Chapter 5

Magneto-thermoelastic Response in a Perfectly Conducting Medium with Three-phase-lag Effect

5.1 Introduction

The generalized thermoelasticity theory with dual-phase-lag effect has been developed by Tzou [221] and Chandrasekhariah [44]. Tzou [221] introduced two-phase-lags to both the heat flux vector and temperature gradient. The next generalization is known as three-phase-lag thermoelasticity which is due to Roychoudhari [184].

The purpose of the present work is to study magneto-thermoelastic interaction due to the presence of periodically varying heat sources in a perfectly conducting medium in the context of linear theory of generalized thermoelasticity (GNII, GNIII and 3P models). The governing equations are expressed in Laplace-Fourier transform domain. The solution for displacement, temperature, stress and strain in the Laplace transform domain are obtained by taking the Fourier inversion which is carried out by using residual calculus, where the poles of the integrand are obtained numerically in the complex domain by using Laguerre’s method. The inversion of the Laplace transform is computed

Magneto-thermoelastic Response in a Perfectly Conducting Medium with Three-phase-lag Effect 78 numerically by using a method based on the Fourier series expansion technique [82]. The results obtained theoretically have been computed numerically and are presented graphically to show the comparison of results of the above theories and also the effect of the magnetic field and damping coefficient on the physical quantities.

5.2 Basic Equations

For a perfectly conducting medium, the constitutive equations are

\[ \sigma_{ij} = 2\mu e_{ij} + [\lambda \Delta - \gamma(T - T_0)] \delta_{ij}, \]  

where

\[ e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \Delta = e_{ii}. \]

The stress equations of motion in the presence of body forces \( F_i \) are

\[ \sigma_{ij,j} + F_i = \rho \ddot{u}_i. \]

The heat equation corresponding to generalized thermoelasticity for the three-phase-lag model with energy dissipation in the presence of a heat source is

\[ \rho c_v(\dot{T} + \tau_q \dddot{T} + \frac{1}{2} \tau_q \dot{T}^2) + \gamma T_0(\dot{\Delta} + \tau_q \dddot{\Delta} + \frac{1}{2} \tau_q \dot{\Delta}^2) = \]

\[ K(\nabla^2 \dot{T} + \tau_T \nabla^2 \dddot{T}) + K^* (\nabla^2 T + \tau_v \nabla^2 \dot{T}) + \rho \ddot{Q}, \]

where \( \gamma = (3\lambda + 2\mu)\alpha_1, \) \( K \) is the thermal conductivity, \( K^* \) the material constant, \( \tau_T \) the delay time caused by the microstructural interactions and is called the phase lag of the temperature gradient, \( \tau_q \) the delay time due to the fast transient effects of thermal inertia and is called the phase-lag of the heat flux and \( \tau_v \) the phase-lag for thermal displacement gradient.
5.3 Formulation of the Problem

We now consider an unbounded, perfectly conducting thermoelastic medium at a uniform reference temperature $T_0$ in the presence of periodically varying heat sources distributed over a plane area. We shall consider an one-dimensional disturbance of the medium, so that the displacement vector $u$ and temperature field $T$ can be expressed in the following form:

$$u = (u(x,t), 0, 0), \quad (5.3.1)$$

$$T = T(x,t). \quad (5.3.2)$$

The electromagnetic field is governed by Maxwell's equations (in the absence of the displacement current and charge density) as

$$\text{curl } H = J, \quad (5.3.3)$$

$$\text{curl } E = -\frac{\partial B}{\partial t}, \quad (5.3.4)$$

$$\text{div } B = 0, \quad (5.3.5)$$

$$B = \mu_0 H. \quad (5.3.6)$$

The generalized Ohm's law in the deformable continua is

$$J = \kappa (E + \mathbf{u} \times B), \quad (5.3.7)$$

where the small effect of a temperature gradient on the conduction current $J$ is neglected.

