6. **Summary and Conclusions.**

In this thesis, we have discussed the importance of extended Higgs structures as a sign of physics beyond the Standard Model. The most viable extensions of the Standard Model in the scalar sector are those that involve extra $SU(2)$ doublets and singlets — and this discussion has been devoted solely to such cases. We have considered possible effects of these extra particles — in some specific models — in the prediction of observables which are either measured already or are expected to become measurable in the near future.

These effects are obtained at three energy scales, namely

(i) *low energies* of the order of a few GeV or less;

(ii) *intermediate energies* at the scale of electroweak symmetry-breaking, specifically at the mass of the $Z^0$ boson (91 GeV);

(iii) *high energies* of the order of a few tens of TeV, which may turn out to be the supersymmetry-breaking scale.

Two specific instances of extra scalars in models with extended Higgs sectors have been considered in detail. One is that of physical charged Higgs bosons arising in models with two doublets. The other is the scenario when there is an extra singlet Higgs coupling to extra vectorlike fermions. At low energies, all the effects considered are described by one-loop Feynman diagrams and hence are somewhat suppressed. Their measurement requires high precision such as is now becoming increasingly available in results from high energy machines. The calculations are also rather complicated and most of these details have been set out in the text and appendices.

At low energies, we have examined a supergravity model where there are stringent constraints on the scalar sector parameters coming from the specific pattern of electroweak symmetry-breaking considered. With these constraints (which essentially amount to those in the Minimal Supersymmetric Standard Model with the tan$\beta$ pa-
rameter restricted to the range 1.0 — 2.0 provided we consider the fermion sector only)
we have presented two representative studies. One is the phenomenon of \( B_d^0 \rightarrow \bar{B}_d^0 \)
mixing and the other is the interesting rare decay process \( B \rightarrow K^* \gamma \). In either case
we find loops with internal charged Higgses making a significant contribution. While
these contributions may be masked by uncertainties in the hadronic matrix element
for \( B_d^0 \rightarrow \bar{B}_d^0 \) mixing, in the case of \( B \rightarrow K^* \gamma \) they contribute significantly to bring the
prediction to the margin of the current experimental upper bound. As this bound is
expected to decrease substantially within the next year or two, the \( B \rightarrow K^* \gamma \) process
seems to be an excellent probe of new physics, particularly charged Higgs bosons, at
low energies such as those available at \( B \)-factories.

At intermediate energies of the order of the electroweak symmetry-breaking scale,
which are available at the \( LEP \) \( e^+ e^- \) collider, we have considered two possible rare
decay modes of the \( Z^0 \) boson produced in these collisions. One is the rare flavour-
changing decay \( Z^0 \rightarrow \bar{b}s + b\bar{s} \) which is strongly GIM-suppressed in the Standard Model.
Charged Higgs contributions (in the supergravity model described above) can provide
reasonably large enhancements of the effect. The most optimistic predictions are well
within the range of detectability when \( LEP-I \) completes its run. The other process
is the decay of a \( Z^0 \) to a singlet Higgs \( \chi^0 \) and a photon, which is a process mediated
by exotic fermion loops only. If the distinctive signals, such as a hard monoenergetic
photon, accompanied by two jets, (or otherwise, depending on the particular scenario
involved) for this process are observed, one would, at the same stroke, be able to deduce
the existence of a singlet Higgs as well as exotic vectorlike fermions. Even with the
most favourable range of parameters, however, this process, in order to be observable,
requires a rather high resolution in the photon energy and the full catch of 10 million
\( Z^0 \)'s projected at \( LEP-I \) to be at all decipherable from the large backgrounds. The
prospects of detecting a singlet Higgs at \( LEP-I \) are not, therefore, very promising, and
we have to look more to LEP-II or TeV range hadron colliders to observe this particle.

Finally, we have considered a particular rare process $H^\pm \rightarrow W^\pm \gamma$ which may turn out to be of great importance in the identification of charged Higgses produced directly at high energy $pp$ colliders such as the LHC or SSC, which are planned to be run at centre-of-mass energies of 15.6 and 40 TeV respectively. Large QCD and other backgrounds hinder detection of the $H^\pm$ through the usual dominant decay modes. The rare process $H^\pm \rightarrow W^\pm \gamma$ is likely to have a clean signature, though a detailed study of the backgrounds still requires to be made. We have presented the first complete calculation of this decay width to one-loop in a two-Higgs doublet model which mimics the scalar sector of the Minimal Supersymmetric Standard Model. This highly nontrivial exercise involves the evaluation of over a hundred one-loop diagrams and a great deal of accurate book-keeping. We ultimately find that for a charged Higgs boson lighter than the top quark, there is a substantially large number of $H^\pm \rightarrow W^\pm \gamma$ events at the SSC and even more at the LHC, while for a charged Higgs boson heavier than the top quark, the number is somewhat marginal for detection. Given our calculation of the decay width — which shows, rather unexpectedly, that it is enough to consider loops with top and bottom quarks only — it should now be possible to conduct a proper study of the signal vis-à-vis background for this process.

In these considerations, we have tried to focus on the phenomenological importance of Higgs bosons as a probe of physics beyond the Standard Model. It also appears that rare processes mediated by one-loop diagrams may well prove the Rosetta stone of Higgs physics in spite of the complicated, often daunting, technology involved in their evaluation. A number of new questions have also been raised, such as QCD corrections to $H^\pm \rightarrow W^\pm \gamma$, LEP-II signatures of the $\chi^0$ and background elimination techniques. These could form the nucleus of a body of future investigations in this area. In any case, the motivation for this series of investigations has been to underline the possibilities of
nonstandard Higgs bosons heralding the advent of a new age of discovery in particle physics.
Appendix A

Calculating two- and three-point functions.

The one-, two- and three-point functions of 't Hooft and Veltman and Passarino and Veltman are defined below. All integrations are in Euclidean space.

One-point function:

\[ A(m) \equiv \frac{1}{\pi^2} \int d^4q \frac{1}{q^2 + m^2} \]  
(A.1)

Scalar two-point function:

\[ B_0(m_1, m_2; M) \equiv \frac{1}{\pi^2} \int d^4q \frac{1}{(q^2 + m_1^2)((q + p)^2 + m_2^2)} \]  
(A.2)

where \( p^2 = M^2 \).

