Chapter 2

Perspectives of the magnetization dynamics and localized modes
2.1 Introduction

Magnetism is certainly one of the cornerstones of what we today call the information technology era. Following the huge development of electronics devices such as the personal computer, this era has been strongly driven by the growing demand to increase the density and speed of writing and retrieving data in memory devices. Today, the demand for information storage is enormous and expected to increase even further as new technologies such as high-definition video and on-demand TV are established in the market. In this quest, driven by the two words smaller and faster, magnetic recording remains the dominant data storage technology. The reason for this is the unmatched combination offered by magnetic recording: large storage capacity, small physical size, random and fast access to data, nonvolatility, radiation hardness and low cost. Consequently, the science and technology of magnetic materials is strongly fuelled by the global market for magnetic storage devices, which was estimated at $20 billion in 2005, and had to grow on $40 billion in 2010. Some examples of storage devices that are based on magnetism such as hard disk drive (HDD), magneto-optical disk and magnetic random access memory (MRAM).

In magnetic memory devices, logical bits (one and zero) are stored by setting the magnetization vector of individual magnetic domains either ‘up’ or ‘down’. The size of these domains is what defines the density of information in a memory device. In other words, the smaller the bit size is, the higher is the storage capacity of data storage media. The evolution of the bit size has been impressive since 1955, when IBM built the first hard disk drive featuring a storage capacity about 5MB with areal recording density 2 kbit/in².

Besides to disk storage, a tremendous research effort has been devoted in recent years to MRAM. This magnetic memory has the potential to store data at a relatively high density, high speed, and to have a low power consumption. Traditionally, in order to reverse the magnetization, and thus write or rewrite the information, an external magnetic field pulse is applied. The operation time for this magnetization reversal mechanism lies in the nanosecond regime. In-
creasing the strength of the magnetic field, the magnetization reversal time can be pushed into the picosecond range. However, by trying to do this new challenges appear. In particular, the write poles approach their limits in achieving strong and short field pulses for HDD. In addition, it must be noted that the increase in the density of the recorded information is achieved by using materials with very high magnetic anisotropy. In these conditions, the strength of the writing field must increase even more. There are more challenges regarding the actual writing process in which a coil is used. For example, in order to affect the neighboring data the writing field distribution must be scaled down as the density of information increases. On top of all these challenges, it has been recently predicted that no matter how short and strong the magnetic-field pulse, magnetic recording cannot be made ever faster than about 2 picoseconds [110]. Having all this in mind, it becomes clear that the actual magnetic storage technology is fastly approaching its speed limitations.

With the recent developments of ultrafast femtosecond lasers, the study of ultra-fast magnetization dynamics has become one of the most active fields of magnetism fuelled by both scientific and technological interest. In this quest, the ultrafast optical manipulation of the magnetization promises to become a real alternative to the magnetic field pulses. Note that the time-scale offered by femtosecond laser pulses for manipulating the magnetization is of orders of magnitude shorter than the magnetization reversal time in actual memory devices. Indeed, the first experimental studies on magnetization dynamics using femtosecond lasers uncovered a sub-picosecond demagnetization of magnetic metals [111]. Following this experiment, a wave of exciting results appeared: excitation of coherent spin waves via optically changing the magnetic anisotropy fields [112, 113]; optical excitation of high frequency spin oscillations (400 GHz) in antiferromagnets [114, 115] and even their control via the opto-magnetic inverse Faraday effect [116]; small-angle ultrafast switching of magnetization in garnets via the opto-magnetic inverse Faraday effect [117] and laser-induced coherent spin dynamics at a frequency of several THz [118, 119]. In all these experiments, though fast, the laser excitation only brings the
spins out of equilibrium (about several degrees) for a certain amount of time but does not accomplish a complete ultrafast magnetization reversal, as required for data storage. Consequently, one of the biggest challenge in the field of ultrafast magnetization dynamics is to find ways to ultrafast induced (180°) magnetization reversal.

Even in nowadays science, light interaction with solids is an intriguing area of research. In particular, if the light can magnetize or demagnetize a ferromagnet and if, in which timescales, are challenging question to be yet answered. The time scales of the magnetic recording, limited to the motion of the magnetic precession, are approaching the nanosecond timescale. Hence, fundamental research on the magnetization dynamics has become major interest for the storage industry.