In the context of the linear theory of generalized thermoelasticity based on the three-phase-lag model, the equation of motion, heat equation and constitutive equations can be written as

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} + F_x = \rho \frac{\partial^2 u}{\partial t^2} \quad (5.3.8)$$
where

\[ F = (J \times B), \quad F = (F_x, F_y, F_z) \]

\[
\begin{align*}
K \left( \frac{\partial^2 T}{\partial x^2} + \tau_f \frac{\partial^2 T}{\partial x^2 \partial t^2} + \frac{1}{2} \tau_q \frac{\partial^4 T}{\partial x^4} \right) + K' \left( \frac{\partial^2 T}{\partial x^2} + \tau_f \frac{\partial^2 T}{\partial x^2 \partial t^2} + \frac{1}{2} \tau_q \frac{\partial^4 T}{\partial x^4} \right) + \rho \dot{Q} = \\
\rho c_v \left( \frac{\partial^2 T}{\partial t^2} + \tau_f \frac{\partial^2 T}{\partial t^2} + \frac{1}{2} \tau_q \frac{\partial^4 T}{\partial t^4} \right) + \gamma T_0 \left( \frac{\partial^2 u}{\partial t^2} + \tau_f \frac{\partial^2 u}{\partial t^2} + \frac{1}{2} \tau_q \frac{\partial^4 u}{\partial t^4} \right)
\end{align*}
\]  \hspace{1cm} (5.3.9)

\[ \sigma_{xx} = (\lambda + 2\mu)e_{xx} - \gamma (T - T_0) \]  \hspace{1cm} (5.3.10)

where

\[ e_{xx} = \frac{\partial u}{\partial x} \]  \hspace{1cm} (5.3.11)

We set \( H = H_0 + h \), where \( H_0 = (0, 0, H_0) \). The perturbed magnetic field \( h \) is so small that the product of \( h \) and \( u \) and their derivatives can be neglected for linearization of the field equations.

We assume that all the vector and scalar functions depend only on the spatial coordinate \( x \) and time \( t \) and are independent of the \( y \) and \( z \) coordinates.

Eq. (5.3.3) gives

\[ J_x = 0, \quad J_y = -\frac{\partial H_z}{\partial x}, \quad J_z = \frac{\partial H_x}{\partial x}, \]  \hspace{1cm} (5.3.12)

where \( J = (J_x, J_y, J_z), H = (H_x, H_y, H_z) \).

Eqs. (5.3.4) and (5.3.6) yield

\[ \frac{\partial H_x}{\partial t} = 0, \quad \frac{\partial E_z}{\partial x} = \mu_e \frac{\partial H_y}{\partial t}, \quad \frac{\partial H_y}{\partial x} = -\mu_e \frac{\partial E_z}{\partial t}, \quad E = (E_x, E_y, E_z). \]  \hspace{1cm} (5.3.13)

Eq. (5.3.5) gives \( \frac{\partial h_z}{\partial x} = 0 \), which implies that \( h_z = 0 \), since initially no perturbed field was applied along the \( x \)-axis.
The modified Ohm’s law gives

\[ J_x = \kappa E_x, \quad J_y = \kappa [E_y - \mu \epsilon H_z \frac{\partial u}{\partial t}], \quad J_z = \kappa [E_z + \mu \epsilon H_y \frac{\partial u}{\partial t}] \quad (5.3.14) \]

Now \( J_z = 0 \) implies \( E_z = 0 \).

By eliminating \( J_x, J_y, J_z \) and using Eqs. (5.3.12), (5.3.13), and (5.3.14), we get

\[ \begin{align*}
\frac{\partial H_z}{\partial t} &= \frac{\nu_H}{\rho_p} \frac{\partial^2 H_z}{\partial x^2} - \frac{\partial}{\partial x} \left( H_z \frac{\partial u}{\partial t} \right), \quad (5.3.15) \\
\frac{\partial H_y}{\partial t} &= \frac{\nu_H}{\rho_p} \frac{\partial^2 H_y}{\partial x^2} - \frac{\partial}{\partial x} \left( H_y \frac{\partial u}{\partial t} \right), \quad (5.3.16)
\end{align*} \]

where \( \nu_H = (\kappa \mu_e)^{-1} \) is called the magnetic viscosity.