Scalar three-point function:

\[ C_0(m_1, m_2, m_3; M_1, M_2, M_3) \equiv \frac{1}{\pi^2} \int d^4q \frac{1}{(q^2 + m_1^2)((q + p_1)^2 + m_2^2)((q + p_1 + p_2)^2 + m_3^2)} \]  
(A.3)

where \( (p_1 + p_2)^2 = M_1^2; p_1^2 = M_2^2; p_2^2 = M_3^2 \).

Vector two-point function:

\[ B_\mu(m_1, m_2; M) \equiv \frac{1}{\pi^2} \int d^4q \frac{q_\mu}{(q^2 + m_1^2)((q + p)^2 + m_2^2)} \]  
(A.4)

\[ \equiv p_\mu B_1(m_1, m_2; M) \]

Vector three-point functions (\( C \equiv C_\mu(m_1, m_2, m_3; M_1, M_2, M_3) \))

\[ C_\mu \equiv \frac{1}{\pi^2} \int d^4q \frac{q_\mu}{(q^2 + m_1^2)((q + p_1)^2 + m_2^2)((q + p_1 + p_2)^2 + m_3^2)} \]  
(A.5)

\[ \equiv p_{1\mu} C_{11} + p_{2\mu} C_{12} \]
Tensor three-point functions:
\[ C_{\mu \nu} = \frac{1}{x^2} \int d^4q \frac{q_\mu q_\nu}{[q^2 + m_i^2][(q + p_1)^2 + m_i^2][(q + p_1 + p_2)^2 + m_i^2]} \]
\[ \equiv p_{1\mu}p_{1\nu}C_{21} + p_{2\mu}p_{2\nu}C_{22} + [p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu}]C_{23} + \delta_{\mu \nu}C_{24} \]  
(A.6)

The form factors \( B_1, C_{11}, C_{12}, C_{21}, C_{22}, C_{23} \) and \( C_{24} \) can all be written as linear combinations of the basic functions \( A, B_0 \) and \( C_0 \). The general formulae for these are very complicated. Fortunately, for all the processes considered in the text, one of the external legs is a photon, whose vanishing mass renders the calculation considerably easier. A list of the relevant linear combinations is given below.

For Chapter 4:

We consider the C-functions with arguments

\[ C = C_0(m_f, m_f, m_f; m_2, m_3, m_4) \]  
(A.7)

arising from the triangle diagram in Fig. 11 (a). Then

\[
\begin{pmatrix}
C_{11} \\
C_{12} \\
C_{21} \\
C_{22} \\
C_{23} \\
C_{24}
\end{pmatrix} = M
\begin{pmatrix}
1 \\
C_0 \\
B_0(m_f, m_f; m_3) \\
B_0(m_f, m_f; m_2) \\
B_0(m_f, m_f; m_4)
\end{pmatrix}
\]  
(A.8)

where the 5 × 6 matrix \( M \) is given by

\[
M = \begin{pmatrix}
0 & \frac{m_f^2}{m_f^2 - m_3^2} & \frac{1}{m_f^2 - m_3^2} & \frac{m_f^2 + m_f^2}{(m_f^2 - m_3^2)^2} & -\frac{2m_f^2}{(m_f^2 - m_3^2)^3} \\
0 & 0 & 0 & -\frac{1}{m_f^2 - m_3^2} & \frac{m_f^2 - m_f^2}{(m_f^2 - m_3^2)^2} \\
\frac{m_f^2}{m_f^2 - m_2^2} & \frac{2m_f^2 - m_f^2}{(m_f^2 - m_2^2)^2} & \frac{m_f^2 - 3m_f^2}{2(m_f^2 - m_2^2)^3} & -\frac{m_f^2 - 4m_f^2 + 3m_f^2}{2(m_f^2 - m_2^2)^4} & \frac{m_f^2 - 2m_f^2}{(m_f^2 - m_2^2)^5} \\
0 & 0 & 0 & -\frac{1}{m_f^2 - m_2^2} & \frac{m_f^2 - m_f^2}{(m_f^2 - m_2^2)^2} \\
\frac{1}{4}(1 + \Delta) & -\frac{1}{2}m_f^2 & 0 & -\frac{m_f^2}{4(m_f^2 - m_2^2)} & \frac{m_f^2}{m_f^2 - m_2^2}
\end{pmatrix}
\]  
(A.9)

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In order to prove gauge invariance of the amplitude given in the text, we require the identity:

$$4C_{24} - B_0(m_f, m_f; m_x) = 4q_p(C_{22} - C_{23})$$  \hspace{1cm} (A.10)

which can be easily established using the above formulae for $C_{22}, C_{23}, C_{24}$.

For Chapter 5:

We first consider the C-functions with arguments

$$C = C_0(m_W, m_W, m_H; m_W, m_\gamma, m_+)$$  \hspace{1cm} (A.11)

arising from the triangle diagrams in Fig. 14 (a). Then

$$\begin{pmatrix} C_{11} \\ C_{12} \\ C_{21} \\ C_{22} \\ C_{23} \\ C_{24} \end{pmatrix} = M_1 \begin{pmatrix} 1 \\ C_0 \\ B_0(m_W, m_W; m_+) \\ B_0(m_W, m_H; m_W) \\ B_0(m_W, m_H; m_+) \\ a_H - a_W \end{pmatrix}$$  \hspace{1cm} (A.12)

where $z_i = m_i^2/m_W^2$, $\alpha_i = 1/m_i^2 A(m_i)$ and the 6 x 6 matrix $M_1$ is given by

$$M_1 = \begin{pmatrix} 0 & \frac{2-x_H}{x_+ -1} & \frac{1}{x_+ - 1} & \frac{x_++1}{(x_+ - 1)^2} & \frac{-x_+}{(x_+ - 1)^2} & 0 \\ 0 & 0 & 0 & \frac{1}{x_+ - 1} & \frac{-1}{x_+ - 1} & 0 \\ -\frac{x_+}{(x_+ - 1)^3} & \frac{x_+ - 2x_H + 2x_+ + 4}{(x_+ - 1)^3} & \frac{x_+ + 2x_H - 5}{2(x_+ - 1)^3} & \frac{1}{2(x_+ - 1)^3} & \frac{x_++1}{2(x_+ - 1)^3} \\ 0 & 0 & 0 & \frac{x_++2}{2(x_+ - 1)} & \frac{x_++1}{2(x_+ - 1)} & 0 \\ 0 & \frac{1}{x_+ - 1} & 0 & \frac{1}{2(x_+ - 1)} & \frac{x_++2}{2(x_+ - 1)} & \frac{-1}{2(x_+ - 1)} \\ \frac{1}{4}(1 + \Delta)m_W^2 & -\frac{1}{4}m_W^2 & 0 & \frac{1}{4}m_W^2 & \frac{1}{4}(x_+ - 1) & \frac{1}{4}(x_+ - 1) \end{pmatrix}$$  \hspace{1cm} (A.13)

Similar results follow on replacing $m_H$ with $m_h$ everywhere.

