

\[
U (J+1, 1), U (J+1, 2), ..., U (J+1, N), P (J+1)
\]

\[
\theta (J+1, 1), \theta (J+1, 2), ..., \theta (J+1, N)
\]

These equations are be solved by Gauss Elimination method.

**NUMERICAL DISCUSSION:**

In the present study, quantitative information on the effects of buoyancy and asymmetric heating have been obtained for \( P_r = 0.72 \). Since parabolic partial differential equations have been utilised in the present investigation, the solutions immediately down the stream of separation are not examined in great detail. Boundary layer equations have been traditionally applied, in external flow situations up to the point of separation but not beyond [84]. However, several studies have been reported where boundary layer equations, with the neglect of the stream-wise convective term, have been used to obtain quantitative or qualitative information across the point of separation in duct flow.

For a channel with symmetric heating at uniform wall temperature ( \( r_r = 1 \) ). This stream-wise variation of the centre line velocity is indicated in figure (1.4) it can
be seen that buoyancy effects are felt very close to the channel entrance. We observe that when X value increases, centre line velocity decreases.

Figure (1.6) shows the variation of pressure with stream-wise distance for fixed \( r_T = 1 \). We observe that the magnetic field M increases when the pressure value (P) decreases near the wall at \( Y = 0 \). This effect is illustrated in figure (1.3). The development of temperature field is illustrated in figure (1.5).

**Velocity for fixed** \( r_T = 0.5 \) and \( X = 0.05 \)

<table>
<thead>
<tr>
<th>Y</th>
<th>( M = 0 )</th>
<th>( M = 1 )</th>
<th>( M = 3 )</th>
<th>( M = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.37</td>
<td>1.36</td>
<td>1.303</td>
<td>1.226</td>
</tr>
<tr>
<td>0.1</td>
<td>1.367</td>
<td>1.358</td>
<td>1.299</td>
<td>1.224</td>
</tr>
<tr>
<td>0.2</td>
<td>1.35</td>
<td>1.342</td>
<td>1.289</td>
<td>1.220</td>
</tr>
<tr>
<td>0.3</td>
<td>1.315</td>
<td>1.309</td>
<td>1.267</td>
<td>1.209</td>
</tr>
<tr>
<td>0.4</td>
<td>1.255</td>
<td>1.252</td>
<td>1.227</td>
<td>1.189</td>
</tr>
<tr>
<td>0.5</td>
<td>1.162</td>
<td>1.162</td>
<td>1.161</td>
<td>1.151</td>
</tr>
<tr>
<td>0.6</td>
<td>1.028</td>
<td>1.032</td>
<td>1.058</td>
<td>1.084</td>
</tr>
<tr>
<td>0.7</td>
<td>0.848</td>
<td>0.856</td>
<td>0.907</td>
<td>0.973</td>
</tr>
<tr>
<td>0.8</td>
<td>0.620</td>
<td>0.629</td>
<td>0.696</td>
<td>0.791</td>
</tr>
<tr>
<td>0.9</td>
<td>0.339</td>
<td>0.347</td>
<td>0.405</td>
<td>0.494</td>
</tr>
<tr>
<td>1.0</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Temperature values for fixed $r_T = 0.5$ and $X = 0.05$

Table - II

<table>
<thead>
<tr>
<th>Y</th>
<th>M = 0</th>
<th>M = 1</th>
<th>M = 3</th>
<th>M = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.042</td>
<td>0.042</td>
<td>0.041</td>
<td>0.039</td>
</tr>
<tr>
<td>0.1</td>
<td>0.050</td>
<td>0.050</td>
<td>0.048</td>
<td>0.045</td>
</tr>
<tr>
<td>0.2</td>
<td>0.073</td>
<td>0.073</td>
<td>0.069</td>
<td>0.065</td>
</tr>
<tr>
<td>0.3</td>
<td>0.116</td>
<td>0.115</td>
<td>0.109</td>
<td>0.101</td>
</tr>
<tr>
<td>0.4</td>
<td>0.181</td>
<td>0.180</td>
<td>0.170</td>
<td>0.158</td>
</tr>
<tr>
<td>0.5</td>
<td>0.272</td>
<td>0.270</td>
<td>0.257</td>
<td>0.240</td>
</tr>
<tr>
<td>0.6</td>
<td>0.388</td>
<td>0.386</td>
<td>0.371</td>
<td>0.345</td>
</tr>
<tr>
<td>0.7</td>
<td>0.526</td>
<td>0.524</td>
<td>0.509</td>
<td>0.488</td>
</tr>
<tr>
<td>0.8</td>
<td>0.679</td>
<td>0.677</td>
<td>0.665</td>
<td>0.648</td>
</tr>
<tr>
<td>0.9</td>
<td>0.839</td>
<td>0.838</td>
<td>0.831</td>
<td>0.822</td>
</tr>
<tr>
<td>1.0</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

From Table - I, we observe that the velocity decreases with increase in $M$ for fixed $Y$ up to $Y = 0.5$. For $Y > 0.5$ the value of velocity increases with increase in $M$.

