CHAPTER 2
REVIEW OF METHODS OF FAULT ANALYSIS OF SIX PHASE SYSTEM

2.1 INTRODUCTION:

Modern power systems have become very complex in character, both for investigation and for operation and control. This is mainly due to continued growth of interconnection and use of higher and higher voltages for transmission purpose, to meet the increasing power demand. This led to the upgrading of existing lines to higher voltages or installing new lines. An alternative to this have been discussed by S.S.Venkata, et al. [2] in the form of six phase transmission.

The concept of n-phase system has been outlined by Fortescue in his symmetrical components theory. Symmetrical components method was extended to six phase fault analysis by S.S.Venkata, et al. [5]. In their subsequent paper [13], it has been stated that there are 120 possible combinations of which 23 fault combinations are distinct and prominent. It has been reported that as the number of phases increases, the application of symmetrical components theory is tedious and becomes unwieldy or too cumbersome [5].

Six phase symmetrical components transformation posed some difficulties in the analysis of phase to phase faults, requiring complex turns ratio transformers even for short circuit on adjacent phases. Hence Dual Three Phase Transformation in terms of three phase symmetrical components had been proposed by P.S.Subramanyam treating the six phase system as two mutually coupled three phase systems [6,14].

In this work, both the six phase symmetrical components (wherever possible) and Dual Three Phase Transformation (DTPT) methods are employed for analysing various types of faults occurring on six phase transmission system. Hence these two methods have been explained in this chapter.
2.2 SIX PHASE SYMMETRICAL COMPONENTS METHOD:

The faults that can occur on six-phase power system are of symmetrical or unsymmetrical type, which may consist of short circuits, faults through impedances or open conductors. The fault analysis in six-phase power system is more complicated than in the conventional three-phase power systems. This is due to the fact that there are six phases, each fault is subjected to a different voltage and the presence of a neutral makes the number of faults types to be much larger. The total number of possible faults in three-phase system is eleven, while that in six-phase system is one hundred and twenty. But the possible number of significant faults in three-phase and six-phase are five and twenty three respectively [13]. Further these twenty three faults are reduced to thirteen by P. S. Subrahmanyam et al [14]. The phases of a six-phase system A, B, C, D, E, F are shown in fig 2.1

![Fig 2.1 Phasors of six-phase system](image)

![Fig 2.2 A typical six phase unbalanced phasors](image)

The method of symmetrical components is the most powerful tool for dealing with unbalanced polyphase circuits. Fortsue's work has proved that an unbalanced system of n phasors can be resolved into n system of balanced phasors called the symmetrical components of the original phasors [5]. This method is commonly used in unsymmetrical faults.

In this method the six-phase system of unbalanced phasors A, B, C, D, E, F is resolved into six balanced sets of phasors [5]. If the original phasors are voltages, they may be designated $V_A$, $V_B$, $V_C$, $V_D$, $V_E$, $V_F$. Fig 2.2 shows the original six-phase unbalanced phasors. While Fig 2.3 shows the six sets of balanced phasors that are the symmetrical components of the unbalanced phasors.
Fig 2.3 six balanced sets of symmetrical components of six unbalanced phasors.
Let the operator 'b' for the six-phase be used. Where $b = 1 \angle 60^\circ$. The six-phase operator 'b' is related to the three-phase operator 'a' as $b = -a^2$. The following Fig 2.4 shows the phasor diagram of various powers of operator 'b'.

![Diagram of various powers of operator b]

**Fig 2.4 Phasor diagram of the various powers of operator b**

According to Fortescue Theorem, the six-phase unbalanced phasors of a six-phase system can be resolved into a six balanced system of phasors.

Each of the $i^{th}$ sequence components ($i = 0, 1, 2, 3, 4, 5$) consists of six phasors equal in magnitude and displaced from each other by $\angle 60^\circ$ in phase, represented by the operator $b = 1 \angle 60^\circ$.

The original unbalanced phasors can be expressed in terms of their symmetrical components by the following equation.

$$V_{p6} = [T_6] V_{s6} \quad \quad \quad \quad \quad (2.1)$$

Where $\bar{V}_{p6} = [V_A \ V_B \ V_C \ V_D \ V_E \ V_F]^T$

$\bar{V}_{s6} = [V_{A0} \ V_{A1} \ V_{A2} \ V_{A3} \ V_{A4} \ V_{A5}]^T$. 

