Chapter II

Sagnac Interferometer

Changes in the states of polarization of two interfering beams affect the fringe pattern formed. Pancharatnam's phase is easily revealed as the phase associated with such changes. To study the effects of Pancharatnam's phase one is required to choose an interferometer. Sagnac Interferometer [2.1- 9] turns out to be the one made for this type of study. Here, the changes in the interference pattern can solely be attributed to changes in the state of polarization created by a polarization changing optical element placed inside the interferometer. Aim of this work is to study the effects of Pancharatnam's phase on the output of Sagnac interferometer and its nonlinear behaviour. To do that, it is necessary to look into how Sagnac interferometer works and what will be its output in its different configurations (The matrices giving the output for different configurations of Sagnac interferometer are given in appendix B). The best part of Sagnac interferometer is that the polarization states of the two counter-propagating beams can be altered by a single element. Both the beams are made to travel through the same optical element, which changes the polarization. Because of this, both the beams see the same propagation but different polarization effects. Sagnac interferometer is already known for its inherent stability towards vibrations and against any changes in optical path. All these factors make Sagnac interferometer suitable for this study.

The states of polarization of the two beams passing through the interferometer are altered by placing an optical element like a retarder, polarizer and an optically active medium in one of the arms of Sagnac interferometer. As a retarder or a polarizer is rotated, the polarization states change continuously and as a result the fringe pattern changes continuously. These modifications in the fringe pattern depend on the following factors. (i) Number of arms in the interferometer, (ii) type of the beam splitter (BS), (iii) state of polarization of the input beam (iv) nature of the optical element used to alter the polarization. Hence, as a part of the study different configurations of Sagnac interferometer are studied and are supported by experiments wherever possible. The configurations studied are:
Chapter II. Sagnac Interferometer

I. Four-arm Sagnac interferometer with non-polarizing 50:50 beam splitter.
II. 3-arm Sagnac interferometer with non-polarizing 50:50 beam splitter.
III. Both the above configurations with polarizing beam splitter.

In each of the above cases, different optical elements are placed inside one of the arms and the output is observed for different states of polarization of the input beam. Though, some of these cases have been discussed in literature earlier, for the sake of completeness they have been included.

I. Four-arm Sagnac Interferometer.

Consider the four-arm Sagnac interferometer shown in fig 2.1 consisting of a 50:50 non-polarizing beam splitter (BS), three 100% reflecting mirrors (M_1, M_2 & M_3). The four elements of four-arm Sagnac interferometer, i.e., beam splitter and the three mirrors are arranged at the corners of a quadrilateral of suitable perimeter. One of the two faces marked 1 & 2 acts as an entrance for the input beam and the other acts as an exit for the output (though output beams emerge out of both the faces, the face other than the input face is considered keeping in view the experimental convenience).

Let a laser beam be incident on face-1 of the beam splitter. This beam gets split into two at the beam splitting surface of the beam splitter. These beams marked \( B_1 \) and \( B_2 \) counter-propagate in the interferometer travelling the same path and meet again at the beam splitter. The beam travelling clockwise will take the path beam splitter - \( M_3 \) - \( M_2 \) - \( M_1 \) - beam splitter, while the counter-clockwise beam just takes the reverse path. Each of these beams is split into two at beam splitter once again, one exiting (not shown) from face-1 and other (shown by outgoing arrow) from face-2. Hence, the output consists portions of the two beams \( B_1 \) and \( B_2 \) traversing the interferometer. Plane of interferometer (POI) is defined as the plane containing the axes of propagation. It is useful now to define and associate with each beam a right handed Cartesian coordinate system. A beam travels along the z-axis of its coordinate system. The x-axis of the coordinate systems of different beams in fig.2.1 are perpendicular to, and point out from the paper. Consequently the y-z plane in fig.2.1 coincides with
POL It is readily seen that a polarization vector \( \mathbf{E}(\mathbf{e}_x, \mathbf{e}_y) \) at the input face-1 transforms at the output stage on face-2 as \( \mathbf{E}(\mathbf{e}_x, -\mathbf{e}_y) \) with respect to the coordinate system of the beams. This is due to the change in the sign of the component parallel to POI at each reflection. Thus a linearly polarized input coherent beam produces two coherent beams with similar polarization at the exit face-2.

