CHAPTER 6

APPLICATION OF FEAROM MODEL IN A MEDIUM SIZED FOUNDRY MANUFACTURING CAST COMPONENTS USING CO₂ SAND CASTING METHOD

6.1 INTRODUCTION

FEAROM’s implementation aspects were also investigated in a medium-sized foundry manufacturing cast components using CO₂ sand casting method. During investigation, two specific components that are cast in this foundry were studied. After collecting the relevant data, FEAROM model was applied to prioritize the failure modes of cast components. The outcome of FEAROM model was utilized to prioritize the mould designs of these cast components. The details of this work are briefly presented in this chapter.

6.2 LOCATION AND BRIEF DETAILS ABOUT THE FOUNDRY

The practical application and implementation of FEAROM model were conducted at Shree Harie Steel & Alloys (hereunder called, Harie steel), a sand casting foundry located at Coimbatore city, Tamil Nadu State, India. Harie Steel was established in the year 2005 as a steel foundry producing castings made of ASTM-WCB, WCC, WC6 and WC9 steels. The strict in-house quality control, starting from raw materials to spectrographic analysis ensures highest quality of the castings produced in the Harie steel.
In Harie steel, infrastructure is well established and a comprehensive range of inspection and testing facilities are available. In Harie steel, mandatory systems and procedures have been installed and necessary certifications have been obtained to instill confidence in the minds of its customers. The annual turnover of Harie steel is at around INR 18 crores and has a net production ranges from 110 tonnes to 120 tonnes of casting products in a month. The main customers of Harie steel are reputed companies in India as well as in abroad.

The main castings which are produced in Harie steel are, valves, pumps, general machinery, earth moving and construction machinery parts, gate, globe, check, butterfly, ball valve body, plug valve and bonnet castings, strainer valve body and cover castings, bearing housing and cover castings. As mentioned earlier, CO₂ sand casting method is adopted for manufacturing these products in Harie steel. The brief description of the CO₂ sand casting method was presented earlier in chapter 5.

6.3 APPLICATION OF FEAROM MODEL ON A BEARING HOUSING SAND CASTING

To begin with, FEAROM was applied on a cast component called bearing housing. This component is made up of ASTM A216 WCB grade steel. This component finds wide usage because of their adherence to specification and strict control of heat treatment cycle, thereby ensuring high fatigue value and long life. Micro structure inspection is always carried out as a mode of cross checking. FEAROM was applied with the objective of predicting and finalizing the appropriate mould designs that are required to produce the castings. The wooden patterns required for producing this component are made based on the drawing supplied by the customer.
Three mould designs M₁, M₂ and M₃ designed by the engineer working in the design department of Harie steel are shown in Figures 6.1, 6.2 and 6.3 respectively. The patterns of M₁, M₂ and M₃ were used to produce CO₂ sand moulds in preproduction trials of the bearing housing. In mould design 1 (M₁), 22 chills, 3 cores and ingate of size 30 × 20 mm and runner of size 35 × 25 mm were used. Two risers of size φ 90 × 150 mm, three risers of size φ 75 × 150 mm, one riser of size φ 50 × 150 mm and one riser of size φ 100 × 150 mm were incorporated to ensure complete filling of the molten metal in the mould.

![Figure 6.1 Moulds Design 1 (M₁) of bearing housing](image)

In mould design 2 (M₂), 22 chills, 3 cores and ingate of size 30 × 20 mm and runner of size 35 × 25 mm were used. Two risers of size φ 90 × 150 mm, three risers of size φ 75 × 150 mm, one riser of size φ 50 × 150 mm and one riser of size φ 38 × 100 mm were incorporated to ensure complete filling of the molten metal in the mould.
In mould design 3 (M₃), 22 chills, 3 cores and ingate of size 30 × 20 mm and runner of size 35 × 25 mm were used. Two risers of size φ 90 × 150 mm, three risers of size φ 75 × 150 mm, one riser of size φ 50 × 150 mm, one riser of size φ 100 × 150 mm and one put up riser were incorporated to ensure complete filling of the molten metal in the mould.
As a sample, the pattern made up of wood of mould design 3 (M3) is shown in Figure 6.4. A sample inspection-ready fettled castings which were made by using the moulds of M3 are shown in Figure 6.5.

In order to gather the information related to FMEA, the quality control manager, production manager and mould design engineer of Harie steel were consulted. During this consultation, the data required for completing the FMEA table were carefully gathered and recorded. These data were used to complete the FMEA table of bearing housing which is shown in Table 6.1.
Table 6.1 FMEA table of bearing housing

<table>
<thead>
<tr>
<th>Serial number</th>
<th>Brief details of mould designs</th>
<th>Potential Failure Mode</th>
<th>Potential Effect(s) of Failure</th>
<th>Severity</th>
<th>Potential Cause(s)/ Mechanism(s) of Failure</th>
<th>Occurrence</th>
<th>Detection method</th>
<th>Detection</th>
<th>RPN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mould design 1 (M₁) Runner: 35 × 25 mm Ingate: 30 × 20 mm Chills: 22 Numbers No. of cores = 3 Riser: 90 × 150 – 2 Numbers 75 × 150 – 3 Numbers 50 × 150 – 1 Number 100 × 150 – 1 Number Yield: 46% Total number of castings that was produced using M₁ = 3</td>
<td>Shrinkage is in step of bearing</td>
<td>Rejected at customer point</td>
<td>9</td>
<td>Improper directional solidification due to inadequate feeding</td>
<td>5</td>
<td>Radiography/ultrasonic testing/Magnetic particle inspection</td>
<td>3</td>
<td>135</td>
</tr>
<tr>
<td>Serial Number</td>
<td>Brief details of mould designs</td>
<td>Potential Failure Mode</td>
<td>Potential Effect(s) of Failure</td>
<td>Potential Cause(s)/ Mechanism(s) of Failure</td>
<td>Severity</td>
<td>Occurrence Method</td>
<td>Detection</td>
<td>RPN</td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>-------------------------------</td>
<td>------------------------</td>
<td>------------------------------</td>
<td>------------------------------------------</td>
<td>---------</td>
<td>------------------</td>
<td>-----------</td>
<td>-----</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Mould design 2 ($M_2$)</td>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>2</td>
<td>Radiography/Ultrasound testing</td>
<td>6</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>Runner: $35 \times 25$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ingate: $30 \times 20$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chills: 22 Numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. of cores = 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Riser: $90 \times 150 - 2$ Numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$75 \times 150 - 3$ Numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$50 \times 150 - 1$ Number</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$38 \times 100 - 1$ Number</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yield: 49.5%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total number of castings that</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>was produced using $M_2 = 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Serial Number</td>
<td>Brief details of mould designs</td>
<td>Potential Failure Mode</td>
<td>Potential Effect(s) of Failure</td>
<td>Severity</td>
<td>Potential Cause(s)/Mechanism(s) of Failure</td>
<td>Occurrence</td>
<td>Detection method</td>
<td>Detection</td>
<td>RPN</td>
</tr>
<tr>
<td>---------------</td>
<td>--------------------------------</td>
<td>------------------------</td>
<td>-------------------------------</td>
<td>----------</td>
<td>-------------------------------------------</td>
<td>------------</td>
<td>------------------</td>
<td>-----------</td>
<td>-----</td>
</tr>
<tr>
<td>3</td>
<td>Mould design 3 (M₃)</td>
<td></td>
<td></td>
<td>6</td>
<td>Adequate riser is not present at flange portion</td>
<td></td>
<td>Radiography</td>
<td></td>
<td>108</td>
</tr>
</tbody>
</table>
The mould designs were ranked using traditional FMEA and RPC values. The values and rankings of the mould designs are presented in Table 6.2. As shown, S, O, D and RPN pertaining to the three mould designs entered in second, third, fourth and fifth columns in the Table 6.2. The RPN of M₁ is higher while it is lowest and equal in the case of M₂ and M₃. These mould designs were ranked by following the ascending order of RPN values. As mentioned in the chapter 3, the mould design whose RPN value is the least is ranked first. This is due to the reason that the failure in the case of mould design with least RPN may be quickly overcome and thus, the development time of the casting can be shortened.

Subsequent to the ranking of mould designs of the bearing housing, the RPC values were calculated by considering the traditional FMEA and practical conditions. The calculation of RPC in both cases is illustrated below.

**Case (a):** Initially, maximum and equal importance of S, O and D was assigned to all the mould designs. This importance is denoted the code number as L₁₀. Subsequent to this assumption, the following steps were followed.