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\([ \mathbf{T}_6 ] \) = Six phase symmetrical components transformation matrix.

\[
[ \mathbf{T}_6 ] = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
1 & b^5 & b^4 & b^3 & b^2 & b \\
1 & b^4 & b^2 & 1 & b^4 & b^2 \\
1 & b^3 & 1 & b^3 & 1 & b^3 \\
1 & b^2 & b^4 & 1 & b^2 & b^4 \\
1 & b & b^2 & b^3 & b^4 & b^5
\end{pmatrix}
\]

Where \( b \) is a six phase operator of the value \( (0.5 + j 0.866) \) i.e. \( 1 \angle 60^\circ \)

The Inverse relationship is

\[
\overline{V_{S6}} = [\mathbf{T}_6]^{-1} V_{P6}
\]

\[
------------------ (2.2)
\]

where \([ \mathbf{T}_6 ]^{-1} = \frac{1}{6} \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
1 & b & b^2 & b^3 & b^4 & b^5 \\
1 & b^2 & b^4 & 1 & b^2 & b^4 \\
1 & b^3 & 1 & b^3 & 1 & b^3 \\
1 & b^4 & b^2 & 1 & b^4 & b^2 \\
1 & b^5 & b^4 & b^3 & b^2 & b
\end{pmatrix}
\]

The equations (2.1) and (2.2) are very often used for unsymmetrical fault analysis.
Equations (2.1) and (2.2) can similarly be written for line currents as

\[ \overline{I}_{p6} = [T_6] \overline{I}_{s6} \] \hspace{2cm} (2.1a) \n
\[ \overline{I}_{s6} = [T_6]^{-1} \overline{I}_{p6} \] \hspace{2cm} (2.2a) \n
If the line is completely transposed or assumed to transposed, the impedance matrix will reduce to

\[ Z_{p6} = \begin{pmatrix} Z_6 & Z_m \\ Z_m & Z_6 \end{pmatrix} \]

The series voltage drop equation for the transposed transmission line is

\[ \overline{E}_{p6} - \overline{V}_{p6} = \Delta \overline{V}_{p6} = \overline{Z}_{p6} \overline{I}_{p6} \] \hspace{2cm} (2.3) \n
Using equations (2.1) and (2.1a) in equation (2.3) leads to

\[ [T_6] \Delta \overline{V}_{s6} = [Z_{p6}] [T_6] \overline{I}_{s6} \] \hspace{2cm} (2.4) \n
\[ \therefore \Delta \overline{V}_{s6} = [T_6]^{-1} [Z_{p6}] [T_6] \overline{I}_{s6} \]

\[ \Delta \overline{V}_{p6} = [Z_{s6}] \overline{I}_{s6} \] \hspace{2cm} (2.5)
In the expanded form is

\[
\begin{bmatrix}
\Delta V_{A0} \\
\Delta V_{A1} \\
\Delta V_{A2} \\
\Delta V_{A3} \\
\Delta V_{A4} \\
\Delta V_{A5}
\end{bmatrix} = \begin{bmatrix} 0 & -V_{A0} \\
E_{A1} - V_{A1} & 0 \\
0 & -V_{A2} \\
0 & -V_{A3} \\
0 & -V_{A4} \\
0 & -V_{A5}
\end{bmatrix} = \begin{bmatrix} Z_0 \\
Z_1 \\
Z_2 \\
Z_3 \\
Z_4 \\
Z_5
\end{bmatrix} \begin{bmatrix} I_{A0} \\
I_{A1} \\
I_{A2} \\
I_{A3} \\
I_{A4} \\
I_{A5}
\end{bmatrix}
\]

(2.5a)

Where \( Z_0 = Z_d + 5 Z_m \)

\[ Z_1 = Z_2 = Z_3 = Z_4 = Z_5 = (Z_d - Z_m) \]