Magnetization dynamics is an intriguing field of magnetism with a number of open questions. The term magnetization dynamics mainly stands for a magnetization precession, during which the magnetization vector align with external magnetic field. The characteristic of the timescales of the magnetization processes are approaching the technological limit for magnetic devices. Therefore, the relevance of the fundamental timescales of the magnetization relaxation goes from theoretical to practical research.

2.2 Theory of magnetization dynamics

Spin dynamics in magnetic systems is a hot topic of current interest. When the spin system is excited, of course, the spins precess in response to various torques exerted on them. These precession arise from externally applied magnetic fields, anisotropy fields of internal origin, dynamic dipole fields generated by the motions of the spins themselves, and finally torque generated by exchange interactions between the spins, if spatial gradients of the dynamics of magnetization are present. Quite generally speaking, the origin and magnitude of such torques have been very well understood for decades, and our knowledge of these interactions in bulk magnetic materials provides us with knowledge sufficient to address their nature on the nanometer length scale.
2.2 Theory of magnetization dynamics

It is crucial to understand the nature of the damping of spin motion in such structures. There are practical reasons for this, in addition to interest from the perspective of fundamental physics. In the current era, major advances have resulted from devices which incorporate nanoscale magnetic components. All such devices depend for their operation on physical effects associated with the reversal of the magnetization, or on the response of the device to changes in the orientation of the magnetization. The speed by which information may be read, written or extracted is controlled by the damping of spin motions within the device. In the current era, there is a very great interest indeed in obtaining a complete description of the magnetization dynamics in magnetic nanostructures, under conditions where the deviations from equilibrium are very large in amplitude. In real dynamics, the Landau-Lifshitz-Gilbert (LLG) equation forms the basis for the analysis of the spin motion.

2.2.1 Magnetization damping

Damping is due to the interaction of spin waves with each other and also with lattice vibrations and conduction electrons. The phenomenological description can be obtained from the equation of motion for the magnetization, containing the relaxation term. The Landau-Lifshitz equation implies that the magnetization, once taken out of the equilibrium position, precesses around the external field $H$ infinitely long. In reality, though, the magnetization eventually aligns with the external field. This experimentally observable fact demands the introduction of a dissipation term into the Landau-Lifshitz equation. The amplitude of the magnetization precession is gradually reduced until the magnetization aligns with the external field. To estimate the damping term, Gilbert first applied a thermodynamical approach in the following form:

$$\alpha \frac{M}{M_s} \times \frac{dM}{dt},$$

where $\alpha$ denotes the dimensionless Gilbert damping parameter. It determines how fast the energy of the magnetization precession is dissipated from the system. With this, the equation of motion for the magnetization is given by the
Landau-Lifshitz-Gilbert (LLG) equation:

\[
\frac{d}{dt} \mathbf{M} = -\gamma \mu_0 \mathbf{M} \times \mathbf{H} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{d}{dt} \mathbf{M}.
\] (2.1)

The Gilbert damping parameter for transition metals is much smaller than 1, which allows the magnetization to make a number of precessions before it is aligned with the external field. Magnetization dynamics are determined by the effective magnetic field \( H_{\text{eff}} \) in the sample. This field is the derivative of the free magnetic energy over the direction of the magnetization. Later on, it is used to determine the frequency of the magnetization precession. The magnetization is oriented in such a way, that all energy contributions to the free magnetic energy are reduced. The most important contributions, such as exchange energy, anisotropy energy and Zeeman energy are presented in the previous chapter.