Next, we consider the C-functions with arguments

$$C = C_0(m_+, m_+, m_H; m_W, m_\gamma, m_+)$$  \hspace{1cm} (A.14)
also arising in triangle diagrams in Fig. 14 (a). Then
\[
\begin{pmatrix}
C_{11} \\
C_{12} \\
C_{21} \\
C_{22} \\
C_{23} \\
C_{34}
\end{pmatrix}
= M_2
\begin{pmatrix}
1 \\
C_0 \\
B_0(m_+,m_+;m_+;m_+) \\
B_0(m_+,m_H,m_W) \\
B_0(m_+,m_H;m_+) \\
a_H - a_+
\end{pmatrix}
\]  
(A.15)

where the components of the $6 \times 6$ matrix $M_2$ are

\[
(M_2)_{11} = 0 \\
(M_2)_{12} = \frac{x_+ - x_H + 1}{x_+ - 1} \\
(M_2)_{13} = \frac{1}{x_+ - 1} \\
(M_2)_{14} = \frac{x_+ + 1}{(x_+ - 1)^2} \\
(M_2)_{15} = \frac{2x_+}{(x_+ - 1)^2} \\
(M_2)_{16} = 0
\]

\[
(M_2)_{21} = 0 \\
(M_2)_{22} = 0 \\
(M_2)_{23} = 0 \\
(M_2)_{24} = \frac{1}{x_+ - 1} \\
(M_2)_{25} = -\frac{1}{x_+ - 1} \\
(M_2)_{26} = 0
\]

\[
(M_2)_{31} = -\frac{x_+}{(x_+ - 1)^2}
\]
\[(M_2)_{33} = \frac{x_+^2 + x_H^2 + (x_+ + 1)(x_+ - 2x_H + 1)}{(x_+ - 1)^2}\]

\[(M_2)_{34} = \frac{x_+ - 2x_H + 3}{2(x_+ - 1)^2}\]

\[(M_2)_{35} = \frac{x_+(7 - x_+^2) + x_+x_H(x_+ - 2) + 3(x_H^2 - x_H + 1)}{2(x_+ - 1)^3}\]

\[(M_2)_{36} = \frac{x_+ + 1}{2(x_+ - 1)^3}\]

\[(M_2)_{41} = 0\]

\[(M_2)_{42} = 0\]

\[(M_2)_{43} = 0\]

\[(M_2)_{44} = \frac{x_+ - x_H + 2}{2(x_+ - 1)}\]

\[(M_2)_{45} = \frac{x_H - 2x_+}{2x_+(x_+ - 1)}\]

\[(M_2)_{46} = \frac{1}{2x_+}\]

\[(M_2)_{51} = \frac{1}{2(x_+ - 1)}\]

\[(M_2)_{52} = \frac{x_+}{x_+ - 1}\]

\[(M_2)_{53} = 0\]

\[(M_2)_{54} = \frac{(x_+ - 1)(x_+ - x_H) + (x_H - 2)}{2(x_+ - 1)^2}\]

\[(M_2)_{55} = \frac{x_H - 2}{2(x_+ - 1)^2}\]

\[(M_2)_{56} = \frac{1}{2(x_+ - 1)}\]
\begin{align*}
(M_2)_{61} &= \frac{1}{4}(1 + \Delta)m_w^2 \\
(M_2)_{62} &= -x_+m_w^2 \\
(M_2)_{63} &= 0 \\
(M_2)_{64} &= \frac{x_+ + x_H - 1}{4(x_+ - 1)} \\
(M_2)_{65} &= \frac{x_H - 2x_+}{4(x_+ - 1)} \\
(M_2)_{66} &= 0 \quad (A.16)
\end{align*}

As before, we can get similar relations by replacing \(m_H\) by \(m_A\).

In order to prove gauge invariance of the amplitudes given in the text, we require the following two identities:

\begin{align*}
2 B_1(m_1, m_2; M) + B_0(m_1, m_2; M) &= \frac{1}{M^2}[A(m_2) - A(m_1)] - \frac{m_1^2 - m_2^2}{M^2} B_0(m_1, m_2; M) \\
(A.17)
\end{align*}

For \(C_A = C_A(m_1, m_1, m_3, M_1, 0, M_3)\) with \(A = 22, 23, 24\)

\begin{align*}
4 C_{24} + 4 p_1 p_2 (C_{23} - C_{22}) &= \frac{1}{M_3^2}[A(m_3) - A(m_1) + (M_3^2 + m_3^2 - m_1^2)B_0(m_1, m_3; M_3)] \\
(A.18)
\end{align*}

These can be derived as before from the formulae given above.
Appendix B

Detailed Formulae related to Chapter 3.

1. In order to evaluate $B^0_d - \bar{B}^0_d$ mixing one requires to calculate the functions $\mathcal{F}_1(x,y), \mathcal{F}_2(x,y,h)$ which arise in equations (3.27). These functions are defined by

$$\mathcal{F}_1(x,y) = \frac{1}{x-y} [g(x) - g(y)]$$ (B.1)

where

$$g_1(x) = \frac{1}{(1-x)^2} \left[(1-x) + x^2 \log(x)\right]$$ (B.2)

and

$$\mathcal{F}_2(x,y,h) = \frac{1}{x-y} [\mathcal{H}(x,h) - \mathcal{H}(y,h)]$$ (B.3)

where

$$\mathcal{H}_1(x,h) = \frac{1}{x-h} \left[\frac{x \log(x)}{1-x} - \frac{h \log(h)}{1-h}\right]$$ (B.4)

The form of these functions tells us that the case of equal arguments can be obtained by a limiting process, viz.

$$\mathcal{F}_1(x,x) = g'(x)$$
$$\mathcal{F}_2(x,x,h) = \frac{\partial}{\partial x} \mathcal{H}(x,h)$$ (B.5)

These are required to handle the case when the two quark propagators $u_i, u_j$ are identical, as for example, in the crucial case of a pair of $t$-quark propagators.