The equation (2.5a) can be written as

\[
\begin{bmatrix} V_{A0} \\
V_{A1} \\
V_{A2} \\
V_{A3} \\
V_{A4} \\
V_{A5}
\end{bmatrix} = \begin{bmatrix} 0 \\
E_{A1} \\
0 \\
0 \\
0 \\
0
\end{bmatrix} = \begin{bmatrix} Z_0 \\
Z_1 \\
Z_2 \\
Z_3 \\
Z_4 \\
Z_5
\end{bmatrix} \begin{bmatrix} I_{A0} \\
I_{A1} \\
I_{A2} \\
I_{A3} \\
I_{A4} \\
I_{A5}
\end{bmatrix}
\]

(2.6)

2.2.1 Sequence Networks:

The positive sequence network consists of a voltage source \( E_{A1} \) in series with positive sequence impedance. The remaining sequence networks consist of sequence impedances only, sequence networks are shown in fig 2.5.

Fig 2.5 Six phase symmetrical component sequence networks of a six phase line.
2.3 DUAL THREE PHASE TRANSFORMATION (DTPT) METHOD:

The balanced six phase system can be considered as one consisting of two mutually coupled three phase systems.

Fig 2.6 Balanced six phase voltages as a combination of two balanced three phase voltages.

In fig 2.6 the six phase system voltages $V_A$, $V_B$, $V_C$, $V_D$, $V_E$ and $V_F$ are renamed as $V_a$, $V_c^l$, $V_b$, $V_a^l$, $V_c^l$ and $V_b^l$.

Thus the sequence components of six phase unbalanced system consists of two balanced three phase systems $V_{a0}$, $V_{a1}$, $V_{a2}$ and $V_{a0}^l$, $V_{a1}^l$, $V_{a2}^l$. Hence a transformation for the six phase system can be defined as Dual Three Phase Transformation. The three sets of the first system are termed as the first zero, first positive and first negative sequence components. The three sets of second system are termed as second zero, second positive and second negative sequence components. Each of the first zero and second zero components consist of three phasors equal in magnitude and with zero displacement with each other. But the each of the first sequence components are of three phase phasors equal in magnitude and displaced at an $120^o$ from each other. The first positive and second positive have the same phase sequence that as the original phasors. While the first negative and second negative sequences have the
phase sequence opposite to the original phasors [6]. The Fig 2.2 shows the original unbalanced phasors then the six sets of balanced phasors which are three-phase symmetrical of six-phase unbalanced phasors are shown in Fig 2.7.

Fig 2.7 six sets of balanced phasors which are three-phase symmetrical of six-phase unbalanced phasors
Where $T_s$ is the familiar symmetrical component matrix for 3-phase system.

$T_s = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & 1 \\ 1 & 1 & a^2 \end{pmatrix}$ & $T_s^{-1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

\[ T_s^{-1} = \frac{1}{3} \begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \] (2.8)

where 'a' is a three phase operator. \( a = 1 \angle 120^0\)

The proposed transformation leads to two identical sets of three phase symmetrical component voltages or currents. Hence the proposed transformation can be termed as Dual Three Phase Transformation (DTPT). The transpose of the original six phase voltages, after renaming and regrouping will be

$$V_{p0}^T = \begin{pmatrix} V_{p1}^T & V_{p2}^T \end{pmatrix}^T$$

$$= \begin{bmatrix} V_a & V_b & V_c & V_a^i & V_b^i & V_c^i \end{bmatrix}^T \quad (2.9)$$

In the proposed three phase transformed domain with a and \( a^i \) as reference.

$$V_D^T = \begin{pmatrix} V_{s1}^T & V_{s2}^T \end{pmatrix}^T = [V_{a0} V_{a1} V_{a2} V_{a1}^i V_{a2}^i]^T \quad (2.10)$$
In terms of Dual Three Phase Transformation matrix.

$$\overline{V_{p6}} = [T_D] \overline{V_D} = \begin{pmatrix} T_s & 0 \\ 0 & T_s \end{pmatrix} \begin{pmatrix} V_{s1} \\ V_{s2} \end{pmatrix}$$

$$= \begin{pmatrix} T_s V_{s1} \\ T_s V_{s2} \end{pmatrix}$$   \( (2.11) \)

$$\overline{V_D} = [T_D]^{-1} \overline{V_{p6}} = \begin{pmatrix} T_s^{-1} V_{p1} \\ T_s^{-1} V_{p2} \end{pmatrix}$$   \( (2.12) \)

