APPENDIX - B

Taking into consideration the distance 'u' of the laser diode from the UDTL end, 'q' parameters of the Gaussian beams at the input laser facet and output lens fiber interface can be related by the ABCD matrix as follows

\[ q_2 = \frac{(Aq_1 + Au + B)}{(Cq_1 + Cu + D)} \]  

(B.1)

where

\[ \frac{1}{q_{1,2}} = \frac{1}{R_{1,2}} - \frac{j \lambda_0}{(\pi W_{1,2}^2 n_{1,2})} \]  

(B.2)

with A,B,C and D being the matrix elements of the lens matrix and given as

\[ A = r_2(z) - \frac{(1 - n)}{n R_0} r_1(z); \]  

(B.3)

\[ B = \frac{r_1(z)}{n}; \]  

(B.4)

\[ C = \frac{dr_2(z)}{dz} + \frac{(1 - n)}{n R_0} \frac{dr_1(z)}{dz} \]  

(B.5)

and

\[ D = \frac{1}{n} \frac{dr_1(z)}{dz}; \]  

(B.6)
where \( n = \frac{n_2}{n_1} \).

The \( z \)-dependence of the above matrix elements can be explicitly expressed by substituting

\[
\begin{align*}
    r_1(z) &= \frac{L}{\alpha} \frac{1}{(1 - \frac{z}{L})^{1/2}} \sin k(z) \quad \text{(B.7)}
\end{align*}
\]

\[
\begin{align*}
    \frac{dr_1(z)}{dz} &= \frac{1}{(1 - \frac{z}{L})^{1/2}} \left\{ \cos k(z) + \frac{1}{2\alpha} \sin k(z) \right\} \quad \text{(B.8)}
\end{align*}
\]

\[
\begin{align*}
    r_2(z) &= (1 - \frac{z}{L})^{1/2} \left\{ \cos k(z) - \frac{1}{2\alpha} \sin k(z) \right\} \quad \text{(B.9)}
\end{align*}
\]

and

\[
\begin{align*}
    \frac{dr_2}{dz} &= \frac{A_0^2 L}{\alpha(1 - \frac{z}{L})^{1/2}} \sin k(z) \quad \text{(B.10)}
\end{align*}
\]

where \( k(z) = \alpha \ln(1 - \frac{z}{L}) \) and \( \alpha = \left( A_0^2 \frac{L^2}{4} - 1/4 \right)^{1/2} \), \( L \) is the length of the tapered cone and \( A_0 \) is a constant given by \( A_0 = \frac{1}{D} \frac{n_{co}}{n_{cl}} \) for an
UDTL having aperture 2D. In order to obtain $W_{2X,2Y}$ we evaluate the matrix for $Z_L = L(\frac{D-a}{D})$.

The transformed spot sizes and radii of curvature are related through the matrix elements with the input spot size and radius of curvature as follows

$$W_{2X,2Y} = \frac{A_1^2 W_{1X,1Y}^2 + (\lambda_1^2 B_1^2) / (\pi^2 W_{1X,1Y}^2)}{n (A_1 D_1 - B_1 C_1)}$$  \hspace{1cm} \text{(B.11)}$$

$$\frac{1}{R_{2X,2Y}} = \frac{A_1 C_1 W_{1X,1Y}^2 + (\lambda_1 B_1 D_1) / (\pi^2 W_{1X,1Y}^2)}{A_1^2 W_{1X,1Y}^2 + (\lambda_1 B_1^2) / (\pi^2 W_{1X,1Y}^2)}$$  \hspace{1cm} \text{(B.12)}$$

with $B_1 = Au + B$, $D_1 = Cu + D$, $A_1 = A + \frac{B_1}{R_1}$ and $C_1 = C + \frac{D_1}{R_1}$

where $R_1$ is the radius of curvature of the wavefront from the laser facet and in our plane wavefront modal $R_1 \to \infty$ and $\lambda_1 = \frac{\lambda_0}{n_1}$. 