2. The transition amplitude for $b \rightarrow s\gamma$ can be written in the form

$$\mathcal{M} = e^\mu(p) \bar{s}(p_1) \Gamma_\mu b(p_2)$$ (B.6)

where $p_1, p_2, p$ are the momenta of $s, b, \gamma$ respectively and $e^\mu$ is the polarisation vector of the photon. The effective one-loop coupling $\Gamma_\mu$ can be written in the form

$$\Gamma_\mu = \lambda \left[\gamma_\mu (f_1 + f_2 \gamma_5) + i \sigma_{\mu\nu} p^\nu (f_3 + f_4 \gamma_5)\right]$$ (B.7)

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where $\lambda = (2\pi)^{-3/2} G_F \sqrt{\alpha} \tan^2 \beta$ and the form factors $f_1, f_2, f_3, f_4$ receive contributions from all the four diagrams listed in Fig. 7 (and marked A,B,C,D). A list of the form factors obtained from each diagram (neglecting $m_s << m_b$) is given below in terms of two- and three-point functions.

\[
f_1^A = \sum_{i=1}^{3} m_i^2 V_{ts} V_{tb} \left[ 2C_{24}(+, i, i) - \frac{1}{2} m_i^2 C_6(+, i, i) \right] \tag{B.8}
\]

\[
f_2^A = \sum_{i=1}^{3} m_i^2 V_{ts} V_{tb} \left[ 2m_b^2 C_{12}(+, i, i) + 2m_b^2 C_{23}(+, i, i) \right] - f_1^A
\]

\[
f_3^A = \sum_{i=1}^{3} m_i^2 V_{ts} V_{tb} \left[ m_b C_{11}(+, i, i) + m_b C_{25}(+, i, i) \right]
\]

\[
f_4^A = \sum_{i=1}^{3} m_i^2 V_{ts} V_{tb} \left[ m_b C_{11}(+, i, i) + m_b C_{25}(+, i, i) \right]
\]

\[
f_1^B = -\sum_{i=1}^{3} m_i^2 V_{ts} V_{tb} \left[ 3C_{24}(i, +, +) + 3m_b^2 C_{12}(i, +, +) + 3m_b^2 C_{23}(i, +, +) \right] \tag{B.9}
\]

\[
f_2^B = -f_1^B
\]

\[
f_3^B = -\sum_{i=1}^{3} m_i^2 V_{ts} V_{tb} \left[ \frac{3}{2} m_b C_{12}(i, +, +) + \frac{3}{2} m_b C_{23}(i, +, +) \right]
\]

\[
f_4^B = f_3^B
\]

\[
f_1^C = \sum_{i=1}^{3} m_i^2 V_{ts} V_{tb} \left[ \frac{1}{2} B_0(+, i; b) + \frac{1}{2} B_1(+, i; b) \right] \tag{B.10}
\]

\[
f_2^C = -f_1^C
\]

f_3^C = 0

f_4^C = 0

\[
f_1^D = -\sum_{i=1}^{3} m_i^2 V_{ts} V_{tb} \left[ \frac{m_s}{2m_b} B_0(+, i; s) + \frac{m_s}{2m_b} B_1(+, i; s) \right] \approx 0 \tag{B.11}
\]

f_2^D = -f_1^D

f_3^D = 0

f_4^D = 0

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Keeping only $m_t$ and $m_b$ and neglecting all other masses these reduce to

$$f_1^A = m_t^2 V_{tb}^* V_{ub} \left[ 2C_{24}(+,t,t) - \frac{1}{2} m_t^2 C_6(+,t,t) \right]$$

$$f_1^B = -3m_t^2 V_{tb}^* V_{ub} C_{24}(t,+,+)[(+)$$

$$f_1^C = \frac{3}{2} m_t^2 V_{tb}^* V_{ub} \left[ B_0(+,t;b) + B_1(+,t;b) \right]$$

$$f_1^D = 0$$

$$f_2^{A,B,C,D} = -f_1^{A,B,C,D}$$

$$f_3^{A,B,C,D} = 0$$

$$f_4^{A,B,C,D} = 0$$

keeping in mind that $C_{11}, C_{12}, C_{21}, C_{22}, C_{23}$ are suppressed by $(\frac{m_t}{m_b})^2$ compared to $C_{24}$.

Thus, in this approximation,

$$\Gamma_\mu = \lambda f_1 \gamma_5 (1 - \gamma_5)$$

where

$$f_1 = m_t^2 V_{tb}^* V_{ub} \left[ 2C_{24}(+,t,t) - 3C_{24}(t,+,+) - m_t^2 C_6(+,t,t) + \frac{1}{2} (B_0(+,t;b) + B_1(+,t;b)) \right]$$

so that

$$\sum_{\text{states}} | \mathcal{M} |^2 = 32m_t m_b \lambda^2 | f_1 |^2$$

which leads to the expression for the decay width given in the text (Equation 3.35).

For \textit{ab initio} evaluation of the loop integrals, keeping only $m_t, m_b$ and neglecting all masses as before, we need the functions

$$J_1(x) = \frac{x}{(x-1)^4} \left[ 8x^3 + 6x^2 - 48x + 43 + 6x(2 - 3x) \log(x) \right]$$

$$J_2(x) = \frac{2x}{(x-1)^3} \left[ 15x^2 - 24x + 9 + (2 - 3x) \log(x) \right].$$

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3. The transition amplitude for \( Z^0 \rightarrow b\bar{s} + b\bar{s} \) can be written in the form

\[
\mathcal{M} = e^\mu(p)\bar{\sigma}(p_1)\Gamma_\mu b(p_2)
\]

(B.20)

where \( p_1, p_2, p \) are the momenta of \( s, b, Z^0 \) respectively and \( e^\mu \) is the polarisation vector of the \( Z^0 \)-boson. The effective one-loop coupling \( \Gamma_\mu \) can be written in the form

\[
\Gamma_\mu = \lambda [\gamma_\mu(g_1 + g_2 \gamma_5) + i\sigma_{\mu\nu}p^\nu(g_3 + g_4 \gamma_5)]
\]