From Table - II, we observe that the temperature decreases with increase in $M$. 

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Fig. 3.1.2: Mesh Network for different representations
Fig 1.3(a) Stream wise Velocity distribution for fixed $M=0$ & $r_1=1$
Fig 1.3(c) Stream wise Velocity distribution for fixed $M=3$ & $r_1=1$
Fig 1.3(d): Streamwise velocity distribution for fixed $M=5$ & $\Gamma_1=1$
Fig 1. 4 streamwise variation of the centreline velocity for symmetric heating \( r_T = 1.0 \)
Fig 1.5(b) Dimensionless Temperature distribution at r=0.5, M=1
Fig 1.5(d) Dimensionless Temperature distribution at $r_1=0.5$ & $M=0$
Fig 1.6 (a) Variation of Pressure with streamwise distance for fixed $r_f=1.0$
Fig 1.6 (b) Variation of Pressure with streamwise distance for fixed $r_1=1.0$
Problem - [1]

The system of non-linear equations (2.2.9) to (2.2.11) are solved by a numerical method based on finite difference approximation. An implicit difference technique is employed where by the differential equations are transformed into a set of simultaneous linear algebraic equations.

Following the methods of Bodola and Osterle [19] a variable mesh net work is introduced across the range of the problem as shown in figure 3.2.2. The finite difference approximation to the derivatives (2.2.10), (2.2.11) & (2.2.9) are as follows:

\[
U(j,k)\frac{U(J+1,k)-U(J,k)}{\Delta X} + V(j,k)\frac{U(J+1,k+1)-U(J+1,k-1)}{2\Delta y}
\]

\[
= \frac{U(J+1,k+1)-2U(J+1,k)+U(J+1,k-1)}{(\Delta Y)^2}
\]

\[-\frac{P(j+1)-P(j)}{\Delta X} - M^2 U(j+1,k) \]

----------------------------- (3.2.1)
The finite difference approximations are not perfectly symmetrical nor are they of the same form in all equations. This is necessary to ensure stability of the computer solution and to enable the equations to be coupled momentum and energy equations. All of these forms approach the real derivative if a small mesh spacing is used. From the nature of the last three expressions it can be seen that (2.2.11) is the only expression involving the temperature and therefore it may be solved separately from (2.2.10) and (2.2.9). By determining an additional involving only unknowns which appear in (2.2.10) this equation may be uncoupled from (2.2.9) and solved separately.

The equation (2.2.13) and the simultaneous equations (2.2.10) taken about each mesh point column wise, using Simpson's 1/3 rule give the following equations
\[ 4 \left[ U(j+1, 1) + U(j+1, 3) + \cdots + U(j+1, n) \right] \\
12 \left[ U(j+1, 2) + U(j+1, 4) + \cdots + U(j+1, n-1) \right] = 3(n+1) \]

\[ \beta_1 U(j+1, 1) + \gamma_1 U(j+1, 2) + \cdots + n p(j+1) = \Phi_1 \]

\[ \alpha_2 U(j+1, 1) + \beta_2 U(j+1, 2) + \gamma_2 U(j+1, 3) + \cdots + n p(j+1) = \Phi_2 \]

\[ \alpha_N U(j+1, N-1) + \beta_N U(j+1, N) + n p(j+1) = \Phi_N \]

where

\[ \alpha_k = \frac{1}{(\Delta y)^2} + \frac{V(j, k)}{2 \Delta y} \]

\[ \beta_k = -\left[ \frac{2}{(\Delta y)^2} + M^2 + \frac{U(j, k)}{\Delta x} \right] \]

\[ \gamma_k = \frac{1}{(\Delta y)^2} - \frac{V(j, k)}{2 \Delta y} \]

\[ n = \frac{-1}{\Delta x} \]

\[ \Phi_k = -\left[ \frac{P(J) + U^2(j, k)}{\Delta x} \right] \]
The set of equations are solved by Gauss-Jordan method. Application of the newly found axial velocities \((U's)\) together with those in equation (2.2.9) to determine the new transverse velocities \((V's)\).