2.2.2 The Gilbert damping phenomenology

The magnetization, disturbed from equilibrium by an intensive laser pulse, precesses around a magnetic field \( \mathbf{H} \) and tries to align with it on timescale
τ, given by the macroscopic damping parameter α. The magnetization vector obeys the Landau-Lifshitz equation of motion:
\[
\frac{dM}{dt} = -\gamma_0 M \times H + \frac{\alpha}{M_s} M \times \frac{dM}{dt},
\] (2.2)
with a Gilbert form of damping. This equation can be rewritten to the explicit equation for magnetization:
\[
\frac{dM}{dt} = -\frac{\gamma_0}{1 + \alpha^2} M \times H + \frac{\gamma_0 \alpha}{M_s(1 + \alpha^2)} M \times (M \times H),
\] (2.3)
with a Landau-Lifshitz form of damping. Microscopically, the Landau-Lifshitz form of damping can be derived to agree with the principles of the statistical physics, contrary to the Gilbert form of damping. These slight differences between Landau-Lifshitz and Gilbert damping, explain the current induced domain wall motion experiments by Hayashi et. al. [120]. In this special case, the Gilbert form of damping does not imply energy minimization and introduces some additional non-adiabatic term in the equation of motion. The Landau-Lifshitz form of damping explains the experiment appropriately. This effect appears only for large magnetization precessions, therefore, the analysis in this work starts from Eq. (2.2) with the consideration of only Gilbert damping parameter α. This presents the Gilbert damping parameter as follows: first, the microscopic origin of the damping is introduced with the associated classification of the magnetic dissipation processes.

The Landau-Lifshitz-Gilbert (LLG) equation is used widely in device design to describe spin motion in magnetic nanoscale structures. The damping term in this equation plays an essential role in the description of the magnetization dynamics. The form of this term is simple and appealing, but it is derived through use of elementary phenomenological considerations.

### 2.3 Theoretical modeling of damping

The derivation of the Gilbert damping parameter α is a challenging task for theorist. The direct signature of the damping is written in the equation of motion. Even without damping, there are no experimental variables that are
constants of motion on the microscopic scale. Therefore, it is hard to define an
appropriate equation of motion for the precessing magnetization. The choice
of dynamic variables will primarily influence the quantity and the origin of the
damping parameter. Magnetic damping is considered mainly within two types
of theory. The first class of theory is based on the direct transfer of magnetic
energy to the lattice, mainly due to magnon-phonon scattering. Spin-orbit cou-
pling is considered phenomenologically via magnetoelastic coupling between
the magnetization M(r,t) and lattice strain (r,t). The main damping mecha-
nism is the scattering of magnons at phonons and defects. The second class
of the theory is based on the energy transfer from magnons to electrons and
then to the lattice via electronic scattering. In this case, electrons inherit the
magnetic moments from the spin waves, and then pass them to the lattice.
The transfer from spin to electrons can be based on spin-current interactions
(damping by Eddy currents), coulomb interactions (Breathing Fermi Surface
model) and magnon-electron interactions (s-d model for damping). Electronic
scattering transfers energy and magnetic momentum from the electrons to the
lattice. The electronic scattering takes place between the spin-up and spin-
down states, which are not orthogonal to each other due to spin-orbit coupling.
This results in non-vanishing amplitudes of the scattering potential at inhom-
genieties such as phonons or defects.

For the magnetic materials, the approximation of a defect in the form of a flat
(or lamellar) magnetic inclusion of a finite thickness is usually employed. The
influence of planar inhomogeneities of magnetic anisotropy on the static and
some dynamic properties of magnetic inhomogeneities in ferro- and antiferro-
magnets were studied both analytically [121-127] and numerically [128-131].
There are also experimental works [132, 133], which show the presence of
defects, for example, in weak ferromagnetic materials such as rare earth ortho-
ferrites, can lead to the inhomogeneity of the magnetic anisotropy constant.
2.4 Magnetization reversal processes

The magnetization reversal process is one of the fundamental questions of magnetism. The investigation of this process is important to elucidate the internal nature of the magnetic interactions as well as their modification in a confined geometry. It is also quite relevant in technological applications for the improvement of the functionalizing of magnetic memory media and spintronics devices. We will now concentrate on the magnetization reversal process. Magnetization reversal, or switching, represents the process that leads to a 180° reorientation of the magnetization vector with respect to its initial direction, from one stable orientation to the opposite one along the easy axis. Technologically, this is one of the most important processes in magnetism that is linked to the magnetic data storage process. As we have previously discussed, the speed of storing data into a memory device is decided by how fast one can switch the magnetization. However, it is very important to stress here that the time defining the speed of storing the information is the time required to initialize a magnetization reversal that will lead to its complete switching into the new desired direction. In this view, as long as the magnetization reversal process is reliable, it is acceptable to have a longer settling down process such as the magnetization recovery process following laser heating [134]. As it is known today, there are only few possible ways to reverses the magnetization of a metallic magnet: reversal in an applied magnetic field and reversal by spin injection. Only theoretically demonstrated until now, all-optical switching is also sometimes regarded as a possible way to switch the magnetization [134].