**Step 1** : Calculation of RPCs of mould designs

The following formula was used to calculate the RPCs.

\[ \text{RPC}(M_i) = \max_j \left[ \min \{ (I(K_j), g_j(M_i)) \} \right] \]

In case of M₁, I(K_j) refer to L₁₀. Then, in the same case, g_j(M_i) is equal to L₀, L₅ and L₃. Suffices in L refer to S, O and D respectively. These values are indicated in Table 6.2.
Table 6.2 Ranking of mould designs of bearing housing

<table>
<thead>
<tr>
<th>Mould Designs</th>
<th>Values of</th>
<th>RPN</th>
<th>Traditional FMEA Rank Order</th>
<th>FEAROM Case (a)</th>
<th>FEAROM Case (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>S  O  D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₁</td>
<td>9  5  3</td>
<td>135</td>
<td>3</td>
<td>L₀   3</td>
<td>L₀  3</td>
</tr>
<tr>
<td>M₂</td>
<td>9  2  6</td>
<td>108</td>
<td>1</td>
<td>L₀   2</td>
<td>L₀  2</td>
</tr>
<tr>
<td>M₃</td>
<td>6  3  6</td>
<td>108</td>
<td>1</td>
<td>L₆   1</td>
<td>L₆  1</td>
</tr>
</tbody>
</table>

Note: I(S), I(O), I(D) are the importance associated with each index.

Hence,

\[
RPC (M₁) = \max \left[ \min \{ (I(K_j), g_j (M₁)) \} \right]_j
\]

\[
= \max \left[ \min (L_{10}, L₀), \min (L_{10}, L₅), \min (L_{10}, L₃) \right]
\]

\[
= \max [L₀, L₅, L₃] = L₀
\]

The calculation of RPCs in the case of M₂ and M₃ is presented below.

\[
RPC (M₂) = \max \left[ \min \{ (I(K_j), g_j (M₂)) \} \right]_j = \max [L₀, L₂, L₆] = L₀
\]

\[
RPC (M₃) = \max \left[ \min \{ (I(K_j), g_j (M₃)) \} \right]_j = \max [L₆, L₃, L₆] = L₆
\]
In order to rank the mould designs, it is required to calculate CPM. The steps followed to calculate CFM is presented below.

**Step 2**: Ranking using Critical Failure Mode (CFM)

The formula used to calculate CFM is presented below:

\[
CFM(M^*) = \min_{M_i \in A} \{RPC(M_1), RPC(M_2), RPC(M_3)\}
\]

where \(A\) refers to the set of mould designs.

Now,

\[
CFM(M^*) = \min\{L_9, L_9, L_6\}
\]

\[
= L_6 = RPC(M_3)
\]

As revealed above, preferred mould design is \(M_3\). But still a tie exists between the other two mould designs. This tie could be overcome by using the tie ranking rule as follows.

Since tie occurs in case \(M_1\) versus \(M_2\), the same (tie) is to be broken by using the following formula.

\[
T(M_i) = N(M_i)
\]

where \(N(M_i)\) is the number of occurrences of S, O and D in each mould designs that are lesser than \(L_6\).

As shown in Table 6.2, the values of O and D of \(M_1\) are 5 and 3 respectively. These values are lower than \(L_6\) (that is 6). Hence:

\[
T(M_1) = 2
\]
Likewise, \[ T(M_2) = 1 \]

As shown above, \( T(M_2) < T(M_1) \). Hence, the preferred mould design is \( M_2 \). Hence the order of preference of mould designs is \( M_3, M_2 \) and \( M_1 \). These details are presented in Table 6.2.

**Case (b):** In case (a) equal importance was assigned to \( S, O \) and \( D \), in reality, these values may differ. In the case of DFC valve mould designs the values assigned by the FMEA team members of Harie steel varied against \( S, O \) and \( D \) were gathered. These values are indicated in Figure 6.6.

![Graph showing the importance of S, O, and D](image)

**Figure 6.6 Importance of S, O and D in case (b) of bearing housing**

The same information is mathematically denoted below:

\[ I(S) = L_{10}, \ I(O) = L_8, \ I(D) = L_6 \]

The ranking method that progressed through two steps given below:
Step 1 : Calculation of RPC

The calculation of RPCs is shown below.

RPC \( (M_1) = \max \left[ \min (L_{10}, L_9), \min (L_{8}, L_5), \min (L_6, L_3) \right] \)

\[ = \max [L_9, L_5, L_3] = L_9 \]

RPC \( (M_2) = \max [L_9, L_2, L_6] = L_9 \)

RPC \( (M_3) = \max [L_6, L_3, L_6] = L_6 \)

Step 2 : Ranking using CFM

As the RPC values of the three mould designs are same, the ranking order as determined under case (a) remained the same.

6.3.2 Validation using MFTOPSIS Method Hybrid with AHP

The ranking of mould designs was validated using the MFTOPSIS method hybrid with AHP under the cases (a) and (b). The details of this validation are presented in the following subsections.

6.3.2.1 Validation under Case (a)

While applying MFTOPSIS method hybrid with AHP, the mould designs \( M_1, M_2 \) and \( M_3 \) were considered as alternatives and the indices S, O, and D as the evaluation criteria. Subsequently ten steps were followed. These details are presented in the following steps.
Step 1 : Determination of relative weights

The relative weight of indices and mould designs were calculated using Equation 3.5 (given in chapter 3).

The calculations are shown below.

Relative weight between S and O: \((L_n - L_i) + 1 = (10 - 10) + 1 = 1\)

Relative weight between S and D: \((L_n - L_i) + 1 = (10 - 10) + 1 = 1\)

Relative weight between O and D: \((L_n - L_i) + 1 = (10 - 10) + 1 = 1\)

As shown above, an equal importance with value \(L_{10}\) was assigned to the indices S, O and D, the \(L_n\) and \(L_i\) values were 10.

Step 2 : Determination of pairwise comparison matrix (A)

The relative weights were used to develop pairwise matrix. The method of developing this pairwise matrix is described here. When the same index is compared, then the weight is 1. When different indices compared, then the weight is reciprocal to one another. This is to maintain consistency of the matrix. Hence, the weight of S when compared with O is 1. Then the weight of O when compared with S is \(1 \div 1 = 1\). Likewise, O when compared with D is 1 and hence D when compared with O is \(1 \div 1\). These values are shown below as pairwise matrix.

\[
\begin{bmatrix}
S & O & D \\
S & 1 & 1 & 1 \\
O & 1 & 1 & 1 \\
D & 1 & 1 & 1 \\
\end{bmatrix} \Rightarrow A = \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}
\]
Step 3 : Formation of normalized matrix

The values of the elements of the normalized matrix were calculated using the following formula.

\[
\text{Value of the element of the normalized matrix} = \frac{\text{Original value from pairwise matrix}}{\text{Total column value}}
\]

The method of calculating the elements of normalized matrix with the above equation is illustrated here. For example, in the case of pairwise matrix, the original value against S and S is 1. The total value of the first column is 3. Hence, the value of this element in normalized matrix is \(1 / 3\). The normalized matrix thus formed is presented below:

\[
S \begin{bmatrix} 1/3 & 1/3 & 1/3 \end{bmatrix} O \begin{bmatrix} 1/3 & 1/3 & 1/3 \end{bmatrix} D \begin{bmatrix} 1/3 & 1/3 & 1/3 \end{bmatrix} \text{ Row average} = 1/3
\]

The weights of evaluation criteria were same as their row average. Accordingly, the weights of the evaluation criteria are presented below.

Severity (S) \(= 0.33\)
Occurrence (O) \(= 0.33\)
Detection (D) \(= 0.33\)

The same is expressed in matrix form as shown below.

\[
W = [S \ O \ D] = [0.33 \ 0.33 \ 0.33]
\]
Step 4: Checking for consistency

The consistency matrix was developed by multiplying pairwise comparison matrix (A) and evaluation criteria column matrix ($W^T$).

\[
AW^T = \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 
\end{bmatrix}
\begin{bmatrix}
0.33 \\
0.33 \\
0.33 
\end{bmatrix}
= \begin{bmatrix}
0.99 \\
0.99 \\
0.99 
\end{bmatrix}
\]

Subsequently, the largest eigen value ($\mu$) was calculated. The formula used to calculate the largest eigen value ($\mu$) is,

\[
\mu = \frac{1}{n} \sum_{i=1}^{n} \frac{i^{th} \text{ entry in } AW^T}{i^{th} \text{ entry in } W^T}
\]

where $n =$ order of $A$

The calculation of the larger eigen value ($\mu$) using the above formula is presented below.

\[
\mu = \frac{1}{3} \left( \frac{0.99}{0.33} + \frac{1.99}{0.33} + \frac{0.99}{0.33} \right) = 3
\]

The calculation of the consistency ratio (CR) is shown below.

\[
CR = \frac{\text{Consistency index (CT)}}{\text{Average index of randomly generated weights (ACI)}} = \frac{\mu - n}{n - 1} \frac{\text{ACI}}{\text{ACI}}
\]

where the values of ACI is depended on the order of the matrix and was taken from Table 3.9 (given in chapter 3).

\[
CR = \frac{\left( \frac{3 - 3}{3 - 1} \right)}{0.58} = 0
\]
Because CR value is less than 10%, the normalized matrix formed was consistent. After checking the consistency, MFTOPSIS method was applied by considering the weight calculated in step 3. The weight and the values of S, O and D are presented in decision matrix shown in Table 6.3.

