CHAPTER VII

POSSIBLE FURTHER STUDIES
§ 15. SUGGESTIONS FOR FURTHER STUDIES

In conclusion we propose some problems for further studies which may reveal some new aspects that could not be unveiled in our short discussion.

1) We have established in § 9 a new technique suitable for application to problem of atomic scattering. This remedies some of the deficiencies of ordinary wave mechanical treatment. But still there are many aspects which call for further studies.

Firstly our formalism does not provide any scope for taking into account the pair creation and annihilation effects. In most scattering problems at ultra relativistic energies this effect might be important. So generalization of the technique along this line is suggested. Convergence problem of the series expansion as also the renormalizability of the theory needs investigation. Many of the phenomena in atomic physics are important at low energies such as resonance scatterings. So one may try to modify the theory in such a way that it becomes useful in discussing low energy phenomena. Finally we may suggest for immediate study in our formalism a number of interesting problems -

(a) ionization of $^a$ proton or a alpha particle, (b) ionization of $^a$ He, Li or more complex atoms by electron, proton or alpha particle, (c) $^1$-$^1$ scattering problem, (d) some simple rearrangement collisions like

$$
H^+ + H(1s) \rightarrow H(nl) + H^+ ; \quad H^+ + He^+ \rightarrow H + He^{++} ;
$$

$$
H^+ + He \rightarrow H + He^+ ; \quad H^+ + H^- \rightarrow H + H ; \text{etc.}
$$
(e) some simple problems of electron detachment,

(f) some rearrangement collisions accompanied with radiation

ii) We have presented in § 10 an improved calculation on the low-energy electron ejection cross-section in electron-atom collisions at relativistic energies. Recently some experimental results are available [Missoni et al, 1970] for this problem. So the cross-section result (10.8) may be evaluated for relevant situations and compared with the available data. The calculation needs also improvement in the low energy region.

iii) In §11 we have derived results which are suitable for application to the inner-shell ionization of light- to medium heavy atoms (formula 11.9) as also an expression for heavy atom ionization (eq. 11.8). In these formulae exchange effect has been included in Ochkur approximation. The result has been applied to the study of inner-shell ionization of $^{55}$Fe. A systematic study of ionization of various atoms may be made.

It is known that the semi-relativistic results of Arthurs and Moiseiwitsch are only a few percent higher than the experimental results of Rester and Dance (1966) for Au. From our study on $^{55}$Fe we expect that if one applies the simple formula (11.9) to Au one may find very good agreement. This is a problem suggested for immediate study.
iv) Second order result (eq. 12.3) for the charge exchange scattering of proton on hydrogen has been presented following our theory. A high energy approximation has been used in discussing the result. The problem needs reinvestigation at moderately high energy employing the full matrix element (12.3). Convergence of the series expansion for the present case deserves a rigorous study.

v) The second order result (eq. 13.10) for positronium formation may be attempted for numerical results.

vi) Finally we suggest a few calculations in strong interaction physics which may be done as an extension of our investigation. We have seen that on proceeding to next higher order approximation in Balazs' equivalent potential approach we get improvements over the lowest order result considered by Balazs and Vaidya. Using a computer one may explore how the parameters (determined from bootstrap consideration) changes from one approximation to the next in the strip approximation. A study of low energy scattering cross sections may also be made following Balazs equivalent potential approach.
APPENDIX A

EQUIVALENCE OF THE BETHE-MOLLER SCATTERING AMPLITUDE AND THAT OF FANO IN ELECTRON HYDROGEN DIRECT SCATTERING

One may derive the Bethe-Moller scattering amplitude for the inelastic scattering (direct) of relativistic electrons by hydrogen atom by sandwiching the Bethe-Moller equivalent potential given by eq. (4.4) between the initial and final states. In momentum space this potential is

\[
\mathcal{V}_{BM}(\mathbf{t}) = \frac{4\pi\alpha}{(2\pi)^3} \left[ 1 - \frac{\mathbf{x} \cdot \mathbf{p}}{\mathbf{p}^2} \right] / (t^2 + \mathbf{w}_{ij}^2),
\]

(A1)

where \( \mathbf{w}_{ij} \) is the energy loss in the transition.