From the all above section in this chapter, we discussed about the connection with the enormous advances in the magnetic storage technologies in the magnetization dynamics of one-dimensional continuum Heisenberg model. The magnetic soliton, which describes localized magnetization, is an important excitation in the classical Heisenberg spin chain. In particular, the continuum limit for the nonlinear dynamics of magnetization in the classical ferromagnet is governed by the Landau-Lifshitz equation. This equation governs a classical nonlinear dynamical system with novel properties. In the one dimensional
case, some types of L-L equation is completely integrable. In magnetic systems, the magnetization reversal process was discussed analytically in terms of LL equation with spin torque. Therefore, we study the interaction of nonlinear spin wave and magnetic soliton in a ferromagnetic system which in terms of a reasonable assumption of L-L equation in the continuum model. However, since in the theory nearly all theoretical approaches involved the continuum approximation, valid only for zone center spin wave modes (i.e., for $q=0$, where $q$ is the wave number of the spin waves), some important nonlinear modes of rather short wavelength have been lost. We know that the Heisenberg model for describing magnetic phenomena is inherently discrete, with the lattice spacing being a fundamental physical parameter. For such discrete systems an accurate microscopic description involves a set of nonlinear difference-differential equations and the intrinsic discreteness may drastically modify the nonlinear dynamics of the systems. The discreteness makes the systems lose continuum wavelength and a finite upper bound for the frequency spectrum of the linear spin waves. Due to the interplay between the discreteness and nonlinearity, new types of nonlinear excitations, which have no direct analogy in continuum models, may exist. In nonlinear atomic lattices, recent studies have shown that some novel nonlinear localized excitations, say the intrinsic localized modes or called the discrete breathers and the intrinsic gap modes can be natural nonlinear excitations of the systems. These nonlinear localized modes in perfectly periodic lattices have shorter wavelength in carrier waves and a somewhat similar character of the previously studied force constant or mass defects occurring in purely harmonic lattices, but they can appear at any lattice site because of the discrete translational symmetry of the systems.

The analogy between lattices and spin waves has stimulated a series of studies of intrinsic localized spin-wavemodes (ILSMs) in semiclassical and classical magnetic models [82, 85, 135-144]. For Heisenberg ferromagnetic and antiferromagnetic chains with on-site easy-axis anisotropy, an ILSM with the vibrating frequency above spin wave bands we call it the upper-cutoff ILSM is impossible due to the softness of the intrinsic nonlinearity in exchange interactions, but
ILSMs with the vibrating frequency below the spin wave bands the lower-cutoff ILSMs may exist [135-142]. The lower-cutoff ILSMs are related to the modulational instability of corresponding lower-cutoff spin waves [143]. If a strong magnetic field is applied which is perpendicular to the easy plane, an upper-cutoff ILSM may appear in easy-plane Heisenberg ferromagnetic chains when the strength of the single-ion anisotropy exceeds a certain value [82, 85]. At this stage, there is a need to consider all possible nonlinear localized excitations in Heisenberg chains in a simple and systematic way.