Eq. (5.3.8) reduces to

\[ (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} - \frac{1}{2} \mu_e (H_y^2 + H_z^2) = \frac{\partial^2 u}{\partial t^2}, \quad (5.3.17) \]

We set \( H_z = H_0 + h_z \), where the perturbed magnetic field \( h_z \) is small compared to the strong initial magnetic field \( H_0 \). Then from Eqs. (5.3.15), (5.3.16) and (5.3.17) after linearization, we get

\[ \begin{align*}
\frac{\partial h_z}{\partial t} &= \frac{\nu_H}{\rho_p} \frac{\partial^2 h_z}{\partial x^2} - H_0 \frac{\partial^2 u}{\partial x \partial t}, \quad (5.3.18) \\
\frac{\partial h_y}{\partial t} &= \frac{\nu_H}{\rho_p} \frac{\partial^2 h_y}{\partial x^2}, \quad (5.3.19)
\end{align*} \]

and

\[ (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} - \mu_e H_0 \frac{\partial h_z}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \quad (5.3.20) \]

Now for a perfect electrical conductor, \( \nu_H \to 0 \) as \( \kappa \to \infty \). Eq. (5.3.18) leads to \( h_z = -H_0 \frac{\partial u}{\partial x} \), since there is no perturbation at \( \infty \). Then Eq. (5.3.20) reduces to

\[ c_1^2 (1 + R_H) - \frac{\gamma}{\rho} \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2} \quad (5.3.21) \]
Magneto-thermoelastic Response in a Perfectly Conducting Medium with Three-phase-lag Effect

where \( R_H = \frac{\mu_e H_0^2}{\rho c_1^2} \) and \( v_A = \frac{x}{c_1} \), \( c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}} \), and \( v_A = \sqrt{\frac{\mu_e}{\rho}} H_0 \) is the Alfven wave velocity of the medium. The coefficient \( R_H \) represents the effect of an external magnetic field in the thermoelastic processes proceeding in the body.

We introduce the following dimensionless quantities:

\[
x' = \frac{x}{l}, \quad u' = \frac{\lambda + 2\mu}{\gamma T_0 l} u, \quad t' = \frac{c_1 t}{l}, \quad T' = \frac{T - T_0}{T_0}, \quad \sigma'_{xx} = \frac{\sigma_{xx}}{T_0}, \quad e'_{xx} = e_{xx},
\]

\[1 + R_H = R^2_M, \quad \tau_T' = \frac{c_1 \tau_T}{l}, \quad \tau_T' = \frac{c_1 ^2 \tau_T}{l}, \quad \tau_T' = \frac{c_1 \tau_T}{l},\]

where \( l = \) some standard length and \( c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}} \) is the standard speed, and omitting primes, Eqs. (5.3.21), (5.3.9), (5.3.10) and (5.3.11) can be re-written in dimensionless form as

\[
R^2_M \frac{\partial^2 u}{\partial x^2} - \frac{\partial T}{\partial t} = \frac{\partial^2 u}{\partial t^2}
\]

\[
\left(1 + \tau_T \frac{\partial}{\partial t}\right) \left[ c_T^2 \frac{\partial^2 T}{\partial x^2} \right] + \left(1 + \tau_T \frac{\partial}{\partial t}\right) \left[ \kappa_0 \frac{\partial^2}{\partial x^2} \left( \frac{\partial T}{\partial t} \right) \right] + Q_0 = \left(1 + \tau_T \frac{\partial}{\partial t} + \frac{1}{2} \tau_T \frac{\partial^2}{\partial t^2} \right) \left[ \frac{\partial^2 T}{\partial t^2} + c_T \frac{\partial^2}{\partial t^2} \left( \frac{\partial u}{\partial t} \right) \right]
\]

\[
\sigma_{xx} = \frac{\partial u}{\partial x} - T
\]

\[
e_{xx} = \frac{\gamma T_0}{\lambda + 2\mu} \frac{\partial u}{\partial x}
\]

where

\[
c_T^2 = \frac{K^*}{\rho c_v c_1^2}, \quad \epsilon_T = \frac{\gamma^2 T_0}{(\lambda + 2\mu)\rho c_v}, \quad \kappa_0 = \frac{K}{\rho c_v c_1^2}, \quad Q_0 = \frac{l}{T_0 c_v c_1} \frac{\partial Q}{\partial t}.
\]

We assume that the medium is initially at rest. The undisturbed state is maintained at a reference temperature. Then we have

\[
u(x, 0) = \dot{u}(x, 0) = T(x, 0) = \dot{T}(x, 0) = 0.
\]
Varying Heat source

Let us assume that the heat source is distributed over the plane \( x = 0 \) in the following form:

\[
Q_0 = Q_0^* \delta(x) \sin \frac{\pi t}{\tau}, \quad 0 \leq t \leq \tau
\]
\[
= 0, \quad t > \tau
\]  

(5.3.27)