Now consider the case when an optical element with an axis is placed in one of the arms of Sagnac interferometer as shown in fig.2.2. Let \( \varphi \) be the angle made by the axis of the optical element with x-axis. The counter-propagating beams see this optical element differently. One of the beams sees the optical element's axis making an angle +\( \varphi \) (plus \( \varphi \)) with x-axis and the other beam sees the optical element to make an angle -\( \varphi \) (minus \( \varphi \)). Hence, the polarization of the output beams will be different and will depend on the polarization of the input and the nature of the optical element.

The intensity pattern of the output of a four-arm Sagnac interferometer for an arbitrary input polarization and any non-reflecting optical element is given by

\[
I_o = |E_o^x|^2 + |E_o^y|^2
\]

\[
E_o^x = [\omega e_{11} - \omega e_{21} e^{i\Delta}] E_i^x + [\omega e_{12} - \omega e_{22} e^{i\Delta}] E_i^y
\]

\[
E_o^y = -[\omega e_{13} - \omega e_{23} e^{i\Delta}] E_i^x - [\omega e_{14} - \omega e_{24} e^{i\Delta}] E_i^y
\]

\[
E_i^x = a
\]

\[
E_i^y = b e^{i\mu}
\]

(2.1)

\( A \) is the phase difference due to optical path and the input is elliptically polarized with the component parallel to x-axis being \( 'a' (= E_i^x) \) and the component parallel to y-axis is equal to \( 'b \exp(i\mu)' (= E_i^y) \), \( \mu \) being the phase advance of the y-component over the x-component. \( '\omega e_{ij}' \) (i, j=1,2) are the elements of the transformation matrix of the optical element.
Chapter II. Sagnac Interferometer

1) For retarder plates the transformation matrix elements are given by the following.

\[ o_{e11} = o_{e21} = \cos^2 \varphi + e^5 \sin^2 \varphi; \quad o_{e14} = o_{e24} = \sin^2 \varphi + e^5 \cos^2 (\varphi) \]

where, \( e \) is the retardation strength. \( e = \frac{\lambda}{2} \) represents a quarter wave plate while \( e = \frac{\lambda}{4} \) represents a half wave plate and \( \varphi \) is the angle made by the fast axis of the wave plates with the x-axis.

2) For a linear polarizer the transformation matrix elements are

\[ o_{e11} = o_{e21} = \cos \varphi \quad o_{e14} = o_{e24} = \sin \varphi \]

where \( \varphi \) is the angle made by the axis of the polarizer with the x-axis.

3) For optically active medium the transformation matrix elements are

\[ o_{e11} = o_{e21} = o_{e14} = o_{e24} = \cos \varphi; \quad o_{e12} = -o_{e13} = o_{e22} = -o_{e23} = \sin \varphi \]

where \( \varphi \) is the angle by which the input polarization is rotated.

In the following work we consider a few representative cases of the above general expression given in eq.2.1.

A: Input beam is circularly polarized \( (a = b, \mu=\pm\pi/2) \)

The intensity of the fringe pattern, when the incident beam is circularly polarized and when a half wave plate \( (\delta=\pi) \) is placed in one of the arms of four-arm Sagnac interferometer is given by

\[ I_{\text{hwp}(\mu=\pm\pi/2)} = \frac{1 - \cos(\Delta \mp 4\varphi)}{2} \quad (2.2) \]