2.5 Modulational instability

Nature abounds with example of nonlinear waves between nonlinearity and dispersion is of much interest in recent days. As a result of a balance between nonlinear and dispersive effects, specific nonlinear objects, namely solitary waves, may appear [145,146]. They can be stable (corresponding, e.g., to the stable solitons) or unstable. The latter means that dispersion balances nonlinear steepening only in the stationary case. Small perturbations around the solitary wave may break this balance leading to instability and perhaps collapse. The stability of solitary waves has been studied in many physically different and important nonlinear problems. For continuous systems a rather well-developed formalism exists [147, 148]. The situation is different for discrete systems. Modulational instability (MI) is a nonlinear wave that manifests itself as the breakup of an extended state of a system into a train of highly localized states. In homogeneous nonlinear media, the extended state is a plane wave, which, under the action of a self-focusing nonlinearity, breaks up spontaneously into multiple filaments. As pointed out a long time ago, by Lighthill [149], localized solutions of the discrete equation of motion exist when a wavetrain is unstable with respect to small perturbations (modulations). It is now accepted that the outcome of an unstable wavetrain (plane wave) that is subjected to small perturbations is indeed, a solitary wave, or a set of localized excitations. Later on, Benjamin-Feir showed analytically that Stokes waves of moderate amplitude are unstable to small long-wave perturbations travelling in the same direction.
This instability is called the Benjamin-Feir instability (or modulational instability). Whitham derived the same result independently by using an averaged Lagrangian approach. At the same time, Zakharov, using a Hamiltonian formulation of the water wave problem obtained the same instability result and derived the nonlinear Schrödinger equation (NLS equation). The evolution of a two-dimensional nonlinear wave train on deep water, in the absence of dissipative effects, exhibits the Fermi-Pasta-Ulam recurrence phenomenon. This phenomenon is characterized by a series of modulation-demodulation cycles in which initially nearly uniform wave trains become modulated and then demodulated until they are again nearly uniform. Modulation is caused by the growth of the two dominant sidebands of the Benjamin-Feir instability at the expense of the carrier. During the demodulation, the energy returns to the components of the original wave train. Recently, within the framework of the NLS equation, Segur et al. revisited the Benjamin-Feir instability when dissipation is taken into account. The later authors showed that for waves with narrow bandwidth and moderate amplitude, any amount of dissipation stabilizes the modulational instability. In the wavenumber space, the region of instability shrinks as time increases. This means that any initially unstable mode of perturbation does not grow for ever. Damping can stop the growth of the sidebands before nonlinear interactions become important. Hence, when the perturbations are small initially, they cannot grow large enough for nonlinear resonant interaction between the carrier and the sidebands to become important. The amplitude of the sidebands can grow for a while and then oscillate in time. Segur et al. have confirmed their theoretical predictions by laboratory experiments for waves of small to moderate amplitude. Later, Wu, Liu and Yue developed fully nonlinear numerical simulations which agreed with the theory and experiments of Segur et al. Therefore MI reveals important information. The intrinsically localized states arises as the result of the interplay between lattice discreteness, nonlinearity and dispersive interactions and have attracted much interest over the past years. Due to the fascinating mathematical properties of soliton solutions such as, for example, the elastic mutual interaction, the soliton concept is ap-
plied to many nonlinear systems like plasmas [150, 151], solid state physics [152, 153], biology [154, 155], astrophysics [156], etc., Theoretical and analytical investigations of solitons in real physical systems provoked the intensive examination of solitons in the presence of perturbations. With respect to the sort of perturbations, the soliton stability problem can be naturally separated in two groups [157]. To the first group belongs the soliton stability problem under perturbations which have the same dimensionality as the initial soliton [158-162], while the other group encloses the soliton stability problem with respect to perturbations which change (in fact, increase) the dimensionality of the problem [157, 163-167]. Generally, solitons can be represented by the four parameters namely amplitude, frequency shift, phase, and position. Thus, their behaviour in the presence of perturbations may be depicted by a set of time dependent evolution equations for these parameters.

![Figure 2.2: Plane waves without and with perturbation](image)