**Table 6.3 Decision matrix**

<table>
<thead>
<tr>
<th>Weight, $w_i$</th>
<th>0.33</th>
<th>0.33</th>
<th>0.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluation criteria</td>
<td>S</td>
<td>O</td>
<td>D</td>
</tr>
<tr>
<td>Alternatives</td>
<td>Values of decision matrix ($y_{ij}$) elements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_1$</td>
<td>9</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>$M_2$</td>
<td>9</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>$M_3$</td>
<td>6</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

**Step 5 :** Formulation of normalized decision matrix

The normalized decision matrix was formulated by inputting the correlation values of the elements of decision matrix. The formula used to calculate the correlation of the elements is given below.

$$r_{ij} = \frac{y_{ij}}{\sqrt{\sum y_{ij}^2}}$$

As shown above, the values of $(\sum y_{ij}^2)^{1/2}$ against each column was calculated. Then each column element was divided by that to get $r_{ij}$ values. As a sample, the calculation of the element covering $M_1$ and S in decision matrix is given below. The value of this element is 6. Therefore,

$$r_{ij} = \frac{9}{\sqrt{9^2 + 5^2 + 3^2}} = 0.640$$
Like the above, the correlation values of all elements of the decision matrix are calculated. These values were used to formulate the normalized decision matrix shown in Table 6.4.

**Table 6.4 Normalized decision matrix**

<table>
<thead>
<tr>
<th>Weight, $w_i$</th>
<th>S</th>
<th>O</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attributes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternatives</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Values of normalized decision matrix elements ($r_{ij}$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_1$</td>
<td>0.640</td>
<td>0.811</td>
<td>0.333</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.640</td>
<td>0.324</td>
<td>0.667</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.426</td>
<td>0.487</td>
<td>0.667</td>
</tr>
</tbody>
</table>

**Step 6** : Formulation of weighted normalized matrix

The values of the weighted normalized matrix elements ($v_{ij}$) were obtained by multiplying each column elements by $w_i$. As a sample, the calculation of the element covering $M_1$ and S in normalized decision matrix is given below. The value of this element is 0.64. Therefore,

$$v_{11} = w_1 \times r_{11} = 0.33 \times 0.640 = 0.211$$

Like the above, the correlation values of all elements of the normalized decision matrix were calculated. These values were used to formulate the weighted normalized matrix shown in Table 6.5.

**Step 7 (i)** : Determination of the ideal solution ($A^*$)

In order to determine the values of the ideal solution ($A^*$), the minimum value in column D and maximum values in columns S and O from Table 6.5 were chosen according to the procedure given by Jain (2011).
Thus $A^* = \{0.211, 0.268, 0.110\}$

<table>
<thead>
<tr>
<th>Weight, $w_i$</th>
<th>0.33</th>
<th>0.33</th>
<th>0.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attributes</td>
<td>S</td>
<td>O</td>
<td>D</td>
</tr>
<tr>
<td>Alternatives</td>
<td>Values of weighted normalized matrix elements ($v_{ij}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>0.211</td>
<td>0.268</td>
<td>0.110</td>
</tr>
<tr>
<td>M2</td>
<td>0.211</td>
<td>0.107</td>
<td>0.220</td>
</tr>
<tr>
<td>M3</td>
<td>0.141</td>
<td>0.161</td>
<td>0.220</td>
</tr>
</tbody>
</table>

**Step 7 (ii):** Determination of the negative ideal solution ($A'$)

In order to determine the values of negative ideal solution ($A'$), the maximum value from column D and minimum values from columns S and O from Table 6.5 were chosen according to the procedure given by Jain (2011).

Thus $A' = \{0.141, 0.107, 0.220\}$

**Step 8(i):** Calculation of values of ideal separation ($S^*_i$)

The values of ideal separation measure were determined for each row using the formula given below.

$$S^*_i = \left[\sum (v^*_j - v_{ij})^2\right]^{1/2}$$

where $v^*_j$ are the values of $A^*$ which were substituted with the correlated row elements of weighted normalized matrix elements ($v_{ij}$).

As a sample, the calculation of the value of $S^*_1$ of $M_1$ using row values of $v_{ij}$ elements is given below.
\[ S_1^* = \sqrt{\sum (v_{ij}^* - v_{ij})^2} = [(0.211 - 0.211)^2 + (0.268 - 0.268)^2 + (0.110 - 0.110)^2]^{1/2} = 0.000 \]

Like the above, the values of ideal separation of \( M_2 \) and \( M_3 \) were calculated. These values are shown in Table 6.6.

**Table 6.6 Values of ideal separation (\( S_i^* \))**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of ideal separation (( S_i^* ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>0</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>0.195</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>0.169</td>
</tr>
</tbody>
</table>

**Step 8(ii) : Calculation of values of negative ideal separation (\( S_i' \))**

The values of negative ideal separation were determined for each row using the formula given below.

\[ S_i' = \sqrt{\sum (v_{ij}' - v_{ij})^2} \]

where \( v_{ij}' \) are the values of \( A' \) which were substituted with the correlated row elements of weighted normalized matrix elements \( (v_{ij}) \).

As a sample, the calculation of the value of \( S_1' \) of \( M_1 \) using row values of \( v_{ij} \) elements is given below.

\[ S_1' = \sqrt{\sum (v_{1j}' - v_{1j})^2} = [(0.141 - 0.211)^2 + (0.107 - 0.228)^2 + (0.220 - 0.110)^2]^{1/2} = 0.207 \]
Like the above, the values of negative ideal separation of \( M_2 \) and \( M_3 \) were calculated. These values are shown in Table 6.7.

**Table 6.7 Values of negative ideal separation (\( S_i' \))**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of negative ideal separation (( S_i' ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>0.207</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>0.070</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>0.054</td>
</tr>
</tbody>
</table>

**Step 9**  
Calculation of the relative closeness to the ideal solution (\( C_i^* \))

The value of relative closeness to the ideal solution was calculated using the following formula.

\[
C_i^* = \frac{S_i'}{S_i'^* + S_i'}
\]

As a sample, the calculation of the value of \( C_1^* \) of \( M_1 \) using the values of \( S_1'^* \) and \( S_1' \) is given below.

\[
C_1^* = \frac{S_1'}{S_1'^* + S_1'} = \frac{0.207}{0.000 + 0.207} = 1.00
\]

Like the above, the values of relative closeness to ideal solution of \( M_2 \) and \( M_3 \) were calculated. These values are shown in Table 6.8.
Table 6.8 Values of relative closeness ($C_i^*$)

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of relative closeness ($C_i^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>1.00</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.26</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.24</td>
</tr>
</tbody>
</table>

**Step 10**: Ranking the preference order

As shown in table 6.8, the relative closeness value $C_i^*$ for mould design $M_3$ is 0.24. This is the value closest to 0. Therefore, mould design $M_3$ is the most preferable mould design. This selection corroborates the preference made using FEAROM. This corroboration indicates that the validity of FEAROM model in selecting the most preferable mould design.

**6.3.2.2 Validation under case (b)**

As shown in Figure 6.6, the importance of values of indices S, O and D are 10, 8, and 6 respectively.

**Step 1**: The relative weight of indices and mould designs were calculated using equation (3.5).

The calculations are shown below.

Relative weight between S and O: $(L_n \sim L_i) + 1 = (10 - 8) + 1 = 3$

Relative weight between S and D: $(L_n \sim L_i) + 1 = (10 - 6) + 1 = 5$

Relative weight between O and D: $(L_n \sim L_i) + 1 = (8 - 6) + 1 = 3$
Since the different importance values 10, 8 and 6 were assigned to the indices S, O and D respectively, the $L_n$ values were 10 and 8 and corresponding $L_i$ values were 8 and 6 in the calculations shown above.