The set of velocities are now placed into a set of the finite difference equations written about each mesh point in a column for equation (2.2.11) are

\[
\begin{align*}
\beta \theta (j + 1, 1) + \gamma_1 \theta (j + 1, 2) + \cdots &= \Phi_j - \alpha_j \\
\alpha_2 \theta (j + 1, 1) + \beta_2 \theta (j + 1, 2) + \gamma_2 \theta (j + 1, 3) &= \Phi_j \\
\vdots \quad \vdots \quad \vdots \ &= \vdots \\
\alpha_N \theta (j + 1, N - 1) + \beta_N \theta (j + 1, N) &= \Phi_j - \gamma_N
\end{align*}
\]

where

\[
\alpha_k = \frac{1}{P_r(\Delta y)^2} + \frac{V(j, k)}{2\Delta y}
\]

\[
\beta_k = -\left[\frac{2}{P_r(\Delta y)^2} + \frac{U(j, k)}{\Delta x}\right]
\]
\[ \gamma_k = \frac{1}{P_r (\Delta y)^2} \cdot \frac{V(j, k)}{2\Delta y} \]

\[ \Phi_k = \left[ \frac{U(j, k) - \Theta(j, k)}{\Delta x} \right] \]

The technique used in problem 1 is used to solve this set of equations.

**Numerical Discussion**

In the present study quantitative information on the asymmetric heating have been obtained for \( P_r = 0.72 \). Since parabolic partial differential equations have been utilised in the present investigation the solutions immediately downstream of separation. Boundary layer equations have been traditionally applied, in external flow situations up to the point of separation but not behind; see for instance, Schlichting [84]. However, several studies have been reported where boundary layer equations, with the neglect of the stream wise convective term, have been used to obtained quantitative or qualitative information across the point of separation in that flow. This approach was not used in the present study.

For a channel with a symmetric heating at uniform wall temperature (\( r_T = 1 \)), the stream wise variation of one centre line velocity is indicated in figures (2.3).
A sequence in the velocity profile also appears as the fluid moves towards the hot wall. The development of the temperature profile is explained by figure (2.6).

The axial variation of the bulk temperature for \( r_f = 1 \) is displayed in figures (2.7), the bulk temperature is defined as

\[
\theta_b = \frac{\int_0^1 U \delta dy}{\int_0^1 U dy}
\]

Figures (2.8) shows the variation of the dimensionless pressure parameter for \( r_f = 1 \).
Fig. 3.2.2: Mesh network for different representations
Fig 2.3 Central Line Axial Velocity for $r_t=1$ & $E_k=1$
Fig 2.4 (a) Velocity Values for $r_t=0.5, \lambda=0.04$ & $E_x=1$
Fig 2.4 (b) Velocity Values for $r_1=0.5, X=0.04 & E_x=10$
Fig 2.5(a) Velocity values for fixed $M=1, r_T=1$ and $E_x=1$
Fig 2.5(c) velocity values for fixed $M=5, r=1$ & $E_x=1$
Fig 2.6(a) Temperature Values for $M=0$, $r_T=0.5$ & $E_x=1$
Fig 2.6 (b) Temperature Values for $M=1, r_f=0.5 \& E_x=1$
Fig 2.6 (d) Temperature Values for $M=5$, $r_1=0.5$ & $E_k=1$
Problem - III

Following the method of Bodoia and Osterle [19], a variable mesh network is introduced across the range of the problem as shown in figure (2.3.1). The finite difference approximation to the derivatives (2.3.8) and (2.3.10) are shown as follows:

Continuity equation:

\[ \frac{\partial V}{\partial Y} = \frac{V(j+1,k+1) - V(j+1,k)}{\Delta Y} \]

\[ \frac{\partial U}{\partial X} = \frac{U(j+1,k+1) + U(j+1,k) - U(j,k+1) - U(j,k)}{2\Delta X} \]

Momentum equation:

\[ \frac{\partial U}{\partial X} = \frac{U(j+1,k) - U(j,k)}{\Delta X} \]

\[ \frac{\partial U}{\partial Y} = \frac{U(j+1,k+1) - U(j+1,k-1)}{2\Delta Y} \]

\[ \frac{\partial^2 U}{\partial Y^2} = \frac{U(j+1,k+1) - 2U(j+1,k) + U(j+1,k-1)}{(\Delta Y)^2} \]

\[ \frac{\partial P}{\partial X} = \frac{P(j+1) - P(j)}{\Delta X} \]
Energy equation:

\[
\frac{\partial \theta}{\partial X} = \frac{\theta(j+1,k)-\theta(j,k)}{\Delta X}
\]

\[
\frac{\partial \theta}{\partial Y} = \frac{\theta(j+1,k+1)-\theta(j+1,k-1)}{2\Delta Y}
\]

\[
\frac{\partial^4 \theta}{\partial Y^4} = \frac{\theta(j+1,k+1)-2\theta(j+1,k)+\theta(j+1,k-1)}{(\Delta Y)^4}
\]