Here '+' sign is for \( X = -\frac{\lambda}{2} \) (left circularly polarized) and '-' is for \( \mu = \frac{\pi}{2} \) (right circularly polarized). In fig.2.3 the fringe pattern is plotted as a function of \( \Delta \) (phase difference due to optical path difference between the two counter propagating beams) for different values of \( \varphi \), where \( \varphi \) is the angle made by the fast axis of the half wave plate with x-axis. It is observed from the intensity expression and from fig.2.3 that, as the
half wave plate is rotated the fringes move across the observation plane i.e., along the A axis linearly. To see how Pancharatnam’s phase evolves it is better to do the analysis using the Poincare sphere (fig.2.4). (For the details of the representation of polarization on Poincare sphere see Appendix A). Let the input polarization be right circularly polarized. Therefore the state of polarization of the counter-propagating beams is also circularly polarized just before passing through the half wave plate (the state of polarization of the beams before passing through the optical element of course depends on the arm in which the optical element is placed in Sagnac interferometer). Let the input right circular polarization be represented by the North Pole on the Poincare sphere for both the beams. As the two beams pass through the half wave plate the polarization changes from right circular to left circular. To define the trajectories on the Poincare sphere, along which these changes take place, one has to define an axis with respect to which all the calculations are done. The axis, which is chosen here, is the axis perpendicular to the plane of interferometer i.e., parallel to x-axis. The paths taken for the changes depend on the angle \( \varphi \), which is the angle made by the fast axis of the half wave plate with the x-axis. When \( \varphi = 0^\circ \) the position of the axis of the half wave plate is taken to be lying on the equator at the point representing the linearly polarization along x-axis. Then the transformation occurs along the longitude passing through the linear polarization state, with azimuth \(-45^\circ\) (with respect to x-axis), represented by the \(-90^\circ\) point on the equator of the Poincare sphere. To know why the beams take the trajectories mentioned above, consider the half wave plate to be made up of two quarter wave plates. It is known that when a right circularly polarized light passes through a quarter wave plate, it gets converted into a linearly polarized light with its azimuth at \(-45^\circ\) to the fast axis of the quarter wave plate. In this case, the first quarter wave plate will convert the right circularly polarized beam into a linearly polarized with azimuth at \(-45^\circ\) (cutting the equator accordingly) and the second quarter wave plate will take it from equator to the left circular polarization state. Also, the trajectory to be taken by the beams should be along a circle, which is perpendicular to the axis joining the center of the Poincare sphere and the point representing the axis of half (or quarter) wave plate. In this case, as the input polarization is North Pole the circle along which the transformation takes place is a great circle i.e., a longitude.
When half wave plate is at $\phi = 0^\circ$ the trajectories take the paths mentioned above. As the half wave plate is rotated, $\phi$ changes and the paths taken for the transformation will be different for the two beams. This is because the beams see the axis of the half wave plate differently. One beam sees it at $-\phi$ whereas the second sees it at $+\phi$. Hence, for any arbitrary value of $\phi$ the transformation for one of the beams will be along the longitude passing through the point $-90^\circ + 2\phi$ on equator representing linear polarization state with azimuth $\phi - 45^\circ$, whereas for the other beam it will be along the longitude passing through the point $-90^\circ - 2\phi$ representing linear polarization (coming out of quarter wave plate) with azimuth $-\phi - 45^\circ$. As the half wave plate is rotated, area covered by the two longitudes changes resulting in a change in the Pancharatnam's phase. Solid angle subtended by the area between the two longitudes at the center of the sphere is equal to $8\phi$. Pancharatnam's phase is half of this solid angle i.e., $4\phi$. It is clear that Pancharatnam's phase changes linearly with the change in $\phi$. Therefore the intensity fringe movement is continuous and linear with the change in $\phi$.

Experimental fringes for this case are shown in fig. 2.5 (print out of CCD images). In this figure we show the fringe pattern for ($p = 0^\circ, 45^\circ, 90^\circ, 135^\circ$ etc. The fringes move across the plane of observation towards right as in the theoretical fringe patterns.

When the half wave plate is replaced by a quarter wave plate ($\delta = \pi/2$), it is observed in the experiments that the fringe pattern shows variation of the contrast of the fringes along with the movement of fringes. The fringe pattern is then governed by

