Appendix - 1

Rheological Properties Measurement

APPENDIX-1

The rheological properties of the SCMC solutions are measured by using pipeline viscometer.

Theoretical consideration

In pipeline viscometer, let us consider a short segment in the test section with diameter D and length ΔL, the flow rate through the segment can be calculated as,

$$Q = 2\pi \int_0^R U(r) r dr \tag{A1}$$

where U(r) is the axial velocity profile. Integrating Eq. (A1) and assuming that $U(R) = 0$ (i.e. no-slip condition at the pipe wall), we get,

$$Q = -\pi \int_0^R r^2 \frac{dU}{dr} dr \tag{A2}$$

For steady state flow of fluid with constant density, the momentum balance yields the following expression,

$$\frac{\tau(r)}{r} = \frac{\tau_w}{R} \tag{A3}$$

where τ_w is the shear stress at pipe wall given by,

$$\tau_w = \frac{R}{2}\left(-\frac{\Delta P}{\Delta L}\right) \tag{A4}$$

Changing variables, Eq. (A2) becomes,

$$Q = -\pi \int_0^{\tau_w} \left(\frac{R}{\tau_w}\right)^3 \frac{dU}{dr} \tau^2 d\tau \tag{A5}$$

Eq. (A5) represents a general relationship between flow rate and shear stress. Denoting $\frac{dU}{dr} = f(\tau)$ and differentiating with respect to τ_w upon rearrangement, we get,

$$\frac{d\left(Q\tau_w^3\right)}{d\tau_w} = -\pi R^3 f\left(\tau_w\right)\tau_w^2 \tag{A6}$$

Hence, the shear rate at the pipe wall is,

$$f\left(\tau_w\right) = \left(-\frac{dU}{dr}\right)_R = \gamma_w = \frac{1}{\pi R^3 \tau_w^2}\frac{d(Q\tau_w^3)}{d\tau_w}$$

Or (A7)

$$\gamma_w = \frac{1}{\pi R^3}\tau_w \frac{dQ}{d\tau_w} + \frac{3Q}{\pi R^3} \tag{A8}$$

Since, $\frac{Q}{\pi R^3} = \frac{2U}{D}$ Eq. (A8) can be written in terms of mean velocity (U) and pipe diameter (D) as below:

$$\gamma_w = \frac{\tau_w}{4}\frac{d\left(\frac{8U}{D}\right)}{d\tau_w} + \frac{3}{4}\left(\frac{8U}{D}\right) \tag{A9}$$

Note that the following is true,

$$\tau_w \frac{d\left(\ln \tau_w\right)}{d\tau_w} = \frac{8U}{D}\frac{d\left(\ln \frac{8U}{D}\right)}{d\left(\frac{8U}{D}\right)} \tag{A10}$$

From Eq. (A10), we get,

$$\frac{d\left(\frac{8U}{D}\right)}{d\tau_w} = \frac{\frac{8U}{D}}{\tau_w}\frac{d\left(\ln \frac{8U}{D}\right)}{d\left(\ln \tau_w\right)} \tag{A11}$$

Substituting (A9) to (A11), we obtain:

$$\gamma_w = \frac{1}{4}\left[3 + \frac{d\left(\ln \frac{8U}{D}\right)}{d\left(\ln \tau_w\right)}\right]\left(\frac{8U}{D}\right) \tag{A12}$$

Introducing the flow behavior index (n), the above equation can be written as follows,

$$\gamma_w = \left(\frac{3n+1}{4n}\right)\frac{8U}{D} \tag{A13}$$

where flow behavior index, n, is expressed as,

$$n = \frac{d(\ln \tau_w)}{d\left(\ln \frac{8U}{D}\right)} \tag{A14}$$

Since for the pseudo plastic fluids

$$f(\tau) = \left(\frac{\tau}{K}\right)^{\frac{1}{n}} \tag{A15}$$

$$\tau_W = K\left(\frac{2U}{D}\right)^n \left(\frac{3n+1}{n}\right)^n \tag{A16}$$

Or, $$\tau_W = K\left(\frac{8U}{D}\right)^n \left(\frac{3n+1}{4n}\right)^n \tag{A17}$$

In Eq. (A17) it is clear that if a logarithmic plot is made between τ_w and $\left(\frac{8U}{D}\right)$, a linear relation will result, from the slope and intercept the values on n and K can be calculated.

Experimental Procedure

A horizontal steel tube of 0.635 cm internal diameter is used as the pipeline viscometer with pressure tapping at a distance of 1.85 m. Measurements on pressure drop are made in the fully developed flow region of non-Newtonian liquids, in the laminar flow condition. The developed flow region is ensured by providing the necessary and sufficient straight entry length, i.e., more than 50 pipe diameter length of the tube. The inlet end of the tube is well rounded to ensure smooth and parallel flow at the entrance.

Results and discussion

From the data on pressure drop and flow rate, the values of τ_w and $\left(\frac{8U}{D}\right)$ are calculated for four different SCMC solutions. The rheogram are shown in Fig. A1.1 and represent typical flow curves for pseudoplastic behavior in the shear rate range 32 – 950 s^{-1}. The values of *n*, *K* and other physical properties of the liquids are shown in Table 2.2.

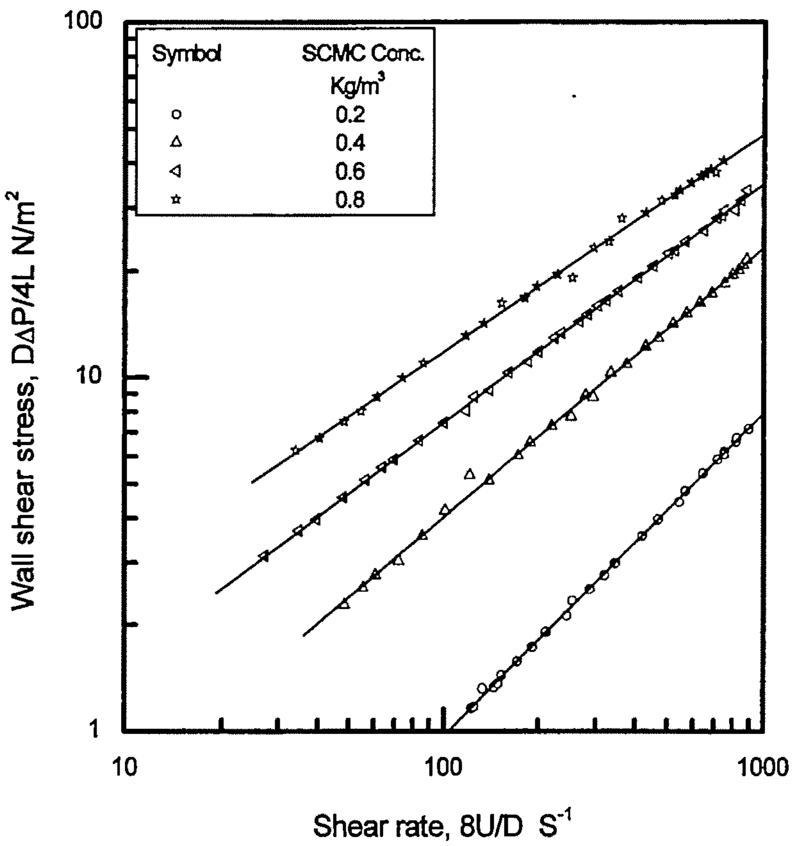

Fig. A1.1 Rheogram of the SCMC solutions.