The resultant equation becomes

\[
U(j+1,0) + 2\sum_{i=1}^{N} U(j+1,k) = U(j,0) + 2\sum_{i=1}^{N} U(j,k) \tag{3.3.11}
\]

The set of simultaneous equations (2.3.9) together with (2.3.11) may be written one equation can be formed about each mesh point in a column as shown:

\[
U(j+1,0) + 2U(j+1,1) + 2U(j+1,2) + \cdots = U(j,0) + 2\sum_{i=1}^{N} U(j,k)
\]

\[
\beta_c U(j+1,0) + (\alpha_o + \gamma_p) U(j+1,1) + \cdots + n_p(j+1) = \phi_c
\]

\[
\alpha_n U(j+1,0) + \beta_n U(j+1,1) + \gamma U(j+1,2) + \cdots + n_p(j+1) = \phi_n
\]

\[
\beta_n U(j+1,N-1) + \beta_n U(j+1,N) + \cdots + n_p(j+1) = \phi_n
\]

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where
\[
\begin{align*}
\alpha_i &= \frac{l}{(\Delta y)^2} + \frac{P(j, k)}{2\Delta y} \\
\beta_i &= \left[ \frac{2}{(\Delta y)^2} + M^2 + \frac{U(j, k)}{\Delta x} \right]^{-1} \\
\gamma_i &= \frac{1}{(\Delta y)^2} - \frac{V(j, k)}{2\Delta y} \\
\eta &= -\frac{1}{\Delta x} \\
\theta_i &= \left[ \frac{P(j) + U^2(j, k)}{\Delta x} \right]^{-1}
\end{align*}
\]

The set of equations are solved by Gauss-Jordan method. Application of the newly found axial velocities (U's) together with those in column behind (2.3.8) determine the new transverse velocities (V's).

The set of velocities are now placed into a set of finite difference equations written about each mesh point in a column for equation (2.3.10) are:
\[
\begin{align*}
\bar{\alpha}_i \theta(j + 1, 0) + (\bar{\alpha}_i + \bar{\gamma}_0) \theta(j + 1, 1) + \cdots &= \bar{\phi}_0 \\
\bar{\alpha}_i \theta(j + 1, 0) + \bar{\beta}_i \theta(j + 1, 1) + \bar{\gamma}_0 \theta(j + 1, 2) + \cdots &= \bar{\phi}_1 \\
\cdots &= \cdots \\
\bar{\alpha}_n \theta(j + 1, N - 1) + \bar{\beta}_n \theta(j + 1, N) &= \bar{\phi}_n - \bar{\gamma}_n
\end{align*}
\]
where

\[ \bar{\alpha}_k = \frac{1}{P_i(\Delta y)^2} + \frac{V(j,k)}{2\Delta y} \]

\[ \bar{p}_k = -\left[ \frac{2}{P_i(\Delta y)^2} + \frac{U(j,k)}{\Delta x} \right] \]

\[ \bar{y}_k = \frac{1}{P_i(\Delta y)^2} - \frac{V(j,k)}{2\Delta y} \]

\[ \bar{\theta}_k = \frac{U(j,k)}{\Delta x} \]

\[ -E_k \left[ \frac{U(j+1,k+1)-U(j+1,k-1)}{2\Delta y} \right]^2 \]

The procedure mentioned earlier was used to solve this set of equations as

were used to solve the momentum equation.