(B.21)

where \( \lambda = \frac{gF^2 \tan^2 \beta}{4\pi m_b \sin 2\theta_W} \) and the form factors \( g_1, g_2, g_3, g_4 \) receive contributions from all the four diagrams listed in Fig. 9 (and marked A,B,C,D). A list of the form factors obtained from each diagram (neglecting all masses except \( m_t, m_b \) and \( m_+ \)) is given below in terms of two- and three-point functions.

\[
g_1^A = V_{ts}^*V_{tb}m_t^2 \left[ 2(3 - 4\sin^2 \theta_W)C_{24}(+,t,t) + \sin^2 \theta_W m_t^2 C_0(+,t,t) \right]
\]

(B.22)

\[
g_2^A = -g_1^A
\]

\[
g_3^A = V_{ts}^*V_{tb}m_t^2 m_b(3 - 4\sin^2 \theta_W) [C_{12}(+,t,t) + C_{23}(+,t,t)]
\]

\[
g_4^A = V_{ts}^*V_{tb}m_t^2 m_b(3 - 4\sin^2 \theta_W) [C_{11}(+,t,t) + C_{21}(+,t,t) + C_{22}(+,t,t) + C_{23}(+,t,t)]
\]

\[
g_1^B = -2V_{ts}^*V_{tb}m_t^2 \cos 2\theta_W C_{24}(t,+,+)
\]

(B.23)

\[
g_2^B = -g_1^B
\]

\[
g_3^B = -V_{ts}^*V_{tb}m_t^2 m_b \cos 2\theta_W [C_{12}(t,+,+) + C_{23}(t,+,+)]
\]

\[
g_4^B = g_3^B
\]

\[
g_1^C = -2V_{ts}^*V_{tb}m_t^2 \sin^2 \theta_W [B_0(+,t;b) + B_1(+,t;b)]
\]

(B.24)

\[
g_2^C = -g_1^C
\]

\[
g_3^C = 0
\]

\[
g_4^C = 0
\]

\[
g_1^D = 2V_{ts}^*V_{tb}m_t^2 \frac{m_+}{m_b} \sin^2 \theta_W [B_0(+,t;s) + B_1(+,t;s)]
\]

(B.25)
$$g_2^D = -V_{ts}^* V_{tb} m_t^2 m_s^2 (3 - 4 \sin^2 \theta_W) \left[ B_0(+,t;s) + B_1(+,t;s) \right]$$

$$g_3^D = 0$$

$$g_4^D = 0$$

Then, noting that $C_{24}$ is the largest three-point function (as in the case of $b \rightarrow s \gamma$), we get

$$\Gamma_\mu = \lambda g_1 \gamma_\mu (1 - \gamma_5)$$

where

$$g_1 = V_{ts}^* V_{tb} m_t^2 \left[ 2(3 - 4 \sin^2 \theta_W) C_{24}(+,t,t) + 8 \sin^2 \theta_W m_t^2 C_0(+,t,t) - 6 \cos 2\theta_W C_{24}(t,+,+) - 2 \sin^2 \theta_W \{ B_0(+,t;b) + B_1(+,t;b) \} \right]$$

so that

$$\sum_{\text{spin}} |\mathcal{M}|^2 = 32 m_s m_b \lambda^2 |g_1|^2$$

which leads to the expression for the decay width given in the text.
Appendix C
Detailed Formulae Related to Chapter 5.

Contributions from Individual Diagrams with Bosonic Loops: The $W^+H^-\gamma$ coupling arising from the bosonic diagram numbered $n$ (see Fig. 14(i)) is given by

$$M_{\mu\nu}^b = \lambda (X_n^b \delta_{\mu\nu} + Y_n^b r_1 r_2)$$

where $\lambda = \frac{g^2 m_W}{\sqrt{\pi} \sin \delta_W} \sin 2(\alpha - \beta)$ and the $X_n^b, Y_n^b$ are listed below. It should be noted that $Y_n^b = 0$ for $n = 11$ to 100, so these have not been listed.

Some prior explanation of the notation used below is called for. All three-point functions have as common arguments, the external masses $m_W, 0, m_\gamma$. These have been omitted for the sake of brevity. The $C$-functions with internal masses $m_i, m_j, m_k$ have been denoted $C(i,j,k)$. For $B$-functions, a similar convention has been adopted. We have denoted by $B(i,j;k)$ the two-point function with internal masses $m_i, m_j$ and external mass $m_k$. For one-point functions, a slightly different notation has been used.

We introduce the symbol $A_i$ which is defined by

$$A_i = \frac{1}{m_i^2} A(m_i)$$

while the symbols $r_i$ are defined as in the text by the formulae $r_i = \frac{m_i^2}{m_W^2}$. The list of bosonic form factors follows.

$$X_1^b = \frac{1}{2} (r_+ + r_H - 2r_W) m_2^2 C_0(W,W,H) + \frac{1}{2} C_{24}(W,W,H) + \frac{1}{2} \{ B_0(W,H;W) - B_0(W,W;H) - B_0(W,H;) \}$$

$$Y_1^b = -2C_0(W,W,H) - C_{11}(W,W,H) - C_{12}(W,W,H) - \frac{1}{2} C_{22}(W,W,H)$$

$$+ \frac{1}{2} C_{23}(W,W,H)$$

$$X_2^b = -\frac{1}{2} (r_+ + r_h - 2r_W) m_2^2 C_0(W,W,h) - \frac{1}{2} C_{24}(W,W,h) - \frac{1}{2} \{ B_0(W,h;W) \}$$

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\[ -B_0(W, W; \gamma) - B_0(W, h; +) \]

\[ Y_2^b = 2C_0(W, W, h) + C_{11}(W, W, h) + C_{12}(W, W, h) + \frac{1}{2} C_{23}(W, W, h) \]

\[ Y_3^b = 0 \]  

\[ X_3^b = \frac{1}{2}(r_+ - r_h) m^2 C_0(W, W, H) \]

\[ Y_4^b = -\frac{1}{2}(r_+ - r_h) m^2 C_0(W, W, h) \]

\[ Y_4^b = 0 \]

\[ X_5^b = \frac{1}{2} C_{24}(W, W, H) \]

\[ Y_6^b = C_{11}(W, W, H) - C_{12}(W, W, H) - \frac{1}{2} C_{22}(W, W, H) + \frac{1}{2} C_{23}(W, W, H) \]  

\[ X_6^b = -\frac{1}{2} C_{24}(W, W, h) \]