5.4 Method of Solution

Let us define the Laplace-Fourier double transform of the function \( g(x,t) \) by

\[
\tilde{g}(x,p) = \int_0^\infty g(x,t)e^{-pt}dt, \quad \text{Re}(p) > 0
\]
\[
\tilde{g}(\alpha, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty \tilde{g}(x,p)e^{i\alpha x}dx.
\]

Applying the Laplace-Fourier double integral transform to the Eqs. (5.3.22)-(5.3.25), we obtain:

\[
(R_M^2 \alpha^2 + p^2)\hat{u}(\alpha, p) = i\alpha\hat{T}(\alpha, p),
\]  

(5.4.1)

\[
[(1 + \tau_p)p + (1 + \tau_p)\rho \kappa_0] \alpha^2 + (1 + p\tau_q + \frac{1}{2} \tau_q \beta^2) \hat{T}(\alpha, p)
\]
\[
= i\alpha \rho \beta (1 + p\tau_q + \frac{1}{2} \tau_q \beta^2) \hat{u}(\alpha, p) + \hat{Q}_0,
\]  

(5.4.2)

\[
\hat{\delta}_{xx}(\alpha, p) = -i\alpha \hat{u}(\alpha, p) - \hat{T}(\alpha, p),
\]  

(5.4.3)

\[
\hat{e}_{xx}(\alpha, p) = -i\alpha \beta_1 \hat{u}(\alpha, p).
\]  

(5.4.4)

Solving Eqs. (5.4.1) and (5.4.2) for \( \hat{u}(\alpha, p) \) and \( \hat{T}(\alpha, p) \), we get

\[
\hat{u}(\alpha, p) = \frac{i\alpha \hat{Q}_0}{M(\alpha)},
\]  

(5.4.5)

\[
\hat{T}(\alpha, p) = \frac{(R_M^2 \alpha^2 + p^2) \hat{Q}_0}{M(\alpha)},
\]  

(5.4.6)
where

\[ M(\alpha) = \left\{(1 + \tau_p p)\phi^2 + (1 + \tau_T p)\kappa_0 R_{st}^2 \alpha^4 + \alpha^2 \left\{ p(1 + \tau_T p)\kappa_0 + (1 + \tau_p p)\phi^2 \right\} + (1 + \tau_T p + \frac{1}{2}\tau_T^2 p^2) \right\} \]

\[ + R_{st}^2 \left\{ (1 + \tau_T p) + p(1 + \tau_T p)\kappa_0 \right\} (\alpha - \alpha_1)(\alpha - \alpha_2)(\alpha - \alpha_3)(\alpha - \alpha_4). \]

(5.4.7)

Now the expressions for the stress and strain in the Laplace-Fourier transform domain can be obtained from Eqs. (5.4.3) and (5.4.4), using Eqs. (5.4.5) and (5.4.6)

\[ \tilde{\sigma}_{xx}(\alpha, p) = \frac{\alpha^2 \hat{Q}_0}{M(\alpha)} - \frac{(R_{st}^2 \alpha^2 + p^2)\hat{Q}_0}{M(\alpha)} \]

(5.4.8)

\[ \tilde{\sigma}_{xx}(\alpha, p) = \frac{\alpha^2 \beta_1 \hat{Q}_0}{M(\alpha)}. \]

(5.4.9)

Thus, the solution for the displacement, temperature, stress, and strain in the Laplace transform domain can be obtained in terms of the following four integrals:

\[ u(x, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{i\alpha \hat{Q}_0}{M(\alpha)} e^{-i\alpha x} d\alpha, \]

(5.4.10)

\[ T(x, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{(R_{st}^2 \alpha^2 + p^2)\hat{Q}_0}{M(\alpha)} e^{-i\alpha x} d\alpha, \]

(5.4.11)

\[ \tilde{\sigma}_{xx}(x, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\alpha^2 (1 - R_{st}^2) - p^2}{M(\alpha)} \hat{Q}_0 e^{-i\alpha x} d\alpha, \]

(5.4.12)

\[ \tilde{\epsilon}_{xx}(x, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\alpha^2 \beta_1 \hat{Q}_0}{M(\alpha)} e^{-i\alpha x} d\alpha. \]

(5.4.13)

where

\[ \hat{Q}_0 = \frac{2\pi \tau (1 + e^{-\tau_T})}{\sqrt{2\pi (\pi^2 + p^2 \tau_T^2)}}. \]