**Numerical discussion:**

The following initial values are taken at entrance: \(X = 0, U = 0.1, P = 0.1357, \)
\(V = 0, \) \(Pr = 0.1 \) and \(E_k = 1.0. \) Calculations have been carried out until a fully
developed flow is obtained. Velocity, temperature and pressure profiles are shown in
figures (3.2) to (3.10). For fixed \(M, \) it is observed that in the region \(0 \leq X \leq 0.02, \) the
velocity increases for $Y=0$ to $Y=0.6$ and decreases for $Y>0.6$ figure (3.3). In region $X > 0.02$, velocity decreases as $Y$ increases. For $Y = 0.2$, the velocity decreases as $M$ increases figure (3.4 (a)). For $Y=0.8$, the velocity increases as $M$ increases figure (3.4 (b)). For fixed $X$, the velocity profile is illustrated in figure (3.5). For fixed $M$, the velocity increases as $X$ increases up to $Y=0.5$ and then decreases. For fixed $M$, the velocity increases as $Y$ increases attains the maximum velocity near $Y=0.1$ and decreases with further increasing $Y$. For fixed $M$ and $Y$, the temperature increases as $X$ increases figure (3.6). For fixed $M$, and $X$, the temperature decreases from $Y=0$ to $Y=0.2$ and then increases when $X \leq 0.01$ figure (3.7). For fixed $M$, increasing the Eckart number is not affected with velocity and pressure values. For fixed $M$, increasing Eckart number, temperature values are effected figure (3.8) and (3.9).
Fig. 3.3.1. Mesh Scheme
Fig 3.2(b) Velocity Profile for $M=1$ & $E_x=1$
Fig 3.2(c) Velocity Profile for M=3 & E_k=1
Fig 3.3(c)  Velocity Profile for $M=5$ & $E_s=10$
Fig 3.4(a) Velocity Profile for $Y=0.2$ & $E_r=1$
Fig 3.4 (c) Velocity Profile for Y=0.8 & Ek=1
Fig 3.5 (b) Velocity Profile for $M=18E_x=1$
Fig 3.5 (c) Velocity Profile for $M=5$ & $E_x = 1$
Fig 3.6(a) Temperature Profile for $M=0$ & $E_r=1$
Fig 3.7(c) Temperature Profile for $M=5$ & $E_x=1$
Fig 3.9(a) Temperature Profile for M=0 & E_k=10
Fig 3.9(b) Temperature Profile for $M=1$ & $E_k=10$
Following the method of Shohet mesh network is introduced across the range of the problem figure (3.4.1). There is no center line symmetry in this problem. The finite difference approximations to the derivatives (2.4.8),(2.4.9) and (2.4.10) are shown as follows:

Continuity equation:

$$\frac{\partial V}{\partial R} = \frac{V(j, k+1) - V(j, k)}{\Delta R}$$

$$\frac{\partial U}{\partial Z} = \frac{U(j + 1, k + 1) + U(j + 1, k) - U(j, k + 1) - U(j, k)}{2\Delta Z}$$

Momentum equation:

$$\frac{\partial U}{\partial Z} = \frac{U(j + 1, k) - U(j, k)}{\Delta Z}$$

$$\frac{\partial U}{\partial R} = \frac{U(j + 1, k + 1) - U(j, k + 1)}{2\Delta R}$$

$$\frac{\partial^2 U}{\partial R^2} = \frac{U(j + 1, k + 1) - 2U(j + 1, k) + U(j + 1, k - 1)}{(\Delta R)^2}$$

$$\frac{\partial P}{\partial Z} = \frac{P(j + 1) - P(j)}{\Delta Z}$$
Energy equation:

\[
\frac{\partial T}{\partial Z} = \frac{T(j+1,k) - T(j,k)}{\Delta Z}
\]

\[
\frac{\partial T}{\partial R} = \frac{T(j+1,k+1) - T(j+1,k-1)}{2\Delta R}
\]

\[
\frac{\partial^2 T}{\partial R^2} = \frac{T(j+1,k+1) - 2T(j+1,k) + T(j+1,k-1)}{(\Delta R)^2}
\]

From the nature of the last three expressions it can be seen that (2.4.10) is the only expression involving the temperature and therefore, it may be solved separately from (2.4.8) and (2.4.9). By determining an additional equation involving only unknowns which appear in (2.4.9), this equation may be uncoupled from (2.4.8) and solved separately. This additional equation or equation of constraint, may be obtained by solving (2.4.8) to obtain the velocity at the outer wall \( V( j+1, N+1 ) \) which is zero, in terms of the inner wall velocity \( V( j+1, 0 ) \) which is also zero. The resultant equation becomes

\[
\sum_{k=1}^{N} R_k U(j+1,k) = \sum_{k=1}^{N} R_k U(j,k)
\]

\[\text{--------------------------(3.4.1)}\]

A set of simultaneous equations (2.4.9) together with (3.4.1) may be written. One can be formed about each mesh point in column as shown become:
\[ R_i U(j+1,l) + R_j U(j+1,2) + R_k U(j+1,3) + \ldots = \sum_{k} R_k U(j,k) \]

\[ \beta_1 U(j+1,l) + \gamma_1 U(j+1,2) + \ldots + nP(j+1) = \phi_1 \]

\[ \alpha_2 U(j+1,l) + \beta_2 U(j+1,2) + \gamma_2 U(j+1,3) + \ldots + nR(j+1) = \phi_2 \]

\[ \ldots \]

\[ \alpha_N U(j+1,N-1) + \beta_N U(j+1,N) + \gamma N U(j+1,N) + \ldots + nR(j+1) = \phi_N \]

where

\[ \alpha_k = \left[ \frac{1}{(\Delta R)^2} + \frac{V(j,k)}{2\Delta R} - \frac{1}{2\Delta RR_k} \right] \]

\[ \beta_k = \left[ \frac{2}{(\Delta R)^2} + \frac{U(j,k)}{\Delta Z} + \dot{M}^2 \right] \]

\[ \gamma_k = \left[ \frac{1}{(\Delta R)^2} + \frac{\dot{V}(j,k)}{2\Delta R} - \frac{1}{2\Delta RR_k} \right] \]

\[ n = \frac{-1}{\Delta Z} \]

\[ \Phi_k = \left[ \frac{P(j) + U^2(j,k)}{\Delta Z} \right] \]
These sets of equations are solved by Gauss-Jordan method. Application of the newly found axial velocities (U's) together with those in column behind determine the new transverse velocities (V's).