$$I_{qwp(\mu=\pm\pi/2)} = [4 - 2 \cos \Delta - 2 \cos(\Delta \mp 4\phi)]/4$$

(2.3)
wave plate. The third term results in a moving fringe pattern with the rotation of $\phi$. Thus, a fringe pattern moves over a stationary fringe pattern having same width and intensity of the fringes. This effect shows a variation in the contrast of the fringes along with the movement of fringes (fig.2.6). There are four positions where the fringe pattern disappears. At these points the states of polarization of the output beams are orthogonal to one another. This happens when $\phi = 45^\circ$, $135^\circ$, $225^\circ$ and $315^\circ$. A similar variation of the contrast is observed when a linear polarizer is used instead of the quarter wave plate. The fringe pattern in this case has the same expression as for quarter wave plate except that the intensity gets reduced by 50\%. To look at the role played by Pancharatnam's phase in both the quarter wave plate case and the linear polarizer case, Poincare sphere (fig.2.7) is used. Let the input beam be represented by the North Pole. In both cases, the trajectories of transformation end up on the equator but at different points. The trajectories followed for an arbitrary angle $\phi$ of quarter wave plate, are the longitudes which pass through the points $-90^\circ + 2\phi$ ($P_2$) and $-90^\circ - 2\phi$ ($P_1$) representing the linear polarization states with azimuths $\phi - 45^\circ$ and $-\phi - 45^\circ$ respectively. In the case of linear polarizer, the trajectories are longitudes passing through the points $+2\phi$ ($P_3$) and $-2\phi$ ($P_4$) representing the linear polarization states with azimuths $+\phi$ and $-\phi$ respectively.

Now consider the case where an optically active medium is placed as an optical element. The intensity expression for the fringe pattern is given by

$$I_{\text{om}(\mu=\pm\pi/2)} = \frac{(a^2 + b^2)}{4} [1 - \cos(\Delta)]$$  \hspace{1cm} (2.4)

The above expression doesn't contain any term involving the optical rotation angle. Also the counter propagating beams see the optically active medium in the same way unlike in the earlier cases of wave plates. Because of this the output beams end with same polarization whatever be the angle of optical rotation. Therefore, there will not be any changes in the fringe pattern as one varies the optical rotation in a four-arm Sagnac interferometer.
**B: Input beam is linearly polarized ($\mu = 0$)**

Intensity of the fringe pattern when the input beam is linearly polarized parallel to x-axis ($b=0$) and a half wave plate is placed in one of the arms of a four-arm Sagnac interferometer, is given by the expression

$$I_{\text{hwp}(\mu=0)} = a^2 \left[ 2 - \cos(\Delta - 4\varphi) - \cos(\Delta + 4\varphi) \right] / 4$$  \hspace{1cm} (2.5)

Where $A$ is the phase due to optical path difference between the two beams and $\varphi$ is the angle made by the fast axis of the half wave plate with the x-axis. On rotation of half wave plate the intensity of the fringes (fig.2.8) is observed to vary. This is in contrast to the movement of the fringes observed when input beam is circularly polarized, discussed earlier. Experimentally, in a full $2\pi$ rotation of half wave plate, fringe pattern disappears eight times. This observation can be explained by the above expression. Here superposition of two fringe patterns moving in opposite directions takes place resulting in the variation in the intensity of the fringes. There is no difference with the change in the azimuth of the linear polarization of the input beam. From fig.2.8 it is observed that the intensity of the peaks start reducing as the half wave plate is rotated while the intensity of the minimum intensity points start increasing. At a point, ($\varphi = 22.5$), the intensities of the peaks and the minimum intensity points become equal resulting in the disappearance of the fringes. As the half wave plate is further rotated, the intensities of the earlier minimum points keep on increasing while the intensities of the earlier peaks keep on decreasing. As a result there is a swapping of the peaks with minima, after the disappearance of the fringe pattern. After reaching the maximum intensity point at $\varphi = 45$, the intensity of the new peaks start reducing and that of the new minimum points start increasing leading to another disappearance of the fringes and swapping of the peaks. This happens a total of eight time in a $2\pi$ rotation of the half wave plate and nowhere the total intensity becomes zero. In fig.2.9 experimental fringe pattern is shown for $\varphi = 0$, 22.5, 45, 67.5, 90 etc. These patterns agree with the theoretical fringe patterns. Observe the swapping of the peak after the zero contrast patterns.
Replacing half wave plate with a quarter wave plate, it is observed that the fringe pattern when \( b = 0 \) follows the expression given by

\[
I_{\text{qwp}(\mu=0)} = \frac{(a^2)}{8}[4 - 2 \cos(\Delta) - \cos(\Delta - 4\varphi) - \cos(\Delta + 4\varphi)]
\]

(2.6)