(5.4.14)
Thus, the expressions for the displacement, temperature, stress, and strain in the Laplace transform domain take the following form:

\[
\tilde{u}(x, p) = \int_{-\infty}^{\infty} \frac{\text{i}\alpha Q_0 \tau (1 + e^{-\text{pt}})}{2(\pi^2 + p^2\tau^2) M(\alpha)} e^{-\text{i}\alpha x} \, d\alpha ,
\quad (5.4.15)
\]

\[
\bar{T}(x, p) = \int_{-\infty}^{\infty} \frac{(R_M^2 \alpha^2 + p^2) Q_0 \tau (1 + e^{-\text{pt}})}{2(\pi^2 + p^2\tau^2) M(\alpha)} e^{-\text{i}\alpha x} \, d\alpha ,
\quad (5.4.16)
\]

\[
\bar{\sigma}_{zz}(x, p) = \int_{-\infty}^{\infty} \frac{[\alpha^2(1 - R_M^2) - p^2] Q_0 \tau (1 + e^{-\text{pt}})}{2(\pi^2 + p^2\tau^2) M(\alpha)} e^{-\text{i}\alpha x} \, d\alpha ,
\quad (5.4.17)
\]

\[
\bar{\varepsilon}_{zz}(x, p) = \int_{-\infty}^{\infty} \frac{\alpha^2 \beta \lambda Q_0 \tau (1 + e^{-\text{pt}})}{2(\pi^2 + p^2\tau^2) M(\alpha)} e^{-\text{i}\alpha x} \, d\alpha .
\quad (5.4.18)
\]

Applying contour integration to the Eqs. (5.4.15)-(5.4.18) we obtain

\[
\tilde{u}(x, p) = -\frac{\text{i}Q_0 \tau (1 + e^{-\text{pt}})}{R_M^2 (\pi^2 + p^2\tau^2) N(p)} \sum_{j=1}^{4} \frac{\text{i}\alpha_j A_j e^{-\text{i}\alpha_j x}}{\text{Im}(\alpha_j)} \quad \text{for } x > 0
\]

\[
\tilde{T}(x, p) = -\frac{\text{i}Q_0 \tau (1 + e^{-\text{pt}})}{R_M^2 (\pi^2 + p^2\tau^2) N(p)} \sum_{j=1}^{4} \frac{\text{i}\alpha_j A_j e^{-\text{i}\alpha_j x}}{\text{Im}(\alpha_j) < 0} \quad \text{for } x < 0
\quad (5.4.19)
\]

\[
\bar{\sigma}_{zz}(x, p) = \frac{(R_M^2 \alpha^2 + p^2) \beta \lambda Q_0 \tau (1 + e^{-\text{pt}})}{R_M^2 (\pi^2 + p^2\tau^2) N(p)} \sum_{j=1}^{4} \frac{\text{i}\alpha_j A_j e^{-\text{i}\alpha_j x}}{\text{Im}(\alpha_j) < 0} \quad \text{for } x > 0
\]

\[
\bar{\varepsilon}_{zz}(x, p) = \frac{\alpha^2 \beta \lambda Q_0 \tau (1 + e^{-\text{pt}})}{R_M^2 (\pi^2 + p^2\tau^2) N(p)} \sum_{j=1}^{4} \frac{(R_M^2 \alpha_j^2 + p^2) A_j e^{-\text{i}\alpha_j x}}{\text{Im}(\alpha_j) > 0} \quad \text{for } x < 0
\quad (5.4.20)
\]
\[ \bar{d}_{zz}(x, p) = -\frac{i Q_0^2 \pi \tau (1 + e^{-p r})}{R_M^2 \left( \pi^2 + p^2 \tau^2 \right) N(p)} \sum_{j=1}^{4} \left[ \frac{\alpha_j^2 \left( 1 - R_M^2 \right) - p^2}{\text{Im} (\alpha_j) < 0} \right] A_j e^{-i \alpha_j x} \quad \text{for } x > 0 \]

\[ = \frac{i Q_0^2 \pi \tau (1 + e^{-p r})}{R_M^2 \left( \pi^2 + p^2 \tau^2 \right) N(p)} \sum_{j=1}^{4} \left[ \frac{\alpha_j^2 \left( 1 - R_M^2 \right) - p^2}{\text{Im} (\alpha_j) > 0} \right] A_j e^{-i \alpha_j x} \quad \text{for } x < 0 \quad (5.4.21) \]