Similarly for currents we have

$$\overline{I_{p6}} = [T_D] \overline{V_D}$$ and

$$\overline{I_D} = [T_D]^{-1} \overline{I_{p6}}$$   \( (2.13) \)

For the voltage and current relationship of the six phase line we have

$$\overline{V_{p6}} = [Z_{p6}] \overline{I_{p6}}$$

Where for a completely transposed line

$$Z_{p6} = \begin{pmatrix} Z_s & Z_m \\ Z_m & Z_m \end{pmatrix}$$

In the DTPT transformed domain

$$\overline{V_D} = [Z_D] \overline{I_D}$$   \( (2.14) \)
Using Dual Three Phase Transformation for $\bar{V}_{p6}$ and $\bar{I}_{p6}$, we have

$$[T_D] \bar{V}_D = [Z_{p6}] [T_D] \bar{I}_D$$

$$\bar{V}_D = [T_D]^i [Z_{p6}] [T_D] \bar{I}_D$$

$$\bar{V}_D = [Z_D] \bar{I}_D \hspace{1cm} (2.15)$$

Where $[Z_D] = [T_D]^i [Z_{p6}] [T_D] \hspace{1cm} (2.16)$

For a completely transposed line with $[Z_{p6}]$ given in DTPT method.

$$Z_D = \begin{bmatrix}
Z_0 & 0 & 0 & Z_{00} & 0 & 0 \\
0 & Z_1 & 0 & 0 & 0 & 0 \\
0 & 0 & Z_2 & 0 & 0 & 0 \\
Z_{00} & 0 & 0 & Z_0 & 0 & 0 \\
0 & 0 & 0 & 0 & Z_1 & 0 \\
0 & 0 & 0 & 0 & 0 & Z_2 \\
\end{bmatrix} \hspace{1cm} (2.16a)$$

with $Z_1 = Z_2 = (Z_4 - Z_m)$

$Z_0 = (Z_4 + 2Z_m)$

$Z_{00} = 3Z_m$

There will be coupling between the two zero sequence components only, which will be absent under normal operating conditions. In DTPT method,

$$\bar{V}_D = [E_D - Z_D \bar{I}_D] \hspace{1cm} (2.17)$$
Yielding the system of equations as given below for a completely transposed line.

\[
\begin{align*}
\begin{bmatrix}
V_{a0} \\
V_{a1} \\
V_{a2} \\
V_{a0}^1 \\
V_{a1}^1 \\
V_{a2}^1
\end{bmatrix}
\cdot
= 
\begin{bmatrix}
-Z_2 l_{a2} \\
-E_{a1} - l_{a1} Z_{a1} \\
-(Z_{a0} l_{a0} + Z_0 l_{a0}^1) \\
(E_{a1} - Z_{a1} i_{a1}^0) \\
-(Z_{a2} i_{a2}^1) \\
-Z_2 i_{a2}^1
\end{bmatrix}
\end{align*}
\]

(2.17a)

2.3.1 Sequence Networks:

The sequence networks for DTPT method are shown in figure 2.8.

Fig. 2.8 Dual Three Phase Transformation sequence networks for a completely transposed line.
In this Chapter, two methods were discussed namely (i) Six phase symmetrical components method and (ii) Dual Three Phase Transformation method (DTPT). But as the number of phases increases, the application of symmetrical components is more tedious and becomes unwieldy too and cumbersome.

Hence the six phase system is considered as one consisting of two coupled three phase systems and a one level transformation is presented in terms of familiar three phase symmetrical components, and is called Dual Three Phase Transformation as it leads two sets of identical three phase symmetrical components.

This dual three phase transformation does not fully diagonalise the impedance matrix of the six phase line, but indicates coupling between zero sequence components only for a completely transposed line.

The assembly of simple sequence networks for all the faults requires no complex ratio transformer or mutual inductance in the DTPT method. The sequence networks are given in terms of the three phase symmetrical components, whose effects are well known. These sequence networks can easily be simulated on network analyzer. Also they give a greater insight into the six phase system.

2.4 SUMMARY:-