\[ Y_6^b = C_{11}(W, W, h) + C_{12}(W, W, h) + \frac{1}{2} C_{22}(W, W, h) - \frac{1}{2} C_{23}(W, W, h) \]

\[ X_7^b = \frac{r_+ - r_H}{r_W} C_{24}(W, W, H) \]

\[ Y_7^b = \frac{r_+ - r_H}{r_W} \{ C_{23}(W, W, H) - C_{22}(W, W, H) \} \]  

\[ X_8^b = -\frac{r_+ - r_h}{r_W} C_{24}(W, W, h) \]

\[ Y_8^b = -\frac{r_+ - r_h}{r_W} \{ C_{23}(W, W, h) - C_{22}(W, W, h) \} \]  

\[ X_9^b = \frac{r_H - 2r_W}{r_W} C_{24}(+, +, H) \]

\[ Y_9^b = \frac{r_H - 2r_W}{r_W} \{ C_{23}(+, +, H) - C_{22}(+, +, H) \} \]

\[ X_{10}^b = -\frac{r_+ - r_h}{r_W} C_{24}(+, +, h) \]

\[ Y_{10}^b = -\frac{r_+ - r_h}{r_W} \{ C_{23}(+, +, h) - C_{22}(+, +, h) \} \]

\[ X_{11}^b = -\frac{r_+ - r_H}{4r_W} B_0(W, H; +) \]

\[ X_{12}^b = \frac{r_+ - r_h}{4r_W} B_0(W, h; +) \]
\[ X_{13}^k = - \frac{r_H - 2r_W}{4r_W} B_0(+, H; +) \]  
(C.13)

\[ X_{14}^k = \frac{r_h - 2r_W}{4r_W} B_0(+, h; +) \]  
(C.14)

\[ X_{15}^k = \frac{1}{2} B_0(W, H; W) \]  
(C.15)

\[ X_{16}^k = - \frac{1}{2} B_0(W, h; W) \]  
(C.16)

\[ X_{17}^k = \frac{r_W}{2r_A} \{ B_1(W, H; +) + 2B_0(W, H; +) \} \]  
(C.17)

\[ X_{18}^k = - \frac{r_W}{2r_A} \{ B_1(W, h; +) + 2B_0(W, h; +) \} \]  
(C.18)

\[ X_{19}^k = \frac{r_+ - r_H}{4r_A} \{ 2B_1(W, H; +) + B_0(W, H; +) \} \]  
(C.19)

\[ X_{20}^k = - \frac{r_+ - r_h}{4r_A} \{ 2B_1(W, h; +) + B_0(W, h; +) \} \]  
(C.20)

\[ X_{21}^k = \frac{r_H - 2r_W}{4r_A} \{ 2B_1(+, H; +) + B_0(+, H; +) \} \]  
(C.21)

\[ X_{22}^k = - \frac{r_h - 2r_W}{4r_A} \{ 2B_1(+, h; +) + B_0(+, h; +) \} \]  
(C.22)

\[ X_{23}^k = \frac{2r_W}{r_A r_H} A_W \]  
(C.23)

\[ X_{24}^k = - \frac{2r_W}{r_A r_h} A_W \]  
(C.24)

\[ X_{25}^k = \frac{1}{4r_A} A_W \]  
(C.25)

\[ X_{26}^k = - \frac{1}{4r_A} A_W \]  
(C.26)

\[ X_{27}^k = - \frac{r_H - 2r_W}{4r_H r_A} A_+ \]  
(C.27)

\[ X_{28}^k = \frac{r_h - 2r_W}{4r_h r_A} A_+ \]  
(C.28)

\[ X_{29}^k = - \frac{r_W}{4r_H r_A} A_W \]  
(C.29)

\[ X_{30}^k = \frac{r_W}{4r_h r_A} A_W \]  
(C.30)

\[ X_{31}^k = - \frac{r_W}{4r_H r_A} A_W \]  
(C.31)
\[ X_{32}^b = \frac{r_W - A_W}{4r_H T_A} \]  
(C.32)

\[ X_{33}^b = -\frac{1}{8r_H T_A} A_Z \]  
(C.33)

\[ X_{34}^b = \frac{1}{8r_H T_A} A_Z \]  
(C.34)

\[ X_{35}^b = \frac{1}{r_H T_A} A_Z \]  
(C.35)

\[ X_{36}^b = -\frac{1}{r_H T_A} A_Z \]  
(C.36)

\[ X_{37}^b = \frac{1}{8r_A} A_Z \]  
(C.37)

\[ X_{38}^b = -\frac{1}{8r_A} A_Z \]  
(C.38)

\[ X_{39}^b = -\frac{3r_H + 3r_h - 6}{8r_A(r_H - r_h)} A_H \]  
(C.39)

\[ X_{40}^b = \frac{3r^2 + 3r_H r_h - 2r_H - 4r_h A_H}{8r_H T_A(r_H - r_h)} \]  
(C.40)

\[ X_{41}^b = \frac{3r^2 + 3r_H r_h - 2r_H - 4r_H A_h}{8r_H T_A(r_H - r_h)} \]  
(C.41)

\[ X_{42}^b = -\frac{3r_H + 3r_h - 6}{8r_A(r_H - r_h)} A_h \]  
(C.42)

\[ X_{43}^b = -\frac{1}{8r_A} A_A \]  
(C.43)

\[ X_{44}^b = \frac{1}{8r_A} A_A \]  
(C.44)

\[ X_{45}^b = \frac{1}{4r_A} \left\{ -A_H + 2A_W + (r_W - 2r_H - 2r_h) B_0(W, H; +) \right\} \]  
(C.45)

\[ X_{46}^b = -\frac{1}{4r_A} \left\{ -A_H + 2A_W + (r_W - 2r_H - 2r_h) B_0(W, h; +) \right\} \]  
(C.46)

\[ X_{47}^b = -\frac{r_H(r_+ - r_H)}{4r_W T_A} B_0(W, H; +) \]  
(C.47)

\[ X_{48}^b = \frac{r_h(r_+ - r_h)}{4r_W T_A} B_0(W, h; +) \]  
(C.48)

\[ X_{49}^b = \frac{(r_+ - r_H)(r_H - 2r_W)}{4r_W T_A} B_0(+, H; +) \]  
(C.49)