The set of velocities are now placed into a set of finite difference equations written about each mesh point in a column for equation (2.4.10) are:

\[
\beta_j T(j + 1, 1) + \gamma_j T(j + 1, 2) + \cdots = \phi_1 - \alpha_1
\]

\[
\alpha_2 T(j + 1, 1) + \beta_2 T(j + 1, 2) + \gamma_2 T(j + 1, 3) = \phi_2
\]

\[\cdots\]

\[
\alpha_N T(j + 1, N - 1) + \beta_N T(j + 1, N) = \phi_N - \gamma_N
\]

where

\[
\alpha_k = \left[\frac{1}{P_r(\Delta R)^2} + \frac{V(j, k)}{2\Delta R} - \frac{1}{2R_i\Delta RP_r}\right]
\]

\[
\beta_k = -\left[\frac{2}{P_r(\Delta R)^2} + \frac{U(j, k)}{\Delta Z}\right]
\]

\[
\gamma_k = \left[\frac{1}{P_r(\Delta R)^2} - \frac{V(j, k)}{2\Delta R} + \frac{1}{2R_i\Delta RP_r}\right]
\]
The same technique may be used to solve this set of equations as were used to solve the momentum equation.

A difficulty with the continuity equation makes itself evident at this point. Any small round off error in the U velocities near the walls of the channel results in this error being propagated upwards to each V velocities as the continuity equation is marched up column. This effect is especially noticeable at those mesh points very near the entrance, where the U velocities are nearly uniform. The result of difficulty is V velocities which adversely affect the succeeding computations.

This effect is very small in the plane channel case for two reasons. The first is that because only half of the channel is considered there are fewer equations in which to have the error build up. The second reason is that the center line V velocity is always identically equal to zero and therefore there is no initial error in the continuity equation for each column.

To correct this effect a second finite difference continuity equation is written which is biased "downward". The V velocities are then computed twice by using the original upward equation (3.4.2) and the downward equation (3.4.3) separately. The velocities are then averaged at each point and the results are used for the next calculation.
The upward equation is

\[ V(j + 1, k + 1) = \frac{1}{R_{j-1}} \left[ R_j V(j + 1, k) - \frac{\Delta R}{2\Delta Z} \left( (U(j + 1, k) - U(j, k + 1))R_{j+1} \right) + R_j (U(j + 1, k) - U(j, k)) \right] \]

\[ + \frac{\Delta R}{2\Delta Z} \left( (R_k U(j + 1, k) - U(j, k)) + R_{k+1} (U(j + 1, k - 1) - U(j, k - 1)) \right) \]  \hspace{1cm} (3.4.2)

and the downward equation is

\[ V(j + 1, k - 1) = \frac{1}{R_{j+1}} \left[ R_j V(j + 1, k) + \frac{\Delta R}{2\Delta Z} \left( (U(j + 1, k) - U(j, k)) + R_k (U(j + 1, k) - U(j, k)) \right) + \right. \]

\[ \left. \frac{\Delta R}{2\Delta Z} \left( (R_{k-1} U(j + 1, k - 1) - U(j, k - 1)) \right) \right] \]  \hspace{1cm} (3.4.3)

**Numerical discussion:**

The following initial values are taken at entrance: \( Z = 0, U = 0.1, P = 0.1357, V = 0, \) \( Pr = 0.1 \) and \( \beta_k = 1.0. \) Calculations have been carried out till a developed flow is obtained. Figures (4.2) to (4.5) show the results of computations for the velocity profile in the annular channel figure (4.5) to (4.9) show the temperature profiles. Figure (4.10) shows the pressure profiles obtained.

For fixed \( M, \) as \( Z \) increases, the velocity decreases for \( R = 0.1, 0.3 \) and then starts increasing, there after for \( R = 0.5 \) the values of the velocity are less than those
of $R = 0.5$. In this process the velocity starts decreasing for $R = 0.9$ as $Z$ increases. The effect of magnetic field in the annular channel is not produce much flattening of the velocity profile. Its primary effect is to shift the position of the point at which the maximum velocity occurs. As the magnetic field increased, this point is shifted towards the outer radius, since the retarding body force due to magnetic field is least at that point. For fixed $R$, the velocity decreases with increasing $M$.