Where \( A \) is the phase due to optical path difference and \( \varphi \) is the angle made by the fast axis of the quarter wave plate with the x-axis. As in the case with circularly polarized input beam, here also the stationary fringe pattern because of the first term (eq.2.6) masks the full effect of the remaining two terms. This causes fringe pattern to disappear four times within \( 2\pi \) rotation of quarter wave plate. The intensity fringe pattern is shown in fig.2.10. Experimental fringe patterns are shown in fig.2.11 for \( \varphi = 0^\circ, 45^\circ, 90^\circ, 135^\circ \) etc. In fig.2.12 a surface graph of the contrast function \( C = 1 - V(\varphi, \delta) \) for linearly polarized incident beam is shown, where \( V(\varphi, \delta) \) is the visibility function given by the eq.2.7 for the fringes around \( A = 0 \). From this graph it is clear that all the wave plates with retardation strength \( 0 < \delta \leq \pi/2 \) and \( 3\pi/2 \leq \delta \leq 2\pi \) show poor visibility four times for a \( 2\pi \) rotation of the wave plate. The transition from four to eight poor visibility positions occurs at \( \delta = \pi/2 \) and changes back to four poor visibility positions at \( \delta = 3\pi/2 \). The poor contrast positions, for \( \pi/2 < \delta < 3\pi/2 \), are not equally placed on the \( \varphi \)-axis except for \( \delta = \pi \) i.e., for half wave plate.

\[
V(\varphi, \delta) = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}
\]

(2.7)

When a linear polarizer is placed inside the Sagnac interferometer, it is observed that the intensity of the fringe pattern is not similar to the fringe pattern with a quarter wave plate, as in the case with circularly polarized input beam, where both with quarter wave plate and linear polarizer the intensity pattern remains similar. The intensity pattern in this case is given by

\[
I_{\text{lp}(\mu=0)} = \frac{1}{4} [(a^2 \cos^4(\varphi) + b^2 \sin^4(\varphi) + (a^2 + b^2) \cos^2(\varphi) \sin^2(\varphi)) - \cos(\Delta) (a^2 \cos^4(\varphi) + b^2 \sin^4(\varphi) + (a^2 + b^2) \cos^2(\varphi) \sin^2(\varphi))] 
\]

(2.8)
Figure 2.13. shows the intensity pattern for different positions of $\varphi$ for x and y polarization states of the input beam. It is observed that here also variation of intensity take place as the linear polarizer is rotated. The fringe pattern disappears four times and the total intensity goes to zero twice within a $2\pi$ rotation of the linear polarizer. It is also found that the peak position swaps as in the case of half wave plate but instead of increase in intensity of the minimum position, the intensities of both the peaks and minimum intensity points start decreasing after they become equal. The decrease in the intensity is more compared to that of the minimum intensity points and this leads to the swapping of peak position. The intensities keep reducing until the total intensity becomes zero.

**II. 3-arm Sagnac Interferometer with non-polarizing beam splitter.**

A 3-arm Sagnac interferometer consists of a beam splitter and two mirrors as shown in fig.2.14. In this case beam splitter is 50:50 non-polarizing and mirrors are 100% reflecting. We can attach a right-handed Cartesian coordinate system to the beams travelling in the interferometer. If we do the polarization analysis for this interferometer one will find that the polarization states of the output beams will be same as that of the input when there is no element placed inside the interferometer. If any element like a wave retarder or a polarizer is placed then also the output beams will have same states of polarization but not same as the input. As a result, there will be no change in the intensity pattern if one rotates the optical element, which will be clear from the analysis given below. The output intensity pattern for an arbitrarily polarized input beam and a non-reflecting optical element is given by

\[
I_0 = \left| E^x_0 \right|^2 + \left| E^y_0 \right|^2
\]

\[
E^x_0 = [\alpha e_{11} - \alpha e_{21} e^{i\Delta}] E^x_i + [\alpha e_{12} + \alpha e_{22} e^{i\Delta}] E^y_i
\]

\[
E^y_0 = [\alpha e_{13} + \alpha e_{23} e^{i\Delta}] E^x_i + [\alpha e_{14} - \alpha e_{24} e^{i\Delta}] E^y_i
\]

\[
E^x_i = a
\]

\[
E^y_i = b e^{i\mu}
\]

(2.9)
All the variables and constants carry the same meaning as in eq. 2.1.