The scattering amplitude of Fano \( \mathcal{F} \) of eqs (9.22) & (9.23) \( \mathcal{F} \), derived by using the Coulomb gauge may be written as

\[
\mathcal{M}_{fi} = \int d\mathbf{k} \chi^{(2)\ast}(\mathbf{P}_{f}) U_{f}^{(0)}(\mathbf{k}) \mathcal{V}_{\mathbf{F}}(\mathbf{t}) \chi^{(2)}(\mathbf{k}) U_{i}^{(0)}(\mathbf{P} - \mathbf{k}),
\]

(A 2)

where

\[
\mathcal{V}_{\mathbf{F}}(\mathbf{t}) = \frac{4\pi\alpha}{(2\pi)^3} \left[ \frac{1}{t^2} - \frac{\mathbf{x} \cdot (\mathbf{x} \times \mathbf{t})}{t^2 \mathbf{w}_{ij}^2} \right].
\]

(A 3)

In the limit of zero nuclear charge (i.e., free free transition) the potential (A 1) gives matrix element identical \( \mathcal{F} \) Heitler 1954 p 235 \( \mathcal{F} \) with that given by the potential (A 3).

\[
\left[ \left< f \left( \chi^{(0)}(\mathbf{P}_{f}) \chi^{(0)}(\mathbf{P} - \mathbf{k}) \right) \right| i > \right> = - \left< 0 | \chi^{(2)\ast}(\mathbf{P}_{2f}) \chi^{(0)}(\mathbf{P}_{2f}) [ \mathbf{x} \cdot (\mathbf{x} \times \mathbf{t}) ] \right| f \left( \chi^{(0)}(\mathbf{P}_{f}) \chi^{(0)}(\mathbf{P} - \mathbf{k}) \right) > \times\left[ \mathbf{w}_{ij} \right] \chi^{(2)}(\mathbf{P}_{2f}) \chi^{(0)}(\mathbf{P}_{f}) = - \mathbf{w}_{ij} \left< f | i > \right. \text{ etc.}
\]

That the same is true for free bound transitions needs proof.
We now give the proof.

For the momentum space wave functions we have

\[
\begin{align*}
(\alpha^2, P^2_j + p^2, \gamma^{(2)}(P^2_j)) &= E_j \chi^{(2)}(P^2_j), \\
(\alpha^2, \vec{P} + p^2, \gamma^{(2)}(\vec{P})) &= W_j \chi^{(2)}(\vec{P}),
\end{align*}
\]

\[A4\]

and \(\gamma\) is the integral operator given by

\[\gamma \psi(\vec{r}) = \int \psi(\vec{r} - \vec{r}') \psi(\vec{r}') d\vec{r}'\]

with

\[\psi(\vec{r} - \vec{r}') = \frac{4\pi \alpha Z}{(2\pi)^3 (\vec{r} - \vec{r}')^2} \]

\[A5\]

Consequently we have

\[
\begin{align*}
\psi^{(2)}(\vec{F}_j) \psi^{(2)}(\vec{F}) (\bar{\alpha}^{(2)}, \bar{\chi}^{(2)}) X_{\chi}(\bar{\alpha}^{(2)}, \bar{\chi}^{(2)}) \psi^{(2)}(\vec{F}_0) \\
= \chi^{(2)}(\vec{F}_j) \left[ \bar{\alpha}^{(2)} \chi^{(2)}(\vec{F}_0) \right] \chi^{(2)}(\vec{F}_0) \psi^{(2)}(\vec{F}_0) \times \\
\times \left[ \bar{\alpha}^{(2)} \chi^{(2)}(\vec{F}_0) \right] \psi^{(2)}(\vec{F}_0)
\end{align*}
\]

\[A6a\]

\[\begin{align*}
= & -W_j \chi^{(2)}(\vec{F}_j) \chi^{(2)}(\vec{F}_0) \psi^{(2)}(\vec{F}_0) \psi^{(2)}(\vec{F}_0) + (A6a) \\
+ & W_j \chi^{(2)}(\vec{F}_j) \chi^{(2)}(\vec{F}_0) \left[ \int d\vec{r}' \psi(\vec{r}' - \vec{r}) \times \\
\end{align*}\]

\[A6b\]
Since the evaluation of the matrix element \( M_{ij} \) involves a final integration over \( \vec{k} \), the two terms in the Curly Bracket of (A 5b) give identical results and so cancel each other. So we are left with the single terms (A 6a). As for free electron case this matrix element with the additional factor

\[
\frac{4\pi\alpha}{(2\pi)^3} \frac{1}{(\vec{t^*} - \vec{W}_{ij})^2}
\]

combine with the matrix element of the remaining part of (A 3) viz.

\[
\frac{4\pi\alpha}{(2\pi)^3} \left[ \frac{1}{t^2 - \frac{\alpha(q^0)}{t^2 + W_{ij}^2}} \right]
\]

to produce exactly the matrix element derived using the \( \beta - M \) potential (A 1).