**Step 2 :** Determination of pairwise comparison matrix (A)

The relative weights were used to develop pairwise matrix. The method of developing this pairwise matrix is described here. When we compare severity with severity the weight is 1. Comparing severity with occurrence, for case (b), the FMEA team of Harie steel estimated the severity was 3 times more than the occurrence. In order to ensure the consistency of the matrix, O should be $1/3$ when compared with S. Likewise, S when compared with D is 5 and hence D when compared with S is $1/5$. These values are shown below.

\[
\begin{bmatrix}
S & O & D \\
S & 1 & 3 & 5 \\
O & 1/3 & 1 & 3 \\
D & 1/5 & 1/3 & 1 \\
\end{bmatrix}
\Rightarrow A = \begin{bmatrix}
1 & 3 & 5 \\
1/3 & 1 & 3 \\
1/5 & 1/3 & 1 \\
\end{bmatrix}
\]

**Step 3 :** Formation of normalized matrix

Value of the element of the normalized matrix = \( \frac{\text{Original value from pairwise matrix}}{\text{Total column value}} \)

The method of calculating the elements of normalized matrix with the above equation is illustrated here. For example, in the case of pairwise matrix, the original value against S and O is 3. The total value of the first column is 4.33. Hence, the value of this element in normalized matrix is \( 3 \div 4.33 = 0.69 \). The normalized matrix thus formed is presented below:
The weights of evaluation criteria were same as their row average. Accordingly, the weights of the evaluation criteria are presented below.

Severity (S) = 0.663  
Occurrence (O) = 0.260  
Detection (D) = 0.107

The same is expressed in matrix form as shown below.

\[ W = \begin{bmatrix} S & O & D \end{bmatrix} = \begin{bmatrix} 0.663 & 0.260 & 0.107 \end{bmatrix} \]

**Step 4**: Checking for consistency

The consistency matrix was developed by multiplying pairwise comparison matrix (A) and evaluation criteria column matrix \((W^T)\).

\[
AW^T = \begin{bmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{bmatrix}\begin{bmatrix} 0.663 \\ 0.260 \\ 0.107 \end{bmatrix} = \begin{bmatrix} 1.95 \\ 0.79 \\ 0.32 \end{bmatrix}
\]

Subsequently, the largest eigen value \((\mu)\) was calculated. The formula used to calculate the largest eigen value \((\mu)\) is,

\[
\mu = \frac{1}{n} \sum_{i=1}^{n} \frac{\text{ith entry in } AW^T}{\text{ith entry in } W^T} \quad \text{where } n = \text{order of } A
\]
The calculation of the larger eigen value (μ) using the above formula is presented below.

$$\mu = \frac{1}{3} \left( \frac{1.95}{0.633} + \frac{0.79}{0.26} + \frac{0.32}{0.107} \right) = 3.04$$

The calculation of the consistency ratio (CR) is shown below.

$$CR = \frac{\text{Consistency index (CT)}}{\text{Average index of randomly generated weights (ACI)}} = \frac{\mu - n}{n-1}$$

where the values of ACI are depended on the order of the matrix and was taken from table 3.9 (given in chapter 3).

$$CR = \frac{0.04}{0.58} = 0.034 = 3.4\%$$

Because CR value is less than 10%, the present normalized matrix was consistent. After checking the consistency, MFTOPSIS method is applied by considering the weight calculated in step 3. The weight and the values of S, O and D are presented in decision matrix shown in Table 6.9.

**Table 6.9 Decision matrix**

<table>
<thead>
<tr>
<th>Weight, ( w_i )</th>
<th>0.633</th>
<th>0.260</th>
<th>0.107</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluation criteria</td>
<td>S</td>
<td>O</td>
<td>D</td>
</tr>
<tr>
<td>Alternatives ↓</td>
<td>Values of decision matrix elements ( y_{ij} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_1 )</td>
<td>9</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>9</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>6</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>
Step 5 : Formulation of normalized decision matrix

The procedure followed for the formulation of normalized matrix is same as that were described in step 5 under case (a). The values of normalized decision matrix elements ($r_{ij}$) presented in the Table 6.4 are in Table 6.10.

Table 6.10 Normalized decision matrix

<table>
<thead>
<tr>
<th>Weight, $w_i$</th>
<th>0.633</th>
<th>0.260</th>
<th>0.107</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attributes</td>
<td>S</td>
<td>O</td>
<td>D</td>
</tr>
<tr>
<td>Alternatives</td>
<td>Values of normalized decision matrix elements ($r_{ij}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_1$</td>
<td>0.640</td>
<td>0.811</td>
<td>0.333</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.640</td>
<td>0.324</td>
<td>0.667</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.426</td>
<td>0.487</td>
<td>0.667</td>
</tr>
</tbody>
</table>

Step 6 : Formulation of weighted normalized matrix

The values of the weighted normalized matrix elements ($v_{ij}$) was obtained by multiplying each column elements by $w_i$. As a sample, the calculation of the element covering $M_1$ and $S$ in normalized decision matrix is given below. The value of this element is 0.640. Therefore,

$$v_{11} = w_1 \times r_{11} = 0.633 \times 0.640 = 0.405$$

Like the above, the correlation values of all elements of the normalized decision matrix were calculated. These values were used to formulate the weighted normalized matrix shown in Table 6.11.
Table 6.11 Weighted normalized matrix

<table>
<thead>
<tr>
<th>Weight, $w_i$</th>
<th>Attributes</th>
<th>Alternatives</th>
<th>Values of weighted normalized matrix elements ($v_{ij}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$w_3$</td>
</tr>
<tr>
<td></td>
<td>$S$</td>
<td>$O$</td>
<td>$D$</td>
</tr>
<tr>
<td>$M_1$</td>
<td>0.405</td>
<td>0.211</td>
<td>0.036</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.405</td>
<td>0.084</td>
<td>0.071</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.270</td>
<td>0.127</td>
<td>0.071</td>
</tr>
</tbody>
</table>

Step 7 (i) : Determination of the ideal solution ($A^*$)

In order to determine the values of the ideal solution ($A^*$), the minimum value in column $D$ and maximum values in columns $S$ and $O$ from Table 6.11 were chosen according to the procedure given by Jain (2011).

Thus $A^* = \{0.405, 0.211, 0.036\}$

Step 7 (ii) : Determination of the negative ideal solution ($A'$)

In order to determine the values of negative ideal solution ($A'$), the maximum value from column $D$ and minimum values from columns $S$ and $O$ from Table 6.11 were chosen according to the procedure given by Jain (2011).

Thus $A' = \{0.270, 0.084, 0.071\}$

Step 8(i) : Calculation of values of ideal separation ($S_i^*$)

The values of ideal separation measure were determined for each row using the formula given below.
\[ S_1^* = \left[ \sum (v_j^* - v_{ij})^2 \right]^{1/2} \]

where \( v_j^* \) are the values of \( A^* \) which were substituted with the correlated row elements of weighted normalized matrix elements \( (v_{ij}) \).

As a sample, the calculation of the value of \( S_1^* \) of \( M_1 \) using row values of \( v_{ij} \) elements is given below.

\[
S_1^* = \left[ (0.405 - 0.405)^2 + (0.211 - 0.211)^2 + (0.036 - 0.036)^2 \right]^{1/2} \\
S_1^* = 0.000
\]

Like the above, the values of ideal separation of measure of \( M_2 \) and \( M_3 \) were calculated. These values are shown in Table 6.12.

**Table 6.12 Values of ideal separation (\( S_i^* \))**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of ideal separation (( S_i^* ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>0.000</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>0.132</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>0.163</td>
</tr>
</tbody>
</table>

**Step 8(ii) : Calculation of negative ideal separation (\( S_i' \))**

The values of negative ideal separation were determined for each row using the formula given below.

\[ S_i' = \left[ \sum (v_j' - v_{ij})^2 \right]^{1/2} \]

where \( v_j' \) are the values of \( A' \) which were substituted with the correlated row elements of weighted normalized matrix elements \( (v_{ij}) \).
As a sample, the calculation of the value of $S_1'$ of $M_1$ using row values of $v_{ij}$ elements is given below.

$$S_1' = \sqrt{\sum (v_{1}' - v_{ij})^2} = \sqrt{(0.270 - 0.405)^2 + (0.084 - 0.211)^2 + (0.071 - 0.036)^2}$$

$$S_1' = 0.189$$

Like the above, the values of negative ideal separation of $M_2$ and $M_3$ were calculated. These values are shown in Table 6.13.

**Table 6.13 Values of negative ideal separation ($S_i'$)**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of negative ideal separation ($S_i'$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>0.189</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.135</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.043</td>
</tr>
</tbody>
</table>

**Step 9** : Calculation of the relative closeness to the ideal solution ($C_i^*$)

The value of relative closeness to the ideal solution was calculated using the following formula.

$$C_i^* = \frac{S_i'}{S_i' + S_i^*}$$

As a sample, the calculation of the value of $C_1^*$ of $M_1$ using the values of $S_1^*$ and $S_1'$ is given below.

$$C_1^* = \frac{S_1'}{S_1' + S_1^*} = \frac{0.189}{0.043 + 0.189} = 1.00$$
Like the above, the values of relative closeness to ideal solution of $M_2$ and $M_3$ were calculated. These values are shown in Table 6.14.