The universality of the underlying mechanisms for nonlinear wave motion in physical systems to be applied in ferromagnetic media. The nonlinear physical
system exhibits an instability, which refers to the exponential growth of certain modulation sidebands of nonlinear plane waves propagating in a dispersive medium as a result of the interplay between nonlinearity and dispersion effects. In most of these cases, MI appears in continuous media where the propagation of nonlinear waves is usually governed by nonlinear Schrödinger-type partial differential equations. Computer simulations and experiments [168-170] have demonstrated that one of the main effects of the modulational instability is the generation of localized pulses. The modulational instability mechanism has recently been proposed and examined as a possible way to produce energy localization in discrete lattices [169-174]. Although many aspects of MI in discrete systems are the same as those in continuous media, the discreteness can drastically modify the modulational instability parameter space as deduced from a continuum or even semi-discrete approximation [171]. The advantage of making use of modulational instability to create localized excitations in discrete lattices is that because of the lack of continuous translational symmetry the localized pulse generated by the nonlinear instability can be trapped by discreteness to form strongly localized long-lived excitations. So that the modulational instability can be considered as the leading mechanism for energy localization. However, recent advances in nanoworld have made it possible to fabricate various low dimensional systems with complicated geometry such as magnetic nanodisks, magnetic nanofilms and magnetic nanowires. Besides geometry, the higher order interactions also play a crucial role in the functioning of these systems. This can influence the disposal of energy in the lattice by creation of energetic, mobile, highly localized lattice excitations that can propagate large distance.

2.6 Discrete breathers

Localized modes in discrete nonlinear systems have been a subject of intensive but mainly numerical investigations are progress during the past years [68, 175-179]. Different types of localized states were founding, and very elegant and efficient schemes have been developed [180] for calculating whole
families of solitary wave solutions. Of course, a broad discrete solution may be described with the help of the continuum approximation. However, there exist other types of discrete modes that definitely will not obey the continuum limit [68, 176]. Some of these solutions show stable behavior in numerical experiments. It should be noticed, however, that numerical simulations cannot prove stability in a strict sense. Thus, analytical criteria are urgently needed and to develop analytical stability criteria for discrete solitary waves. For demonstration, we have to choose a specific model. Besides being of fundamental interest, stability investigations [181] have important consequences for the dynamical features of nonlinear systems. Recently, the nonintegrable dynamics of a discrete system was discussed by introducing a ‘tunable’ nonlinearity [152] into the integrable nonlinear Schrödinger equation. There has also been much interest in the formation of very localized self-trapped states, motivated by an important role that they may play in the nonlinear DNA dynamics [182]. One possible mechanism to obtain narrow large amplitude standing states is modulational instability and subsequent energy localization [152]. Energy, initially broadly distributed in a nonlinear lattice, will be localized into large amplitude excitations by inelastic interactions of the small amplitude solitons [183]. This mechanism evidently depends on the stability properties of steady state solutions. Wave collapse may also provide an explanation for the appearance of very localized self-trapped states [152, 184-186] from an initially wide field distribution. The link between the wave collapse (blowup) in a nonlinear system with the instability of stationary states is well known for continuous models [184]. We shall demonstrate that instability of the stationary states in the discrete system corresponds to the well-known continuum wave collapse in a modified form of the quasicollapse to intrinsically localized modes, i.e., whether it is an extremely spiky stationary mode plus ‘radiation’, or an oscillating localized mode, or something else.

Breather solutions are very special solutions of nonlinear systems. We call breather a time periodic solution which is localized in space or equivalently which decays to zero at infinite distance. These solutions appear to be
quite interesting in numerical simulations [68, 69, 177, 183, 187-193] as long lifetime structures which show up spontaneously. Phenomenologically, this intriguing property, now commonly referred to as self-organization, results in the instability of linear waves leading to the formation of large organized coherent structures. As such continuous media are generically described by nonlinear partial differential equations, one can ask how discrete media will behave. The nonlinear interaction between breather-like excitations gives rise to an energy localization; the world of discrete breather-like excitations is as merciless for the weak as the real world, since a systematic tendency to favour growth of the larger excitations at the expense of the others was emphasized. The question of how the formation of localized structures occurs in such conservative systems logically arises. If the existence and the stability of localized excitations is known about their creation. It is certainly one of the main questions for the future of nonlinear dynamics.