When a wave retarder is placed inside the three-arm Sagnac interferometer the fringe pattern is given by the following expression.

$$I_o = \frac{(a^2 + b^2)}{2} [1 - \cos(\Delta)]$$

(2.10)

It is obvious from this expression that there will not be any variation in the fringe pattern with the rotation of the retarder for any polarization state of the input beam. Similarly when a linear polarizer is placed in one of the arms of three-arm Sagnac interferometer, no change in the fringe pattern is observed. In both these cases the output beams have same polarization state.

Consider the case of an optically active medium placed in one of the arms of Sagnac interferometer. In the four-arm Sagnac interferometer with an optically active medium, it is found that there will not be any change in the fringe pattern. Here, in the three-arm case, the fringe pattern is given by the following expression when the input is linearly polarized.

$$I_{oam} = \frac{(a^2 + b^2)}{4} \left[2 - \cos(\Delta - 2\varphi) - \cos(\Delta + 2\varphi)\right]$$

(2.11)

This expression is similar to that of half wave plate in four-arm Sagnac interferometer with linearly polarized input. This changes the intensity level only but will not shift the fringe pattern. Pancharatnam's phase doesn't come into picture, as the trajectories do not enclose a surface between them as they lie on the same circle, which will be parallel to equator of the Poincare sphere (fig. 2.15). If P represents the input state of polarization then P₁ and P₂ represent the two output polarization states. P₁ and P₂ are separated by an angle 2\(\varphi\) if \(\varphi\) is the amount by which the optically active medium rotates the input polarization states. As the optically active medium only rotates the plane of polarization it doesn't change the ellipticity of the input polarization. Therefore, states of polarization of the output beams will also have the same ellipticity and
hence will lie on a circle parallel to the equator if input arbitrarily polarized. If input is linearly polarized then that circle will be equator itself.

III. Three and four-arm Sagnac interferometers with polarizing beam splitters

Till now the interferometers discussed have a non-polarizing beam splitter. Now the three and four-arm Sagnac interferometer configurations will be studied with a polarizing beam splitter. The intensity fringe pattern is found to be same for both three and four-arm configurations. The fringe patterns for different cases of optical elements are given by the following expressions.

\[ I_{\text{hwp}} = (a^2 + b^2) \cos^2 2\phi \]
\[ I_{\text{qwp}} = (a^2 + b^2) (\cos^4 2\phi + \sin^2 2\phi) \]
\[ I_{\text{lp}} = (a^2 \cos^4 2\phi + b^2 \sin^4 2\phi) \]
\[ I_{\text{oam}} = (a^2 + b^2) \cos^2 \theta \]

\( I_{\text{hwp}}, I_{\text{qwp}}, I_{\text{lp}} \) and \( I_{\text{oam}} \) are the the intensity patterns for half wave plate, quarter wave plate, linear polarizer and an optically active medium as optical elements inside the three and four-arm configurations. \( \phi \) is the angle which the axes of half wave plate, quarter wave plate and the linear polarizer make with the x-axis. Whereas \( \theta \) is the angle of optical rotation of the optically active medium. It is clear from the above expressions (note the absence of A) that one will not have any fringe pattern. There will be change only in the intensity.

We have recorded in this chapter various kinds of variations and changes in the fringe pattern as a result of optical elements placed in a Sagnac interferometer. These changes in the fringe pattern can be used as signals in devices and applications. Also note some of these configurations like a wave plate or polarizer in four-arm configurations with linearly polarized input can show nonlinear behaviour of Pancharatnam's phase. In the next chapter, we study one such device, using this nonlinear behavior of Pancharatnam's phase, called an interferometric switch.
Chapter II. Sagnac Interferometer

References:


2.03 "A simple white-light interferometer operating on the Pancharatnam phase", P. Hariharan and D. N. Rao, Curr. Sci., 65, 483, 1993


2.05 "Geometric phase interferometers Possible optical configurations" P. Hariharan, J. Mod. Opt., 40, 985 (1993)