\[ \bar{e}_{xx}(x, p) = -\frac{i \beta_1 Q_0^2 \pi \tau (1 + e^{-p r})}{R_M^2 \left( \pi^2 + p^2 \tau^2 \right) N(p)} \sum_{j=1}^{4} \frac{\alpha_j^2 A_j e^{-i \alpha_j x}}{\text{Im} (\alpha_j) < 0} \quad \text{for } x > 0 \]

\[ = \frac{i \beta_1 Q_0^2 \pi \tau (1 + e^{-p r})}{R_M^2 \left( \pi^2 + p^2 \tau^2 \right) N(p)} \sum_{j=1}^{4} \frac{\alpha_j^2 A_j e^{-i \alpha_j x}}{\text{Im} (\alpha_j) > 0} \quad (5.4.22) \]

where \( A_j \)’s are given by

\[ A_j = \prod_{n=1}^{4} \frac{1}{\alpha_j - \alpha_n} \quad (5.4.23) \]

and

\[ N(p) = \{ c_T^2 (1 + \tau_p p) + p(1 + \tau_T p) \} \]

### 5.5 Numerical Results and Discussions

To get the solution for the thermal displacement, temperature, stress and strain in the space-time domain, we have to apply the Laplace inversion formula to Eqs. (5.4.19)-(5.4.22), respectively. This has been done numerically using a method based on the Fourier series expansion technique (see Appendix). To get the roots of the polynomial \( M(\alpha) \) in the complex domain, we have used Laguerre’s method. The numerical code has been prepared using Fortran 77 programming language. For computational purpose, a
copper-like material with a material constant [187] has been taken into consideration,

\[ e_T = 0.0168, \quad \lambda = 1.387 \times 10^{11} \text{ Nm}^{-2}, \quad \mu = 0.448 \times 10^{11} \text{ Nm}^{-2}, \]

\[ \alpha_t = 16.7 \times 10^{-6} \text{ K}^{-1} \]

Also, we have taken \( Q_0^* = 1, \quad \tau = 1, \quad \beta_i = 3.7 \times 10^{-8} \) and \( c_T = 2 \) so the faster wave is the thermal wave.

The relaxation time parameters are taken as \( \tau_q = 0.001, \quad \tau_T = 0.05, \quad \tau_\nu = 0.05 \), which agree with the stability condition in [177].

We now present our results to compare the thermal displacement, temperature, stress and strain in the case of the TEWOED (GNII model), TEWED (GNIII model) and three-phase-lag model (3P model) for an unbounded elastic medium in the form of the graphs (Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16). Now the figures (Figs.
1, 2, 3, Figs. 5, 6, 7, Figs. 9, 10, 11 and Figs. 13, 14, 15) are plotted against distance \( x \) for \( t = 0.4 \) and the figures (Figs. 4, 8, 12 and 16) are plotted against time \( t \) for \( x = 0.3 \).

Figure 1 depicts the variation of the thermal displacements \( u \) versus distance \( x \). It is observed that the displacement increases for \( 0.0 \leq x \leq 0.3 \) and then decreases and ultimately tends to zero for \( x \geq 0.9 \) in the presence of a magnetic field (WMF) by taking \( R_M = 2.0 \), and it is also observed that displacement increases for \( 0.0 \leq x \leq 0.2 \) and then decreases and ultimately goes to zero in the absence of a magnetic field (WOMF) by taking \( R_M = 1.0 \) for GNI model, i.e., \( \kappa_0 = 0.0 \). For the case of WOMF, the result complies with that of Roychoudhuri and Dutta [187] where they have used the analytical method. Also in the case when \( \kappa_0 = 1.2 \), both for GNI model and 3P model, \( u \) increases first, then decreases and ultimately approaches to zero as before with the increase of \( x \) for WMF \( (R_M = 2.0) \) as well as WOMF \( (R_M = 1.0) \). As may be seen from the figure.
the rate of decay is slower in the case of 3P model than that of GNIII model and that is again slower than that of GNII model both for WMF and WOMF, this implies that $\kappa_0 = 1.2$ corresponds to a slower rate of decay than the case when $\kappa_0 = 0.0$ in the absence of a magnetic field ($R_M = 1.0$). This result agrees with that of Banik et al. [22]. The magnitude of the displacement $u$ is large in the GNII model in comparison to GNIII model and 3P model, but the qualitative behavior is almost the same for all the three models.