The concept of nonlinear self-localization is of importance for many physical phenomena, and has appeared in a number of different contexts since the pioneering work by Landau [194] on the polaron problem in the 1930s. In recent years, much attention has been devoted to the studies of spatially localized and time-periodic vibrational modes in anharmonic lattices. The general existence of such modes, which have been termed discrete breathers, or intrinsic localized modes, as robust solutions to nonlinear (and in general non-integrable) lattice equations was suggested in 1988 by Takeno et al [187]. Later, their existence was rigorously proven under rather general conditions by MacKay and Aubry [195] by considering the limit of uncoupled oscillators (the so called anticontinuous or anti-integrable limit). By means of the implicit function theorem, they showed that the trivial solution of a single-site localized vibration at the uncoupled limit could be continued into a localized breather solution for non-zero coupling between the oscillators, provided that the individual oscillators are anharmonic, and that no multiples of the breather frequency resonate with the bands of linear excitations (phonons). Since the discrete breathers appear under very general conditions in anharmonic lattices and provide efficient
means of energy localization, they have been proposed as candidates to explain experimentally observed localization of energy in many different physical areas [182]. Although, from a fundamental and mathematical viewpoint,

![Two-spin breather in an easy plane ferromagnet](image)

**Figure 2.3:** Discrete breathers

the existence theorems for discrete breathers provide an important cornerstone for understanding the dynamics of anharmonic lattices, it is probably of even
larger physical importance to understand the behaviour of a system close to an exact breather solution. By linearizing the lattice-equations around the exact solution, one can obtain an approximate description of the dynamics of weakly perturbed breathers, and in particular the linear stability properties determining whether small perturbations will grow exponentially or not. The simplest, single-site, breathers are in general linearly stable close to the uncoupled limit, and numerical investigations using standard Floquet analysis [196] have shown that linearly stable breathers typically exist also for rather large values of the inter-site coupling. However, when considering time-scales large compared to the breather period, the mere linear stability of a breather does no longer guarantee the eternal existence of the breather in the presence of small perturbations, and there are still many questions remaining concerning the different mechanisms by which breathers may grow or decay, or possibly finally may be destroyed. If the breathers have a finite life-time, the determination of this life-time is of large importance for understanding the role of breathers in real systems.

The intrinsic localized spin modes of nonlinear extended spin waves have been devoted to the ferromagnetic system. The investigation of ILMs and modulational instability of excitations which can be extended at nanoscale dimensions as well as for future exploration of the quantum properties of such excitations. The existence and properties of localized solutions in extended discrete systems have attracted interest in a broad range of physical fields [197]. Discrete breathers, sometimes referred to as intrinsic localized modes, are spatially localized and time periodic solutions. These solutions arise in the context of nonlinear discrete systems and are of fundamental interest for varied physical applications such as pulse propagation in nonlinear optics, energy storing and transport in biomolecules, plasma physics, etc. The existence of discrete breathers in these systems has been proven rigorously [195] for a number of equations with physical relevance and, contrary to continuous nonlinear equations, their existence can be regarded as a generic feature of these systems. One of the most important classes of equations are the so-called discrete non-
linear Schrödinger lattices [198, 199]. The existence of discrete breathers has been proven for a wide range of systems belonging to this class of nonlinear difference-differential equations. In particular, the most important example of wide applicability is the standard nonlinear Schrödinger equation. For instance, this equation was employed [200-202] for describing the propagation of localized beams in an array of nonlinear (Kerr type) waveguides, having experimental validation subsequently reported in [203, 204]. Breathers with an exactly compact profile, however, are not expected to exist generically.

2.7 Overview of the thesis

Today the magnetization dynamics in ferromagnetic materials is one of the most exciting issues in magnetism. Driven by strong fundamental and technological achievement of the magnetoelectronics such as disks, tapes, sensors and memories which demand the elaborated exploration on the dynamic properties of magnetic materials having in day to day life electronics. In recent years there have been important and far reaching developments in the study of nonlinear waves and a class of nonlinear wave equations which arise frequently in applications. The wide interest in this field comes from the understanding of special waves called solitons and the associated development of a method of solution to a class of nonlinear wave equations termed as nonlinear Schrödinger equation (NLSE). A soliton phenomenon is an attractive field of present day research in nonlinear physics and mathematics. Essential ingredients in the soliton theory are the nonlinear Schrödinger equation and its variants appearing in a wide spectrum of problems.