For $R = 0.2$ the velocity values decrease and for $R = 0.8$ the velocity values increase as $M$ increases. Figure (4.4). From figure (4.8), it is evident that for fixed $M$ the temperature increases with $Z$, for all cross sections $R = \text{constant}$. For fixed $M$ the temperature profile follows a parabolic path with minimum attained near $R = 0.5$ for $Z = 0.001$. As $Z$ increases the parabolic profile flattens and finally becomes a linear path for $Z = 0.1$. The pressure decreases with increasing magnetic parameter.
Fig. 3.4.1. Mesh Scheme
Fig 4.2(a) Velocity Profile for Fixed $M=0$ & $E_\kappa=1$
Fig 4.2(b) Velocity Profile for Fixed $M=1$ 

$E_x=1$
Fig 4.2(c) Velocity Profile for Fixed $M=5$ & $E_k=1$
Fig 4.3 (a) Velocity Profile for Fixed $M=0$ & $E_k=10$
Fig 4.3(b) Velocity Profile for Fixed $M=1$ & $E_x=10$
Fig 4.3(c) Velocity Profile for Fixed M=5 & E_k=10
Fig 4.4(a) Velocity Profile for Fixed $R=0.2$ & $E_r=1$
Fig 4.5(c) Velocity Profile for Fixed $M=3$ & $E_k=1$
Fig 4.6(a) Temperature Profile for Fixed M=0 & E_k=1
Fig 4.6(d) Temperature Profile for Fixed $M=5$ & $E_r=1$
Fig 4.7 (a) Temperature Profile for Fixed $M=0$ & $E_k=10$
Fig 4.7 (b) Temperature Profile for Fixed $M=1$ & $E_k=10$
Fig 4.7 (c) Temperature Profile for Fixed $M=3$ & $E_k=10$
Fig 4.7 (d) Temperature Profile for Fixed M=5 & E_k=10
Fig 4.8 (a) Temperature Profile for Fixed $M=0$ & $E_k=1$. 
Fig 4.8 (b) Temperature Profile for Fixed $M=1$ & $E_i=1$. 
Fig 4.9(c) Temperature Profile for fixed $M=5$ & $E_k=10$
Problem V

Writing equations (2.5.13), (2.5.12) and (2.5.14) in finite difference form and applying them to the \((i, j)\) mesh point of a rectangular grid Figure (3.5.1), they result in

\[
U(i, j) \frac{U(i+1, j) - U(i, j)}{\Delta X} + V(i, j) \frac{U(i+1, j+1) - U(i+1, j-1)}{2\Delta Y} \\
U(i+1, j+1) \frac{U(i+1, j+1) - 2U(i+1, j) + U(i, j)}{(\Delta Y)^2} - \frac{U(i+1, j)}{K} + \theta(i+1, j) - M^2U(i+1, j)
\]

\[\text{------------------------(3.5.1)}\]

\[
U(i, j) \frac{U(i+1, j+1) - U(i, j+1)}{\Delta X} + V(i, j) \frac{U(i+1, j+1) - U(i+1, j)}{\Delta Y} = 0
\]

\[\text{------------------------(3.5.2)}\]

\[
U(i, j) \frac{0(i+1, j) - 0(i, j)}{\Delta X} + V(i, j) \frac{0(i+1, j+1) - 0(i+1, j-1)}{2\Delta Y}
\]

\[
\frac{1}{\rho} \frac{0(i+1, j+1) - 20(i+1, j) + 0(i+1, j-1)}{(\Delta Y)^2}
\]

\[\text{------------------------(3.5.3)}\]

The difference form selected here is highly implicit, that is, not only all \(Y\)-direction derivatives are evaluated at \(i+1\) but the coefficients of non-linear convective terms are also evaluated at \(i+1\). This representation is necessary since the usual implicit
scheme in which the coefficients are evaluated at \( i \) is inconsistent for these conditions.