Figure 2 depicts the variation of the displacement ($u$) with distance ($x$) taking $R_M = 1, 2, 3, 4$ and keeping the damping coefficient $\kappa_0 = 1.2$ where we have considered the 3P model. Here also a similar qualitative behavior is observed as in the case of Figure 1. But one important thing observed here is that with the increase in the magnetic field the magnitude of displacement decreases, which is quite plausible.
Figure 3 is plotted to show the variation of displacement \( (u) \) with distance \( (z) \) for \( R_M = 2.0 \), where we have again considered the 3P model. Figure 3 depicts the effect of the damping coefficient on the displacement. Now it is observed that as the damping coefficient increases the rate of decay of the displacement becomes slow.

Figure 4 represents the variation of displacement \( (u) \) against time \( (t) \) for \( x = 0.3 \). It is observed that the displacement increases first and then reaches a constant value with the increase in time \( t \) in the presence of a magnetic field (WMF) by taking \( R_M = 2.0 \), and it is also observed that in the absence of a magnetic field (WOMF), i.e., for \( R_M = 1.0 \), the displacement shows the same qualitative behavior for GNI model \( (\kappa_0 = 0.0) \). Also, in the case when \( \kappa_0 = 1.2 \), for both GNI model and 3P model, \( u \) increases first and then ultimately approaches to a constant value as before with the increase of \( t \) for WMF \( (R_M = 2.0) \) and WOMF \( (R_M = 1.0) \). But one important thing is observed here that in the presence of a magnetic field the displacement is smaller in the case of 3P model than...
that of GNIII model which is again smaller than that of GNII model. It is also seen from this figure that with the increase in magnetic field the magnitude of displacement decreases for all the three models. It is observed that the time to reach the steady state for GNII model for WMF and WOMF is faster than for the other two models, which is quite plausible since for GNII model there is no such dissipation of energy.

Figure 5 depicts the variation of temperature ($T$) with distance ($x$) for WMF ($R_M = 2.0$) and WOMF ($R_M = 1.0$). Here, it can be observed that temperature decreases with the increase in distance and finally goes to zero for GNII ($\kappa_0 = 0.0$), GNIII ($\kappa_0 = 1.2$) and 3P model ($\kappa_0 = 1.2$). In the case of 3P model, the rate of decay is slower than that of GNIII model and that is again slower than that of GNII model both for WMF and WOMF. From this figure, it can also be observed that there is no such effect of the magnetic field on temperature.
Magneto-thermoelastic Response in a Perfectly Conducting Medium with Three-phase-lag Effect

Fig. 7 Variation of temperature $T$ with distance $x$ in three-phase-lag model

Fig. 8 Variation of temperature $T$ with distance $x$ in three-phase-lag model
Figure 6 shows the variation of temperature ($T$) with distance ($x$) for the various values of the magnetic field keeping $\kappa_0 = 1.2$. Here, we have considered the 3P model. This figure depicts that there is no such effect of the magnetic field on temperature for this model.

Figure 7 is plotted to show the variation of temperature ($T$) versus distance ($x$) for 3P model in the presence of a magnetic field ($R_M = 2.0$). It is observed from the figure that temperature decreases with the increase in distance and finally goes to zero for all values of the damping coefficient but as the damping coefficient increases, the rate of decay decreases.

Figure 8 depicts the variation of temperature ($T$) with time ($t$) for WMF ($R_M = 2.0$) and WOMF ($R_M = 1.0$). Here, it is observed that temperature increases for $0.0 < x < 1.1$ and then approaches a steady state for GNII ($\kappa_0 = 0.0$) model by taking $R_M = 2.0$.
and $R_M = 1.0$. For GNIII model and 3P model, the displacement shows the same nature for WMF and WOMF. From this figure, it can also be observed that there is no such effect of the magnetic field on temperature. Here also the time to reach the steady state for GNU model for WMF and WOMF is faster than for the other two models, which is quite plausible since for GNII model there is no such dissipation of energy as it is in Figure 4.

Figure 9 exhibits the space variation ($x$) of stress ($\sigma_{xx}$) in the presence of a magnetic field ($R_M = 2.0$) and also in the absence of a magnetic field ($R_M = 1.0$). It is observed that stress is compressive in nature, and the magnitude is maximum near the boundary. Here, the rate of decay is faster in the case of GNII model than in case of GNIII model, which is again faster than 3P model both for $R_M = 2.0$ and $R_M = 1.0$.