**Table 6.14 Values of relative closeness ($C_i^*$)**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of relative closeness ($C_i^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>1.00</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.51</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.21</td>
</tr>
</tbody>
</table>

**Step 10**: Ranking the preference order

As shown in Table 6.14, the relative closeness value $C_i^*$ for mould design $M_3$ is 0.07. This is the value closest to 0. Therefore, mould design $M_3$ is the most preferable mould design. This selection corroborates the preference made using FEAROM. This corroboration indicates that the validity of FEAROM model in selecting the most preferable mould design.

**6.4 APPLICATION OF FEAROM MODEL IN THE FBV VALVE BODY CASTING**

FEAROM was also applied on a component called as FBV valve body with flanged ends of size 2 9/16" with pressure rating of 5000 psi. This component is made up of cast steel A487 – grade 4. FEAROM was applied with the objective of predicating and finalizing the appropriate mould designs that are required to produce the castings of FBV valve body. The wooden patterns required for producing this component is made based on the drawing supplied by the customer.
Three mould designs were designed by the engineer working in the design department of Harie steel. These patterns were used to produce CO$_2$ sand moulds in preproduction trials of the FBV body. In mould design 1 ($M_1$), twelve chills of $70 \times 90 \times 80$ mm size, ingate of size $45 \times 15$ and runner of size $50 \times 20$ were used. Three risers of size $\phi 100 \times 150$ mm and one riser of size $\phi 50 \times 100$ mm were incorporated to ensure complete filling of the molten metal in the mould.

In mould design 2 ($M_2$), twelve chills of $70 \times 90 \times 80$ mm size, ingate of size $35 \times 20$ and runner of size $50 \times 25$ were used. Two risers of size $\phi 100 \times 150$ mm, one blind riser of size $\phi 50 \times 100$ mm and two risers of size $\phi 88 \times 150$ mm were incorporated to ensure complete filling of the molten metal in the mould.

In mould design 3 ($M_3$), twelve chills of size $70 \times 90 \times 80$ mm, ingate of size $45 \times 15$ and runner of size $50 \times 20$ were used. Two risers of size $\phi 100 \times 150$ mm, one riser of size $\phi 50 \times 100$ mm and two risers of size

![Figure 6.7 Mould Designs M1, M2 and M3 of FBV body](image)

$M_1 \rightarrow 3$ risers
$M_2 \rightarrow 5$ risers
$M_3 \rightarrow 5$ risers
\( \phi 75 \times 150 \) mm were incorporated to ensure complete filling of the molten metal in the mould. As a sample, the pattern made up of wood for mould design 3 (M3) is shown in Figure 6.8.

![Image](image1.png)  
**Figure 6.8 Pattern of mould design 3 (M3) of FBV body**

A sample inspection-ready fettled casting of FBV valve body which was made by using the moulds of M3 is shown in Figure 6.9.

![Image](image2.png)  
**Figure 6.9 Fettled inspection-ready FBV valve body**

### 6.4.1 Development of FMEA table of FBV valve body

In order to gather the information related to FMEA, the quality control manager, production manager and mould design engineer of Harie steel were consulted.
Table 6.15 FMEA table of FBV valve body

<table>
<thead>
<tr>
<th>Serial Number</th>
<th>Brief details of mould designs</th>
<th>Potential Failure Mode</th>
<th>Potential Effect(s) of Failure</th>
<th>Severity</th>
<th>Potential Cause(s)/Mechanism(s) of Failure</th>
<th>Occurrence</th>
<th>Detection method</th>
<th>Detection</th>
<th>RPN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mould design 1 (M₁)</td>
<td></td>
<td>Insets (chills, chaplets) - Category F</td>
<td>Rejected at customer point</td>
<td>Improper directional solidification due to inadequate feeding and wrongly placed chills</td>
<td>5</td>
<td>Radiography</td>
<td>7</td>
<td>280</td>
</tr>
<tr>
<td>Serial Number</td>
<td>Brief details of mould designs</td>
<td>Potential Failure Mode</td>
<td>Potential Effect(s) of Failure</td>
<td>Severity</td>
<td>Potential Cause(s)/Mechanism(s) of Failure</td>
<td>Occurrence</td>
<td>Detection Method</td>
<td>Detection</td>
<td>RPN</td>
</tr>
<tr>
<td>---------------</td>
<td>--------------------------------</td>
<td>------------------------</td>
<td>-------------------------------</td>
<td>----------</td>
<td>-------------------------------------------</td>
<td>------------</td>
<td>-----------------</td>
<td>----------</td>
<td>-----</td>
</tr>
<tr>
<td>2</td>
<td>Mould design 2 (M2) Chills: 70 × 90 × 80 – 12 Numbers Core : 1 No Ingate = 35 × 20 mm Runner = 50 × 25 mm Riser: φ100 × 150 – 2 Numbers φ50 × 100 – 1 Number φ88 × 150 – 2 Numbers Yield: 50%</td>
<td>Shrink type 2, Internal shrinkage</td>
<td>Rejected at manufacturing plant</td>
<td>7</td>
<td>Improper directional solidification due to inadequate feeding</td>
<td>4</td>
<td>Radiography/Ultrasound testing</td>
<td>6</td>
<td>168</td>
</tr>
</tbody>
</table>
### Table 6.15 (Continued)

<table>
<thead>
<tr>
<th>Serial Number</th>
<th>Brief details of mould designs</th>
<th>Potential Failure Mode</th>
<th>Potential Effect(s) of Failure</th>
<th>Severity</th>
<th>Potential Cause(s)/Mechanism(s) of Failure</th>
<th>Occurrence</th>
<th>Detection method</th>
<th>Detection</th>
<th>RPN</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Mould design 3 (M₃) Chills: 70 × 90 × 80 – 12 Numbers Core: 1 Number Ingate = 35 × 25 mm Runner = 50 × 30 mm Riser: φ100 × 150 – 2 Numbers φ 50 × 100 – 1 Number φ 75 × 150 – 2 Numbers Yield: 58%</td>
<td>Shrink, type 1 (Surface shrinkage)</td>
<td>Rejected manufacturing plant</td>
<td>7</td>
<td>Improper directional solidification</td>
<td>4</td>
<td>Radiography/Magnetic particle inspection</td>
<td>7</td>
<td>196</td>
</tr>
</tbody>
</table>
During this consultation, the data required for completing the FMEA table were carefully gathered and recorded. These data were used to complete the FMEA table of FBV valve body, which is shown in Table 6.15.

Table 6.16 Rankling of mould designs of FBV valve body

<table>
<thead>
<tr>
<th>Mould Designs</th>
<th>Values of I(S)</th>
<th>I(O)</th>
<th>I(D)</th>
<th>RPC</th>
<th>Traditional FMEA RPN</th>
<th>FEAROM</th>
<th>Rank Order</th>
<th>Case (a)</th>
<th>Case (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>8</td>
<td>5</td>
<td>7</td>
<td>280</td>
<td>3</td>
<td>L_8</td>
<td>3</td>
<td>L_8</td>
<td>L_8</td>
</tr>
<tr>
<td>M2</td>
<td>7</td>
<td>4</td>
<td>6</td>
<td>168</td>
<td>1</td>
<td>L_7</td>
<td>2</td>
<td>L_7</td>
<td>L_7</td>
</tr>
<tr>
<td>M3</td>
<td>7</td>
<td>4</td>
<td>7</td>
<td>196</td>
<td>2</td>
<td>L_7^*</td>
<td>1</td>
<td>L_6</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: I(S), I(O) and I(D) are the importance associated with each index. RPC calculated under two different cases: (a) and (b). When two or more mould designs have the same RPC index, the symbol (*) is used to highlight the most preferable mould design.

The mould designs were ranked using traditional FMEA and RPC values. The values and rankings of the mould designs are presented in Table 6.16. As shown, S, O, D and RPN pertaining to the three mould designs entered in second, third, fourth and fifth columns of the Table 6.16. The RPN of M_1 is higher while it is lowest in the case of M_2. These mould designs were ranked by following the ascending order of RPN values. As mentioned in the chapter 3, the mould design whose RPN value is the least is ranked first. This
is due to the reason that the failure in the case of mould design with least RPN may be quickly overcome and thus, the development time of the casting can be shortened.

Subsequent to the ranking of mould designs of FBV valve body, the RPC values were calculated by considering the traditional FMEA and practical conditions. The calculation of RPC in both cases is illustrated below.

**Case (a):** Initially, maximum and equal importance of S, O and D was assigned to all the mould designs. This importance is denoted the code number \( L_{10} \). Subsequent to this assumption, the following steps were followed.