**Fig. 2.1.** This figure shows the setup of a four-arm Sagnac interferometer. It consists of a 50:50 nonpolarizing beam splitter BS, three 100% reflecting mirrors $M_1$, $M_2$ and $M_3$. A right handed cartesian coordinate system is attached to the beam in all the arms. Observe the change in the vector $E$ after each reflection. It changes sign along the $y$-axis. The output vector is not parallel to the input vector. Both the beams have parallel $\mathbf{E}$-vector. The faces 1, 2, 3 and 4 of the beam splitter are shown by the numbers 1, 2, 3 and 4. $B_1$ and $B_2$ are the two counter propagating beams.
Fig. 2.2: Same as fig. 2.1 with an optical element and with coordinate frames shown in one arm only. OE is the optical element with its axis making an angle, $\varphi$ with the x-axis of the coordinate frames of the counter propagating beams. Observe that the axis of the OE will be at an angle $\varphi$ for the beam 1 (BO and for the beam 2 (B$_2$) it will be at an angle $-\varphi$. As a result the output beams will have their vectors at an angle to each other. This angle will be equal to $4\varphi$ when input beam is linearly polarized along the x-axis.
Fig 2.3a and b: Shown is the fringe pattern (eq. 2.2) for circularly polarized input light and HWP inside 4-arm Sagnac interferometer. It is clear that as HWP is rotated ($\varphi$ is changed) the fringe pattern moves linearly and the direction of movement (along A-axis) depends on the input polarization as well as the sense of rotation of HWP. 

a) Input is right circularly polarized and rotation of HWP is **anti-clockwise**. 
b) Input is left circularly polarized and rotation of HWP is **anti-clockwise**.
Fig. 2.4a and b: Poincare sphere representation of the changes in the polarization states is shown in these figures. The point 'X' represents the linear polarization parallel to x-axis and the point 'Y' represents the linear polarization parallel to y-axis. North Pole represents the right circular polarization while South Pole represents the left circular polarization.

a) In this figure we show the case when the fast axis of the half wave plate is parallel to the x-axis. Observe that both the beams move along the same trajectory N(-90°)S.