Figures 10 and 11 are plotted to show the variation of stress ($\sigma_{xx}$) against distance
Magneto-thermoelastic Response in a Perfectly Conducting Medium with Three-phase-lag Effect

\[ \kappa = 1.2, \sigma_0 = 2.0, \epsilon = 0.0168 \]

Fig. 10 Variation of stress \( \sigma_x \) with distance \( x \) in three-phase-lag model

\[ R_w = 1.0, R_w = 2.0, R_w = 3.0, R_w = 4.0 \]

Fig. 11 Variation of stress \( \sigma_x \) with distance \( x \) in three-phase-lag model

\[ R_w = 2.0, \sigma_0 = 2.0, \epsilon = 0.0168 \]
for 3P model taking various values of the magnetic field and the damping coefficient, respectively. From Figure 10, it is observed that with the increase in the magnetic field the magnitude of stress increases near the boundary, and from Figure 11 it is observed that by increasing the value of the damping coefficient, the magnitude and the rate of decay of stress decrease near the boundary.

Figure 12 depicts the variation of stress ($\sigma_{xx}$) with time ($t$) for GNII, GNIII and 3P model in the presence of a magnetic field ($R_M = 2.0$) and in the absence of a magnetic field ($R_M = 1.0$). It is observed that the magnitudes of stress are large in the case of TEWED (GNII) theory in comparison with the rest of the theories. This figure also shows that by increasing magnetic field the damping of stress is also increasing until reaches a constant value.

Figure 13 gives the variation of strain ($e_{xx}$) against distance ($x$) in the presence of a
magnetic field \((R_M = 2.0)\) and in the absence of a magnetic field \((R_M = 1.0)\). From this figure, we can show for GNII model \((\kappa_0 = 0.0)\) that strain is positive up to a distance \(x = 0.2\), and for GNIII model, \((\kappa_0 = 1.2)\) and 3P model \((\kappa_0 = 1.2)\) strain is positive up to a distance \(x = 0.3\), and then, it is negative and finally diminishes to zero for all the three models in the case of WMF \((R_M = 2.0)\). Now in the absence of a magnetic field (WOMF, i.e., \(R_M = 1.0\)), the magnitude of strain is larger near the boundary than that of WMF for all the three models. The qualitative behavior is nearly the same.

Figure 14 depicts the variation of strain \((e_{xx})\) with distance \((x)\) for 3P model taking \(\kappa_0 = 1.2\) and \(R_M = 1, 2, 3\) and 4. In this figure, it can be observed that with the increase in the magnetic field the magnitude of strain decreases near the boundary and ultimately approaches to zero as distance increases, which is quite plausible since the periodic disturbance is given on the boundary.
Magneto-thermoelastic Response in a Perfectly Conducting Medium with Three-phase-lag Effect

Fig. 14 Variation of strain $\varepsilon_\alpha$ with distance $x$ in three-phase-lag model:

$k_\alpha = 1.2, \varepsilon_i = 2.0, \varepsilon_f = 0.0168$

Fig. 16 Variation of strain $\varepsilon_\alpha$ with distance $x$ in three-phase-lag model:

$R_v = 2.0, \varepsilon_i = 2.0, \varepsilon_f = 0.0168$

$k_\beta = 0.0$
$k_\beta = 0.6$
$k_\beta = 1.2$
$k_\beta = 1.8$
Figure 15 is plotted to show the variation of the strain ($e_{xx}$) versus distance ($x$) in the presence of a magnetic field ($R_M = 2.0$) where we have considered the three-phase-lag model. Here, the effect of the damping coefficient on strain is such that for all values of $\kappa_0$ strain is positive first, then remains negative and finally goes to zero.

Figure 16 depicts the variation of strain ($e_{xx}$) with time ($t$). This figure shows that for GNII, GNIII and 3P model strain is negative first, then increases and ultimately reaches a steady state both for WMF ($R_M = 2.0$) and WOMF ($R_M = 1.0$). In Figures 1, 5, 9, 13 when there is no such magnetic field ($R_M = 1.0$) but there is a dissipation of energy ($\kappa_0 = 1.2$), the result agrees with that of Banik et al. [22] and when $R_M = 1.0$ and $\kappa_0 = 0.0$, the result is confirmed by that of Roychoudhuri and Dutta[187] in which the closed-form solution of the problem has been derived.