**Step 1 :** Calculation of RPCs of mould designs

The following formula was used to calculate the RPCs.

\[
RPC (M_i) = \max \left[ \min \left\{ (I(K_j), g_j(M_i)) \right\} \right] \\
\]

In case of \( M_1 \), \( I(K_j) \) refer to \( L_{10} \). Then, in the same case, \( g_j(M_i) \) is equal to \( L_8, L_5 \) and \( L_7 \). Suffices in L refer to S, O and D respectively. These values are indicated in Table 6.16.

Hence,

\[
RPC (M_1) = \max \left[ \min \left\{ (I(K_j), g_j(M_1)) \right\} \right] \\
= \max \left[ \min (L_{10}, L_8), \min (L_{10}, L_5), \min (L_{10}, L_7) \right] \\
RPC (M_1) = \max [L_8, L_5, L_7] = L_8
\]
The calculation of RPCs in the case of $M_2$ and $M_3$ is presented below.

$$RPC(M_2) = \max \{\min \{I(K_j), g_j(M_2)\}\} = \max \{L_7, L_4, L_6\} = L_7$$

$$RPC(M_3) = \max \{\min \{I(K_j), g_j(M_3)\}\} = \max \{L_7, L_4, L_7\} = L_7$$

In order to rank the mould designs, it is required to calculate CPM. The steps followed to calculate CFM is presented below.

**Step 2**: Ranking using Critical Failure Mode (CFM)

The formula used to calculate CFM is presented below:

$$CFM(M^*) = \min_{M_i \in A} \{RPC(M_1), RPC(M_2), RPC(M_3)\}$$

where $A$ refers to the set of mould designs.

Now,

$$CFM(M^*) = \min \{L_8, L_7, L_7\}$$

$$= L_7 = RPC(M_2) = RPC(M_3)$$

As revealed above, preferred mould design is either $M_3$ or $M_4$ since their ranks are same. But still a tie exists between the two mould designs. This tie could be overcome by using the tie ranking rule as follows.

Since tie occurs in case $M_2$ versus $M_3$, the same (tie) is to be broken by using the following formula.

$$T(M_i) = N(M_i)$$
where \( N(M_i) \) is the number of occurrences of S, O and D in each mould designs that are lesser than \( L_7 \).

As shown in Table 6.16, the values of O and D of \( M_2 \) are 4 and 6 respectively. These values are lower than \( L_7 \) (that is 7). Hence:

\[
T(M_2) = 2
\]

Likewise, \( T(M_3) = 1 \)

As shown above, \( T(M_3) < T(M_2) \). Hence, the preferred mould design is \( M_3 \). Hence the order of preference of mould designs is \( M_3, M_2 \) and \( M_1 \). These details are presented in Table 6.16.

**Case (b):** In case (a) equal importance was assumed to S, O and D, but in reality, these values may differ. In the case of DFC valve mould designs the values assigned by the Quality Control Manager and Managing Director varied against S, O and D were gathered. These values are indicated in Figure 6.10.

![Figure 6.10 Importance of S, O and D in case (b) for FBV valve body](image)

**Figure 6.10 Importance of S, O and D in case (b) for FBV valve body**
The same information is mathematically denoted below:

\[ I(S) = L_{10}, \ I(O) = L_8, \ I(D) = L_6 \]

The ranking method that progressed through two steps is presented below:

**Step 1**  :  Calculation of RPC

The calculation of RPCs is shown below.

RPC \( (M_1) = \text{Max} \left[ \text{Min} (L_{10}, L_8), \text{Min} (L_8, L_5), \text{Min} (L_6, L_6) \right] \)

\[ = \text{Max} \left[ L_8, L_5, L_6 \right] = L_8 \]

RPC \( (M_2) = \text{Max} \left[ L_7, L_4, L_6 \right] = L_7 \)

RPC \( (M_3) = \text{Max} \left[ L_7, L_4, L_6 \right] = L_7 \)

**Step 2**  :  Ranking using CFM

The RPC values of the three mould designs are same as that was obtained in case (a). Hence, the ranking order as determined under case (a) remained the same.

**6.4.2 Validation using MFTOPSIS Method Hybrid with AHP**

The ranking of mould designs was validated using the MFTOPSIS method hybrid with AHP under the cases (a) and (b). The details of this validation are presented in the following subsections.
6.4.2.1 Validation under Case (a)

While applying MFTOPSIS method hybrid with AHP, the mould designs M\(_1\), M\(_2\) and M\(_3\) were considered as alternatives and the indices S, O, and D as the evaluation criteria. Subsequently ten steps were followed. These details are presented in the following steps.

**Step 1**: Determination of relative weights

The relative weight of indices and mould designs were calculated using equation 3.5 (given in chapter 3). These calculations are shown below.

Relative weight between S and O: \((L_n \sim L_i) + 1 = (10 \sim 10) + 1 = 1\)

Relative weight between S and D: \((L_n \sim L_i) + 1 = (10 \sim 10) + 1 = 1\)

Relative weight between O and D: \((L_n \sim L_i) + 1 = (10 \sim 10) + 1 = 1\)

As shown above, an equal importance with value \(L_{10}\) was assigned to the indices S, O and D. Hence, the \(L_n\) and \(L_i\) values were 10.

**Step 2**: Determination of pairwise comparison matrix (A)

The relative weights were used to develop pairwise matrix. The method of developing this pairwise matrix is described here. When the same index is compared, then the weight is 1. When different indices compared, then the weight is reciprocal to one another. This is to maintain consistency of the matrix. Hence, the weight of S when compared with O is 1. Then the weight of O when compared with S is \(1 \div 1 = 1\). Likewise, O when compared with D is 1 and hence D when compared with O is \(1 \div 1 = 1\). These values are shown below as pairwise matrix.
Step 3 : Formation of normalized matrix

The values of the elements of the normalized matrix were calculated using the following formula.

\[
\text{Value of the element of the normalized matrix} = \frac{\text{Original value from pairwise matrix}}{\text{Total column value}}
\]

As a sample, the method of determining the value of the elements using above formula when compared with S of S is mentioned here. For example, in the case of pairwise matrix, the original value against S and S is 1. The total value of the first column is 3. Hence, the value of this element in normalized matrix is \( \frac{1}{3} \). The normalized matrix thus developed is shown below.

\[
\begin{bmatrix}
S & 1/3 & 1/3 & 1/3 \\
O & 1/3 & 1/3 & 1/3 \\
D & 1/3 & 1/3 & 1/3 \\
\end{bmatrix}
\]

The weights of evaluation criteria were same as their row average. Accordingly, the weights of the evaluation criteria are presented below.

Severity (S) = 0.33
Occurrence (O) = 0.33
Detection (D) = 0.33
The above weights are presented as matrix below.

\[ W = [S \quad O \quad D] = [0.33 \quad 0.33 \quad 0.33] \]

**Step 4** : Checking for consistency

The consistency matrix was developed by multiplying pairwise comparison matrix (A) and evaluation criteria column matrix \((W^T)\).

\[
AW^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.33 \\ 0.33 \\ 0.33 \end{bmatrix} = \begin{bmatrix} 0.99 \\ 0.99 \\ 0.99 \end{bmatrix}
\]

Subsequently, the largest eigen value \((\mu)\) was calculated. The formula used to calculate the largest eigen value \((\mu)\) is,

\[
\mu = \frac{1}{n} \sum_{i=1}^{n} i^{th \text{ entry in } AW^T} \quad \text{where} \quad n = \text{order of } A
\]

The calculation of the larger eigen value \((\mu)\) using the above formula is presented below.

\[
\mu = \frac{1}{3} \left( \frac{0.99}{0.33} + \frac{0.99}{0.33} + \frac{0.99}{0.33} \right) = 3
\]

The calculation of the consistency ratio (CR) is presented below.

\[
\text{CR} = \frac{\text{Consistency index (CT)}}{\text{Average index of randomly generated weights (ACI)}} = \frac{\mu - n}{n - 1}
\]
where the values of ACI are depended on the order of the matrix and was taken from table 3.9 (given in chapter 3).

\[
CR = \frac{\frac{2-0}{2-1}}{0.58} = 0.
\]

Because CR value is less than 10\%, the normalized matrix formed was consistent. After checking the consistency, MFTOPSIS method is applied by considering the weight calculated in step 3. The weight and the values of S, O and D are presented in decision matrix shown in Table 6.17.