\[ X_{50}^b = -\frac{(r_+ - r_h)(r_h - 2r_W)}{4r_W T_A} B_0(+, h; +) \]  
(C.50)
\[ x_{51}^b = - \frac{2(t_+ - t_H)}{r_{AH}} A_W \]  
\[ x_{52}^b = \frac{2(t_+ - t_h)}{r_{AH}} A_W \]  
\[ x_{53}^b = - \frac{t_+ - t_H}{4r_W T_A} A_W \]  
\[ x_{54}^b = \frac{t_+ - t_h}{4r_W T_A} A_W \]  
\[ x_{55}^b = \frac{(t_+ - t_h)(t_H - 2t_W)}{4r_W T_A T_A} A_+ \]  
\[ x_{56}^b = - \frac{(t_+ - t_h)(t_h - 2t_W)}{4r_W T_A T_A} A_+ \]  
\[ x_{57}^b = \frac{t_+ - t_H}{4r_H T_A} A_W \]  
\[ x_{58}^b = - \frac{t_+ - t_h}{4r_H T_A} A_W \]  
\[ x_{59}^b = \frac{t_+ - t_H}{4r_H T_A} A_W \]  
\[ x_{60}^b = - \frac{t_+ - t_h}{4r_H T_A} A_W \]  
\[ x_{61}^b = \frac{t_+ - t_H}{8r_W T_A T_A} A_Z \]  
\[ x_{62}^b = - \frac{t_+ - t_h}{8r_W T_A T_A} A_Z \]  
\[ x_{63}^b = - \frac{t_+ - t_H}{r_W T_A T_A} A_Z \]  
\[ x_{64}^b = \frac{t_+ - t_h}{r_W T_A T_A} A_Z \]  
\[ x_{65}^b = - \frac{t_+ - t_H}{8r_W T_A} A_Z \]  
\[ x_{66}^b = \frac{t_+ - t_h}{8r_W T_A} A_Z \]  
\[ x_{67}^b = \frac{(t_+ - t_H)(3t_H + 3t_h - 6)}{8r_W T_A (t_H - t_h)} A_H \]  
\[ x_{68}^b = - \frac{(t_+ - t_h)(3t_h^2 + 3t_H t_h - 2t_H - 4t_h)}{8r_W T_A (t_H - t_h)} A_H \]
\begin{align*}
X_{69}^b &= -\frac{(r_+ - r_H)(3r_H^2 + 3r_H r_h - 2r_h - 4r_H)}{8r_W r_H r_A (r_H - r_h)}A_h \quad \text{(C.69)}
X_{70}^b &= \frac{r_+ - r_h}{8r_W r_A (r_H - r_h)}A_h \quad \text{(C.70)}
X_{71}^b &= \frac{r_+ - r_H}{8r_W r_A}A_A \quad \text{(C.71)}
X_{72}^b &= -\frac{r_+ - r_h}{8r_W r_A}A_A \quad \text{(C.72)}
X_{73}^b &= -\frac{r_H - r_h}{2r_W r_A}A_W \quad \text{(C.73)}
X_{74}^b &= \frac{r_H - r_h}{2r_W r_A}A_+ \quad \text{(C.74)}
X_{75}^b &= \frac{r_H + r_h + 2r_W - 2}{8r_W r_A}A_H \quad \text{(C.75)}
X_{76}^b &= -\frac{r_H + r_h + 2r_W - 2}{8r_W r_A}A_h \quad \text{(C.76)}
X_{77}^b &= \frac{r_H - r_h}{8r_W r_A}A_A \quad \text{(C.77)}
X_{78}^b &= -\frac{r_H - r_h}{8r_W r_A}A_Z \quad \text{(C.78)}
X_{79}^b &= \frac{2}{r_H}A_W \quad \text{(C.79)}
X_{80}^b &= -\frac{2}{r_h}A_W \quad \text{(C.80)}
X_{81}^b &= -\frac{1}{4r_W}A_W \quad \text{(C.81)}
X_{82}^b &= -\frac{1}{4r_W}A_W \quad \text{(C.82)}
X_{83}^b &= -\frac{r_h - 2r_W}{4r_H r_W}A_+ \quad \text{(C.83)}
X_{84}^b &= \frac{r_h - 2r_W}{4r_H r_W}A_+ \quad \text{(C.84)}
X_{85}^b &= -\frac{1}{4r_H}A_W \quad \text{(C.85)}
X_{86}^b &= \frac{1}{4r_h}A_W \quad \text{(C.86)}
\end{align*}
Quite a few of the cancellations among these diagrams are apparent on inspection. Others occur between groups of diagrams on addition. The final sum after all cancellations is given in the text.

Divergent parts of these diagrams are easy to identify. In the dimensional regularisation scheme, the $A, B, C$-functions have divergent parts proportional to $\Delta = 2/(4 - d) + \gamma - \log \pi$ where $d \to 4$ and $\gamma$ is the Euler-Mascheroni constant. The

\[
\begin{align*}
X_{87}^b &= -\frac{1}{4r_h} A_W \\
X_{88}^b &= \frac{1}{4r_h} A_W \\
X_{89}^b &= -\frac{1}{8r_H r_W} A_Z \\
X_{90}^b &= \frac{1}{8r_H r_W} A_Z \\
X_{91}^b &= \frac{1}{r_H r_W} A_Z \\
X_{92}^b &= -\frac{1}{r_H r_W} A_Z \\
X_{93}^b &= \frac{1}{8r_W} A_Z \\
X_{94}^b &= -\frac{1}{8r_W} A_Z \\
X_{95}^b &= -\frac{3r_H + 3r_h - 6}{8r_W(r_H - r_h)} A_H \\
X_{96}^b &= \frac{3r_h^2 + 3r_H r_h - 2r_H - 4r_h}{8r_H r_W(r_H - r_h)} A_H \\
X_{97}^b &= \frac{3r_h^2 + 3r_H r_h - 2r_H - 4r_h}{8r_H r_W(r_H - r_h)} A_H \\
X_{98}^b &= -\frac{3r_H + 3r_h - 6}{8r_W(r_H - r_h)} A_H \\
X_{99}^b &= -\frac{1}{8r_W} A_A \\
X_{100}^b &= \frac{1}{8r_W} A_A
\end{align*}
\]
coefficients of $\Delta$ are

\[
div. A_i = -r_i
\]
\[
div. B_0(i,j;k) = 1
\]
\[
div. B_1(i,j;k) = -1/2
\]
\[
div. C_{24}(i,j,k) = 1/4
\]
while the remaining functions are finite. From these, it is obvious that each $Y_i^k$ is finite by construction. Divergent parts of the $X_i^k$ are easy to identify by substituting the above formulae into Eqns. (C.1 - C.100). It is then straightforward to verify that $\sum_i div. X_i^k = 0$. 