b) In this figure we show the case when the fast axis of the half wave plate is an angle $\phi$ with the x-axis. Observe the trajectories taken by the two beams in this case. In the earlier case the area enclosed is zero. But here it is not equal to zero. The solid angle subtended by the area enclosed is equal to the $8\phi$. 
Fig. 2.5: Experimental fringes recorded using a CCD camera, for the case when a half wave plate is placed inside a four-arm Sagnac interferometer and input is circularly polarized, are shown in this figure. Fringe patterns for different angles of half wave plate are shown as the half wave plate is rotated. a) The half wave plate angle is $\phi = 0^\circ$, b) $\phi = 45^\circ$, c) $\phi = 90^\circ$, d) $\phi = 135^\circ$, e) $\phi = 180^\circ$, f) $\phi = 225^\circ$, g) $\phi = 270^\circ$, h) $\phi = 315^\circ$ and i) $\phi = 360^\circ$. Observe the shift in the fringes along the A-axis as the half wave plate is rotated. Though fringe patterns for intermediate angles are not shown it is obvious that the movement is linear along the A-axis with the rotation of half wave plate angle ($\phi$).
**Figure 2.6a and b:** Fringe pattern for circularly polarized input light and QWP inside the 4-arm Sagnac interferometer. In this case the fringes move and there is change in the visibility (eq.2.7) of fringes as the quarter wave plate is rotated ($\varphi$ changes). $\varphi$ is given different values for different curves. Fringes disappear at $\varphi = 45^\circ + n\pi$ ($n=0,1,2,...$) when the fringe visibility becomes zero (i.e., intensity = constant for all values of $\Delta$)
Fig. 2.7: In this figure we show the Poincare sphere representation of the trajectories taken by the counter propagating beams when a quarter wave plate is placed in a four-arm Sagnac interferometer. If North Pole represents the input polarization then one beam moves along the longitude NP₁ while the other beam take the path along the longitude NP₂. P₁ will be at a point which is -90° - 2φ away from the x-axis and P₂ is at a point -90° + 2φ away from x-axis where φ is the angle made by the fast axis of the quarter wave plate with the x-axis.
Fig. 2.8: Intensity fringe pattern for the case of half wave plate in a four-arm Sagnac interferometer with input light linearly polarized. Observe the variation in intensity for different values of $\varphi$. The fringes disappear at $\varphi = 22.5^\circ$. After this disappearance of fringes the peak position swaps with the minimum intensity position.
Fig. 2.9: In this figure we show the fringe pattern for different angles of half wave plate placed inside a four-arm Sagnac interferometer when the input beam is linearly polarized parallel to x-axis. Observe the zero contrast positions at $\varphi = 22.5^\circ$, $67.5^\circ$, etc. This takes place for every $45^\circ$ and occurs eight times in full $2\pi$ rotation of the half wave plate. a) ($\varphi = 0^\circ$), b) $\varphi = 22.5^\circ$, c) $\varphi = 45^\circ$, d) ($\varphi = 67.5^\circ$), e) $\varphi = 90^\circ$, f) ($\varphi = 112.5^\circ$), g) ($\varphi = 135^\circ$), h) $\varphi = 157.5^\circ$ and i) $\varphi = 180^\circ$. Observe the swapping of the peak after every zero contrast position.
Fig: 2.10a and b: Intensity fringe pattern for a quarter wave plate placed in a four-arm Sagnac interferometer with input light linearly polarized, parallel to x-axis. Observe that the intensity of the fringes vary as $\phi$ is varied. Fringe pattern disappears once in a 90° rotation of quarter wave plate. Note there is no swapping of peaks with minimum intensity points as in the case of half wave plate.
Fig. 2.11: In this figure we show the experimental fringe pattern, recorded using a CCD camera, when a quarter wave plate is placed inside a four-arm Sagnac interferometer and input beam is linearly polarized along x-axis. Fringe patterns for different angles ($\Phi$) of the quarter wave plate are shown. a) $\Phi = 0$, b) $\Phi = 45$, c) ($\Phi = 90^\circ$, d) $\Phi = 135^\circ$, e) $\Phi = 180^\circ$, f) ($\Phi = 225^\circ$, g) $\Phi = 270^\circ$, h) $\Phi = 315^\circ$ and i) $\Phi = 360^\circ$. Observe the disappearance of the fringes for $\Phi = 45$, 135, 225 and 315. There is no fringe movement observed here in this case. Only intensity variations take place.
Fig. 2.12: In this figure we show the surface plot of the contrast function defined as $C(\delta, \phi) = 1 - V(\delta, \phi)$, where $V$ is the visibility function around $\delta = 0$ point. $\phi$ is the angle made by the fast axis of the retarder with x-axis. Here z-axis is not shown but the height at a point on the graph gives the contrast at that point. Observe that the number of contrast positions when $\delta = \pi/2$ are four. For $\delta = \%$ there are eight place where contrast goes to one within a $2\pi$ rotation of the retarder.
Fig. 2.13: Intensity fringe pattern when a linearly polarizer is placed in a four-arm Sagnac interferometer with input light linearly polarized. Here the intensity varies as $\varphi$ is varied and fringe pattern disappears once within a 90° rotation of the polarizer.

a) Input polarization is parallel to x-axis. Total intensity becomes zero at $\varphi = 90°$.

b) Input polarization is perpendicular to x-axis (parallel to y-axis). Total intensity becomes zero at $\varphi = 0°$. 
Fig. 2.14: This figure shows a three-arm Sagnac interferometer. An optical element is placed in one of the arms of the interferometer. One can attach a right handed cartesian coordinate system to the beams here also. The two beams emerge out with same polarization as the input if no optical element is placed inside the interferometer. If any optical element like a wave retarder is placed then the two beams have same polarization outside but not same as the input.
Fig. 2.15: This figure shows the output states of polarization of the counter propagating beams in a three-arm Sagnac interferometer. If $P$ represents the input state of polarization then $P_1$ and $P_2$ represent the two output polarization states. $P_1$ and $P_2$ are separated by an angle $2\phi$ if $\phi$ is the amount by which the optically active medium rotates the input polarization states. As the optically active medium only rotates the plane of polarization it doesn’t change the ellipticity of the input polarization. Therefore states of polarization of the output beams will also have the same ellipticity and hence will lie on a circle parallel to the equator if the input is arbitrarily polarized. If input is linearly polarized then that circle will be equator itself.