**Table 6.17 Decision matrix**

<table>
<thead>
<tr>
<th>Weight , ( w_i )</th>
<th>0.33</th>
<th>0.33</th>
<th>0.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluation criteria</td>
<td>S</td>
<td>O</td>
<td>D</td>
</tr>
<tr>
<td>Alternatives</td>
<td>Values of decision matrix elements ( (y_{ij}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₁</td>
<td>8</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>M₂</td>
<td>7</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>M₃</td>
<td>7</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

**Step 5**: Formulation of normalized decision matrix

The normalized decision matrix was formulated by inputting the correlation values of the elements of decision matrix. The formula used to calculate the correlation of the elements is given below.

\[
r_{ij} = \frac{y_{ij}}{\sqrt{\sum y_{ij}^2}}
\]

As shown, the values of \((\sum y_{ij}^2)^{1/2}\) against each column were calculated. Then each column element was divided by that to get \(r_{ij}\) values.
As a sample, the calculation of the element covering M₁ and S in decision matrix is given below. The value of this element is 8. Therefore,

\[ r_{ij} = \frac{8}{\sqrt{8^2 + 5^2 + 7^2}} = 0.629 \]

Like the above, the correlation values of all elements of the decision matrix were calculated. These values were used to formulate the normalized decision matrix shown in Table 6.18.

**Table 6.18 Normalized decision matrix**

<table>
<thead>
<tr>
<th>Attributes</th>
<th>S</th>
<th>O</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternatives</td>
<td>Values of normalized decision matrix elements (r_{ij})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₁</td>
<td>0.629</td>
<td>0.662</td>
<td>0.605</td>
</tr>
<tr>
<td>M₂</td>
<td>0.550</td>
<td>0.530</td>
<td>0.518</td>
</tr>
<tr>
<td>M₃</td>
<td>0.550</td>
<td>0.530</td>
<td>0.605</td>
</tr>
</tbody>
</table>

**Step 6** : Formulation of weighted normalized matrix

The values of the weighted normalized matrix elements (v_{ij}) were obtained by multiplying each column elements by w₁. As a sample, the calculation of the element covering M₁ and S in normalized decision matrix is given below. The value of this element is 0.629. Therefore,

\[ v_{11} = w₁ \times r_{11} = 0.33 \times 0.629 = 0.208 \]
Like the above, the correlation values of all elements of the normalized decision matrix are calculated. These values were used to formulate the weighted normalized matrix shown in Table 6.19.

**Table 6.19 Weighted normalized matrix**

<table>
<thead>
<tr>
<th>Weight, $w_i$</th>
<th>0.33</th>
<th>0.33</th>
<th>0.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attributes</td>
<td>S</td>
<td>O</td>
<td>D</td>
</tr>
<tr>
<td>Alternatives</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_1$</td>
<td>0.208</td>
<td>0.218</td>
<td>0.200</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.182</td>
<td>0.175</td>
<td>0.171</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.182</td>
<td>0.175</td>
<td>0.200</td>
</tr>
</tbody>
</table>

**Step 7 (i):** Determination of the ideal solution ($A^*$)

According to the procedure given by Jain (2011), the minimum value in column D and maximum values in columns S and O from Table 6.19 were chosen.

Thus $A^* = \{0.208, 0.218, 0.171\}$

**Step 7 (ii):** Determination of the negative ideal solution ($A'$)

According to the procedure given by Jain (2011), the maximum value from column D and minimum values from columns S and O from Table 6.19 were chosen.

Thus $A' = \{0.182, 0.175, 0.200\}$
Step 8(i) : Calculation of values of ideal separation ($S_i^*$)

The values of ideal separation were determined for each row using the formula given below.

$$S_i^* = \left[ \sum (v_j^* - v_{ij})^2 \right]^{1/2}$$

where $v_j^*$ are the values of $A^*$ which were substituted with the correlated row elements of weighted normalized matrix elements ($v_{ij}$).

### Table 6.20 Values of ideal separation ($S_i^*$)

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of ideal separation ($S_i^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>0.029</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.050</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.058</td>
</tr>
</tbody>
</table>

As a sample, the calculation of the value of $S_1^*$ of $M_1$ using row values of $v_{ij}$ elements is given below.

$$S_1^* = \left[ \sum (v_1^* - v_{1j})^2 \right]^{1/2} = [(0.208 - 0.208)^2 + (0.218 - 0.218)^2 + (0.171 - 0.200)^2]^{1/2}$$

$$S_1^* = 0.029$$

Like the above, the values of ideal separation of measure of $M_2$ and $M_3$ were calculated. These values are shown in Table 6.20.

Step 8(ii) : Calculation of negative ideal separation ($S_i'$)

The values of negative ideal separation were determined for each row using the formula given below.
\[ S_i' = [\sum (v_{ij}' - v_{ij})^2]^{1/2} \]

where \( v_{ij}' \) are the values of \( A' \) which were substituted with the correlated row elements of weighted normalized matrix elements \( (v_{ij}) \).

**Table 6.21 Values of negative ideal separation \( (S_i') \)**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of negative ideal separation ( (S_i') )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>0.050</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>0.029</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>0.000</td>
</tr>
</tbody>
</table>

As a sample, the calculation of the value of \( S_1' \) of \( M_1 \) using row values of \( v_{ij} \) elements is given below.

\[ S_1' = [\sum (v_{1j}' - v_{1j})^2]^{1/2} = [(0.182 - 0.208)^2 + (0.175 - 0.218)^2 + (0.200 - 0.200)^2]^{1/2} \]

\[ S_1' = 0.050 \]

Like the above, the values of negative ideal separation of \( M_2 \) and \( M_3 \) were calculated. These values are shown in Table 6.21.

**Step 9** : Calculation of the relative closeness to the ideal solution \( (C_i^*) \)

The value of relative closeness to the ideal solution was calculated using the following formula.

\[ C_i^* = \frac{S_i^*}{S_i^* + S_i} \]
As a sample, the calculation of the value of $C_1^+$ of $M_1$ using the values of $S_1^+$ and $S_1'$ is given below.

$$C_1^+ = \frac{S_1'}{S_1' + S_1} = \frac{0.05}{0.029 + 0.05} = 0.63$$

Like the above, the values of relative closeness to ideal solution of $M_2$ and $M_3$ were calculated. These values are shown in Table 6.22.

**Table 6.22 Values of relative closeness ($C_i^+$)**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of relative closeness ($C_i^+$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>0.63</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.37</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Step 10 :** Ranking the preference order

As shown in Table 6.22, the relative closeness value $C_3^+$ for mould design $M_3$ is 0.00. This is the value closest to 0. Therefore, mould design $M_3$ is the most preferable mould design. This selection corroborates the preference made using FEAROM. This corroboration indicates that the validity of FEAROM model in selecting the most preferable mould design.

**6.4.2.2 Validation under case (b)**

As shown in Figure 6.10, the importance of values of indices $S$, $O$ and $D$ are 10, 8, and 6 respectively.

**Step 1 :** The relative weights of indices were calculated using equation 3.5 (given in chapter 3). These calculations are shown below.
Relative weight between S and O: \( (L_n - L_i) + 1 = (10 - 8) + 1 = 3 \)

Relative weight between S and D: \( (L_n - L_i) + 1 = (10 - 6) + 1 = 5 \)

Relative weight between O and D: \( (L_n - L_i) + 1 = (8 - 6) + 1 = 3 \)

As shown above, an importance with value \( L_{10}, L_8 \) and \( L_6 \) was assigned to the indices S, O and D respectively. The \( L_n \) values were 10 and 8. The \( L_i \) values were 8 and 6.

**Step 2**: Determination of pairwise comparison matrix (A)

The relative weights were used to develop pairwise matrix is shown below. When the same index is compared, the weight is 1. Comparing severity with occurrence, for case (b), the FMEA team of Harie steel estimated the severity was 3 times more than the occurrence. Then for the matrix to be consistent occurrence when compared with severity should be \( 1/3 \). Likewise, O versus D is 5 and hence D versus O is \( 1/5 \). These values are shown below.

\[
\begin{array}{c|ccc}
  & S & O & D \\
\hline
  S & 1 & 3 & 5 \\
  O & 1/3 & 1 & 3 \\
  D & 1/5 & 1/3 & 1 \\
\end{array}
\]

\[ A = \begin{bmatrix}
  1 & 1/3 & 3 \\
  1/3 & 1 & 3 \\
  1/5 & 1/3 & 1 \\
\end{bmatrix} \]

**Step 3**: Formation of normalized matrix

Value of the element of the normalized matrix = \( \frac{\text{Original value from pairwise matrix}}{\text{Total column value}} \)
The method of calculating the elements of normalized matrix with the above equation is illustrated here. For example, in the case of pairwise matrix, the original value against S and O is 3. The total value of the first column is 4.33. Hence, the value of this element in normalized matrix is \( \frac{3}{4.33} = 0.69 \). The normalized matrix thus formed is presented below:

\[
\begin{array}{ccc}
S & O & D \\
\hline
S & 0.65 & 0.69 & 0.56 \\
O & 0.22 & 0.23 & 0.33 \\
D & 0.13 & 0.08 & 0.11 \\
\end{array}
\]

The weights of evaluation criteria were same as their row average. Accordingly, the weights of the evaluation criteria are presented below.