In this thesis two different ways of investigation interms of the solitonic nature in ferromagnetic system with higher order dispersive interactions has been presented. The first way deals with the discrete model of nonlinear like excitations interms of energy localization through the instability analysis, the second way is in continuum model describing the dynamics of the magnetization during precessional switching. As we outlined in the previous sections, topics are of utmost importance in the ongoing research on the ultrafast pre-
cessional process in magnetic materials. We present the original results of our investigation in the following chapters:

In the Chapter 3, we study the nonlinear excitations in one dimensional magnetic system of Heisenberg spin chain with varying exchange interactions of bilinear and biquadratic in nature. We derived the generalized higher order nonlinear Schrödinger (NLS) equation through the space curve formalism. Also, we derived the integrability nature with the help of the nature of Painlevé analysis. Particularly, we study creation and annihilation of solitons under the influence of various kinds of nonlinear inhomogeneities and damping.

A class of one dimensional systems of recent interest is comprised of magnetic chains with spin symmetry. These are systems whose static and dynamic properties may exhibit soliton or soliton like features. In Chapter 4, we presented the energy-momentum transport phenomenon through soliton in an one dimensional ferromagnetic spin chain with relativistic Gilbert damping for linear inhomogeneity. The influence of inhomogeneity and damping on the evolution of energy and current densities of the magnetization is also demonstrated. In this chapter, it is concluded that the presence of inhomogeneity and damping support the loss-less energy-momentum transport along the site-dependent spin chain.

Chapter five is divided into two sections. The first section covers the low dimensional magnetic dispersive interactions in the absence of Dzyaloshinskii-Moriya interaction. Despite its smallness, the long range interaction or short range interaction plays an essential role in 1D magnet. The realization that lattice discreteness can stabilize highly localized excitations that has been perfect in nonlinear systems. We analyzed the instability criteria of the discrete systems and the intrinsic localized spin modes were found.

The second section of this chapter gives a profound understanding of dispersive interactions in the presence of Dzyaloshinskii-Moriya of weak ferromagnets. When the symmetry around the magnetic ions is not high enough, an unfamiliar but important antisymmetrical coupling results due to the combined effect of spin orbit coupling and the exchange interaction leading to the
mechanism of weak ferromagnetism. The Dzyaloshinskii-Moriya interaction is essentially different from the effect on the magnetic properties of the ferromagnetic spin system that leads to a canted spin structure. The ferromagnetic spin system with such kinds of exchange interactions of the Heisenberg type between the spins which are coupled by the constants. We introduce that Hamiltonian of such system and derive the equation of motion by using Bose operator. The effect of the higher order dispersive interactions and its localized structure were obtained both in the presence and absence of Dzyaloshinskii-Moriya interaction and the results are discussed elaborately.

Chapter 6 gives the soliton like and localized excitation of higher order dispersive interactions with the inclusion and exclusion of Dzyaloshinskii-Moriya interaction. In the absence of DM interaction, the system admits the higher order perturbed nonlinear Schrödinger equation by making use of Taylor series expansion. A multiple scale perturbation analysis is adopted for this higher order nonlinear Schrödinger equation governing the dynamics of spin systems with dispersive interactions to obtain the evolution of the amplitude and velocity of the perturbed soliton. The corresponding equations are solved numerically using fourth order Runge-Kutta methods. Moreover, we attest the sine-cosine method to solve the nonlinear evolution equation and obtained the new exact propagating solitary wave solution by using symbolic computation. The results are discussed in the first part of the chapter six.

The second section deals with instability criteria and solitary wave solution of dispersive interactions upon the inclusion of DM interaction. For this particular case, we obtained the Sasa-Satsuma equation by following the continuum approximation. The Sasa-satsuma equation is one of the family of NLS equation and in order to determine whether the system can trap the energy by creating instability in ferromagnetic spin chain which can explain the energy transport among the spin lattice we performed modulational instability analysis. Also, we employ Jacobi elliptic function method and obtained the exact solution by using symbolic computation. Seeking the exact solutions is a crucial task in the study of nonlinear equations. Based on computerized symbolic computation,
the system admits a shape changing property during its evolution. This shape changing property can be exploited to reverse the magnetization without loss of energy which may have potential applications in magnetic memory devices.