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Contributions from Individual Diagrams with Fermionic Loops: As in the previous case, the $W^+H^-\gamma$ coupling arising from the fermionic diagram numbered $n$ (see Fig. 14(ii)) is given by

$$M_{\mu\nu} = \lambda(X_n \eta_{\mu\nu} + Y_n^f P_{1\nu} P_{2\mu} + Z_n^f \epsilon_{\mu\nu\alpha\beta} P_1^\alpha P_2^\beta)$$

where $\lambda = e^2 m_W^2 / 2v^2$, and the $X_n^f, Y_n^f, Z_n^f$ are listed below. It should be noted that $Y_n^f = Z_n^f = 0$ for $n = 3$ to 10, so these have not been listed.

It may be pointed out once more that only the diagrams and amplitudes with $t, b$ quarks have been listed here. Contributions from other quark generations will follow exactly the same pattern if we ignore mixing between the generations. If we wish to include all generations with their mixings, it is only necessary to replace $m_t, m_b$ everywhere by $m_i^t, m_j^b$ and sum over all values $i, j = 1, 2, 3$ of the generation indices after multiplying each diagram by the appropriate elements of the Cabibbo-Kobayashi-Maskawa matrix. For leptons, the picture is somewhat different because the diagram corresponding to no. 2 in Fig. 14(ii) is absent and the factors corresponding to $x_b$ vanish. The rest is easily obtained. Inclusion of all these diagrams changes the result for only $t, b$ quarks by considerably less than 1%. It should also be noted that since the charged Higgs boson couplings are proportional to the masses of the charge $\frac{2}{3}$ quarks, there is no GIM cancellation.

The notation follows most of the conventions set up for bosons. We need to introduce two new symbols $r_t, r_b$ which are defined by

$$r_t = \frac{m_t^2}{m_b^2} \cot \beta$$

$$r_b = \frac{m_b^2}{m_t^2} \tan \beta$$

and in terms of these we write $S, D = (r_t \pm r_b) / r_W$. The list of form factors follows.
\[ X_i' = -\frac{1}{4} \left[ 2^{\frac{\tau_A^2}{\tau_W}} m_2^2 C_0(b, b, t) + S \{ 8C_{24}(b, b, t) - 2B_0(b, t; +) \} - 2DB_0(b, t; W) \right] \]

\[ Y_i' = \frac{\tau_b}{\tau_W} C_0(b, b, t) - 2S \{ C_{23}(b, b, t) - C_{22}(b, b, t) \} - DC_{12}(b, b, t) \]

\[ Z_i' = \frac{\tau_b}{\tau_W} C_0(b, b, t) + 2\frac{\tau_b}{\tau_W} C_{11}(b, b, t) + DC_{12}(b, b, t) \]  \hspace{1cm} (C.101)

\[ X_i' = \frac{\tau_A^2}{\tau_W} m_2^2 C_0(t, t, b) + S \{ 4C_{24}(t, t, b) - B_0(b, t; +) \} + DB_0(b, t; W) \]

\[ Y_i' = -2\frac{\tau_t}{\tau_W} C_0(t, t, b) + 4S \{ C_{23}(t, t, b) - C_{22}(t, t, b) \} - 2DC_{12}(t, t, b) \]

\[ Z_i' = 2\frac{\tau_t}{\tau_W} C_0(t, t, b) + SC_{12}(t, t, b) \]  \hspace{1cm} (C.102)

\[ X_i' = -3\frac{\tau_W}{\tau_A^2} \left\{ SB_1(b, t; +) + \frac{\tau_t}{\tau_W} B_0(b, t; +) \right\} \]  \hspace{1cm} (C.103)

\[ X_i' = -\frac{3}{2\tau_A^2} \left\{ D \{ A_t + A_b + \tau_+ B_0(b, t; +) \} - S \csc 2\beta (D - S \cos 2\beta) B_0(b, t; +) \right\} \]  \hspace{1cm} (C.104)

\[ X_i' = -\frac{3\tau_W}{\tau_A^2 H} \sin(\beta - \alpha) \left( \frac{\tau_t}{\tau_W} \sin \alpha \sec \beta A_t + \frac{\tau_b}{\tau_W} \cos \alpha \csc \beta A_b \right) \]  \hspace{1cm} (C.105)

\[ X_i' = \frac{3\tau_W}{\tau_A^2 H} \cos(\beta - \alpha) \left( \frac{\tau_t}{\tau_W} \cos \alpha \sec \beta A_t - \frac{\tau_b}{\tau_W} \sin \alpha \csc \beta A_b \right) \]  \hspace{1cm} (C.106)

\[ X_i' = \frac{3(\tau_t - \tau_H)}{\tau_A^2 H} \sin(\beta - \alpha) \left( \frac{\tau_t}{\tau_W} \sin \alpha \sec \beta A_t + \frac{\tau_b}{\tau_W} \cos \alpha \csc \beta A_b \right) \]  \hspace{1cm} (C.107)

\[ X_i' = -\frac{3(\tau_t - \tau_h)}{\tau_A^2 h} \cos(\beta - \alpha) \left( \frac{\tau_t}{\tau_W} \cos \alpha \sec \beta A_t - \frac{\tau_b}{\tau_W} \sin \alpha \csc \beta A_b \right) \]  \hspace{1cm} (C.108)

\[ X_i' = -\frac{3}{\tau_h} \sin(\beta - \alpha) \left( \frac{\tau_t}{\tau_W} \sin \alpha \sec \beta A_t + \frac{\tau_b}{\tau_W} \cos \alpha \csc \beta A_b \right) \]  \hspace{1cm} (C.109)

\[ X_i' = \frac{3}{\tau_h} \cos(\beta - \alpha) \left( \frac{\tau_t}{\tau_W} \cos \alpha \sec \beta A_t - \frac{\tau_b}{\tau_W} \sin \alpha \csc \beta A_b \right) \]  \hspace{1cm} (C.110)

Finiteness of the sum \( \sum_i X_i' \) is shown exactly as in the bosonic case, while the \( Y_i', Z_i' \) are finite by construction.