Severity (S) = 0.663  
Occurrence (O) = 0.260  
Detection (D) = 0.107

The same is expressed in matrix form as below.

\[
W = [S \ O \ D] = 0.663 \ 0.260 \ 0.107
\]

**Step 4 : Checking for consistency**

The consistency matrix was developed by multiplying pairwise comparison matrix (A) and evaluation criteria column matrix \((W^T)\).
Subsequently, the largest eigen value (µ) was calculated. The formula used to calculate the largest eigen value (µ) is,

$$\mu = \frac{1}{n} \sum_{i=1}^{n} \frac{i^{th} \text{ entry in } AW^T}{i^{th} \text{ entry in } W^T} \quad \text{where } n = \text{order of } A$$

The calculation of the larger eigen value (µ) using the above formula is presented below

$$\mu = \frac{1}{3} \left( \frac{1.95}{0.663} + \frac{0.79}{0.260} + \frac{0.32}{0.107} \right) = 3.04$$

The calculation of the consistency ratio (CR) is shown below.

$$CR = \frac{\text{Consistency index (CT)}}{\text{Average index of randomly generated weights (ACI)}} = \frac{\mu - n}{n - 1} \quad \text{where the values of ACI are depended on the order of the matrix and was taken from Table 3.9 (given in chapter 3).}$$

$$CR = \frac{\left( \frac{3.04 - 3}{3 - 1} \right)}{0.56} = 0.034 = 3.4\%$$

Because CR value is less than 10%, the normalized matrix formed was consistent. After checking the consistency, MFTOPSIS method is applied by considering the weight calculated in step 3. The weight and the values of S, O and D are presented in decision matrix shown in Table 6.23.
Table 6.23 Decision matrix

<table>
<thead>
<tr>
<th>Evaluation criteria</th>
<th>Alternatives</th>
<th>( y_{ij} ) elements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( M_1 )</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>( M_2 )</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>( M_3 )</td>
<td>7</td>
</tr>
</tbody>
</table>

Step 5: Formulation of normalized decision matrix

The procedure followed for the formulation of normalized matrix is same as that were described in step 5 under case (a). Hence, the values of normalized decision matrix elements \( r_{ij} \) presented in Table 6.18 are in Table 6.24.

Table 6.24 Normalized decision matrix

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of normalized decision matrix elements ( r_{ij} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>0.629</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>0.550</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>0.550</td>
</tr>
</tbody>
</table>

Step 6: Formulation of weighted normalized matrix

The formulation of the weighted normalized matrix elements \( v_{ij} \) was obtained by multiplying each column elements by \( w_i \). As a sample, the
calculation of the element covering \( M_1 \) and \( S \) in normalized decision matrix is given below. The value of this element is 0.629. Therefore,

\[
v_{11} = w_1 \times r_{11} = 0.633 \times 0.629 = 0.398
\]

Like the above, the correlation values of all elements of the normalized decision matrix were calculated. These values were used to formulate the weighted normalized matrix shown in Table 6.25.

**Table 6.25 Weighted normalized matrix**

<table>
<thead>
<tr>
<th>Weight, ( w_i )</th>
<th>0.663</th>
<th>0.260</th>
<th>0.107</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attributes</td>
<td>S</td>
<td>O</td>
<td>D</td>
</tr>
<tr>
<td>Alternatives</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_1 )</td>
<td>0.398</td>
<td>0.172</td>
<td>0.065</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>0.348</td>
<td>0.138</td>
<td>0.055</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>0.348</td>
<td>0.138</td>
<td>0.065</td>
</tr>
</tbody>
</table>

**Step 7 (i) :** Determination of the ideal solution (\( A^* \))

In order to determine the ideal solution (\( A^* \)), the minimum value in column D and maximum values in columns S and O from Table 6.25 were chosen according to the procedure given by Jain (2011).

Thus \( A^* = \{0.398, 0.172, 0.055\} \)

**Step 7 (ii) :** Determination of the negative ideal solution (\( A' \))

In order to determine the negative ideal solution (\( A' \)), the maximum value from column D and minimum values from columns S and O from Table 6.25 was chosen according to the procedure given by Jain (2011).
Thus $A' = \{0.348, 0.138, 0.065\}$

**Step 8(i) :** Calculation of values of ideal separation ($S_i^*$)

The values of ideal separation were determined for each row using the formula given below.

$$S_i^* = \left[ \sum (v_j^* - v_{ij})^2 \right]^{1/2}$$

where $v_j^*$ are the values of $A^*$ which were substituted with the correlated row elements of weighted normalized matrix elements ($v_{ij}$).

As a sample, the calculation of the value of $S_1^*$ of $M_1$ using row values of $v_{ij}$ elements is given below.

$$S_1^* = \left[ \sum (v_1^* - v_{1j})^2 \right]^{1/2} = [(0.398 - 0.398)^2 + (0.172 - 0.172)^2$$
$$+ (0.055 - 0.065)^2]^{1/2}$$

$$S_1^* = 0.010$$

Like the above, the values of ideal separation of $M_2$ and $M_3$ were calculated. These values are shown in Table 6.26.

**Table 6.26 Values of ideal separation ($S_i^*$)**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of ideal separation ($S_i^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>0.010</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.060</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.061</td>
</tr>
</tbody>
</table>
**Step 8(ii)**: Calculation of negative ideal separation ($S_i'$)

The values of negative ideal separation were determined for each row using the formula given below.

$$S_i' = \left[ \sum (v_{ij}' - v_{ij})^2 \right]^{1/2}$$

where $v_{ij}'$ are the values of $A'$ which were substituted with the correlated row elements of weighted normalized matrix elements ($v_{ij}$).

As a sample, the calculation of the value of $S_1'$ of $M_1$ using row values of $v_{ij}$ elements is given below.

$$S_1' = \left[ \sum (v_{1j}' - v_{1j})^2 \right]^{1/2} = [(0.348 - 0.398)^2 + (0.138 - 0.172)^2 + (0.055 - 0.065)^2]^{1/2}$$

$$S_1' = 0.06$$

Like the above, the values of negative ideal separation of $M_2$ and $M_3$ are calculated. These values are shown in Table 6.27.

**Table 6.27 Values of negative ideal separation ($S_i'$)**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of negative ideal separation ($S_i'$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>0.06</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.01</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Step 9 : Calculation of the relative closeness to the ideal solution (C*$_i$)

The value of relative closeness to the ideal solution was calculated using the following formula.

$$C_i^* = \frac{S_i}{S_i^* + S_i'}$$

As a sample, the calculation of the value of C$_i^*$ of M$_1$ using the values of S$_i^*$ and S$_i'$ is given below.

$$C_1^* = \frac{S_1'}{S_1^* + S_1'} = \frac{0.189}{0.800 + 0.189} = 1.00$$

Like the above, the values of relative closeness to ideal solution of M$_2$ and M$_3$ were calculated. These values are shown in Table 6.28.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Values of relative closeness (C$_i^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M$_1$</td>
<td>0.86</td>
</tr>
<tr>
<td>M$_2$</td>
<td>0.14</td>
</tr>
<tr>
<td>M$_3$</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Step 10 : Ranking the preference order

As shown in Table 6.28, the relative closeness value C$_i^*$ for mould design M$_3$ was 0.00. This is the value closest to 0. Therefore, mould design M$_3$ is the most preferable mould design. This selection corroborates the preference made using FEAROM. This corroboration indicates that the validity of FEAROM model in selecting the most preferable mould design.
6.5 CONCLUSION

In this chapter, the experiences of applying FEAROM in Harie steel which is a foundry manufacturing valve body components using CO\textsubscript{2} sand casting method have been reported. First, FEAROM was applied in the case of manufacturing bearing housing casting. Three mould designs that were used for manufacturing this component were considered. On applying FEAROM mould design coded as M\textsubscript{3} was chosen. The same practice was followed in the case of manufacturing another component called FBV valve body casting. On applying the FEAROM in the case of manufacturing this casting, out of the three mould designs that were developed by the engineers of this company, the mould design coded as M\textsubscript{3} was chosen. Subsequent to these selections, the same were validated by applying MFTOPSIS method hybrid with AHP. The results of this validation well agreed with that were obtained by applying FEAROM on bearing housing and FBV valve body castings. Thus, the practical and scientific validations of FEAROM were established by applying MFTOPSIS method hybrid with AHP in the valve body castings in the foundry.