The difference equations (3.5.1) to (3.5.3) now become:

\[
\left[ \frac{-1}{(\Delta Y)^2} \right] + \frac{V(i, j)}{2\Delta Y} \left[ U(i+1, j-1) + \frac{2}{(\Delta Y)^2} + \frac{M^2}{\Delta X} + \frac{1}{K} \right] U(i+1, j)
\]

\[
+ \left[ \frac{-1}{(\Delta Y)^2} \right] + \frac{V(i, j)}{2\Delta Y} \left[ U(i+1, j+1) - 0 \right] (i+1, j) = \frac{U(i, j)U(i, j)}{\Delta X} \quad \text{---(3.5.4)}
\]

\[
V(i+1, j+1) = V(i+1, j) - \frac{\Delta Y}{\Delta X} [U(i+1, j+1) - U(i, j+1)] \quad \text{---(3.5.5)}
\]

\[
\left[ \frac{-1}{(\Delta Y)^2} \right] + \frac{V(i+1, j)}{2\Delta Y} \left[ 0(i+1, j-1) + \frac{2}{(\Delta Y)^2} + \frac{P_r(\Delta Y)}{\Delta X} \right] 0(i+1, j)
\]

\[
+ \left[ \frac{-1}{(\Delta Y)^2} \right] + \frac{V(i+1, j)}{2\Delta Y} \left[ 0(i+1, j+1) = \frac{U(i+1, j)U(i, j)}{\Delta X} \right] \quad \text{---(3.5.6)}
\]

Equations (3.5.4) to (3.5.6) together with the boundary conditions (2.5.15) are solved by Gauss elimination method which consists of solving the set of equations (3.5.4), (3.5.5) and (3.5.6) in that order repeatedly.
The local Nusselt number in non-dimension form is given by

$$N_u = \left. \frac{\partial \theta}{\partial Y} \right|_{Y = 0} = \left[ \frac{30 (i + 1,0) - 4 \theta (i + 1,1) + \theta (i + 1,2)}{2 \Lambda Y} \right]$$

Numerical discussion:

The computation are carried out for the mesh sizes $\Delta x = 0.01$ and $\Delta y = 0.25$, where $Y = 45$ is representing infinity. The numerical solutions are obtained for $Pr = 0.71$, $M = 1, 2, 3$ and $K = 2, 4, 8$. Velocity, $U$ and temperature, $\theta$ are shown as a function of $Y$ for three values of $X = 0.05, 0.10, 0.15$ in figures (5.4) to (5.7) and (5.16) to (5.19) respectively. Figures (5.8) and (5.9) show that the variation of $U - velocity$ with $Y$ for different values of $K$ ($\neq 0$) and it is observed that $U - velocity$ increases significantly as the value of $K$ increases for fixed value of magnetic parameter $M$. Figure (5.11) shows that for fixed value of $K$, the variation of $U - velocity$ with $Y$ for different values of $M$ ($\neq 0$) and it is observed that $U - velocity$ decreases significantly as the value of $M$ increases. Figure (5.13) shows that for fixed value of $K$, the variation of $\theta - temperature$ with $Y$ for different values of $M$ ($\neq 0$) and it is observed that $\theta - temperature$ increases as the value of $M$ increases. Figures (5.10) and (5.11) show that for fixed value of $1/k = 0$ the
variation of $U$ - velocity and $\theta$ - temperature with $Y$ for different values of $M$

and it is observed that $U$ - velocity decreases significantly as the value of $M$ increases and $\theta$ - temperature increases as the value of $M$ increases.

Figures (5.2) and (5.3) show that for fixed values of $1/k (= 0)$ and magnetic parameter $M (= 1,3)$, the variation of $U$ - velocity with $Y$ for different values of $X$ and it is observed that $U$ - velocity increases as the value of $X$ increases.

Figures (5.14) and (5.15) show that for fixed values of $1/k (= 0)$ and $M (= 1,3)$, the variation $\theta$ - temperature with $Y$ for different values of $X$ and it is observed that $\theta$ - temperature increases as the value of $X$ increases.
Fig. 3.5.1. Mesh Scheme
Fig 5.2 Velocity Profile for fixed $M=1$, $K=0$ & $P_r = 0.71$
Fig 5.3 Velocity Profile for fixed $M=3$, $K=0$ & $P_r=0.71$
Fig 5.4 Velocity Profile for fixed M=1, K=2, & P=0.71
Fig 5.6 Velocity Profile for fixed M=1, k=4 & P=0.71
Fig 5.9 Velocity Profile for fixed $M=3$, Fixed $X=0.10$ & $P_r=0.71$
Fig 5.10 Velocity Profile for fixed K=0 & Fixed X=0.10
Fig 5.11 Velocity Profile for fixed K=4 & Fixed X=0.10
Fig 5.15 Temperature Profile for fixed $M=3$, $K=0$ & $Pr=0.71$
Fig 5.16 Temperature Profile for fixed $M=1$, $X=2$ & $P=0.71$
Fig 5.17 Temperature Profile for fixed $M=3$, $K=2$ & $Pr=0.71$
Fig 5.18 Temperature Profile for fixed $M=1$, $K=4$ & $Pr=0.71$