Chapter 9

Two-Temperature Magneto-thermoelastic Response in a Perfectly Conducting Medium With Finite Wave Speed

9.1 Introduction

The linearized version of the two-temperature theory (2TT) has been studied by many authors [228, 122, 173, 175]. Sur and Kanoria [213] have studied fractional order two-temperature thermoelasticity with finite wave speed. Banik and Kanoria [23] have studied generalized thermoelastic interactions in an infinite isotropic elastic body with a spherical cavity in the context of the two-temperature generalized thermoelasticity theory. A half space problem filled with an elastic material in the context of the two temperature generalized thermoelasticity theory using state space approach has been solved by Youssef and Al-Lehaibi [237]. Youssef [238] has solved the two dimensional problem of two-temperature generalized thermoelastic half-space subjected to ramp-type heating.

Different thermoelastic problems have been solved by employing the three-phase-

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lag model of generalized thermoelasticity by several authors [104, 109, 110, 116]. Kar and Kanoria [109] have studied the thermo-visco-elastic stresses in an isotropic visco-thermoelastic homogeneous spherical shell due to step input of temperature on the stress free boundaries of the shell in the context of generalized theories of thermoelasticity.

The aim of this chapter is to present magneto-thermoelastic interaction due to the thermal shock on a stress free boundary of a half-space in the context of two-temperature generalized thermoelasticity for three-phase-lag model. The governing equations of the problem are solved in Laplace transform domain by using state-space approach. The inversion of the Laplace transform is computed numerically by using a method based on Fourier series expansion technique [82]. A comparison of results of two different theories (2T3P and 2TGNIII) is shown and the effect of applied magnetic field, damping coefficient and two-temperature parameter on the physical quantities are studied.

9.2 Basic Equations

For perfectly conducting medium the constitutive equations are

\[
\sigma_{ij} = 2\mu e_{ij} + [\lambda \Delta - \gamma(T - T_0)]\delta_{ij}, \quad i, j = 1, 2, 3
\]

(9.2.1)

where

\[
e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \Delta = e_{ii}.
\]

(9.2.2)

Stress equations of motion in the presence of body forces \( F_i \) are

\[
\sigma_{ij,j} + F_i = \rho \ddot{u}_i.
\]

(9.2.3)

The heat equation corresponding to 2T3P theory in the absence of heat source is

\[
\rho c_v (\ddot{T} + \tau_q \dot{T} + \frac{1}{2} \tau_q^2 \dddot{T}) + \gamma T_0 (\dddot{\Delta} + \tau_q \dddot{\Delta} + \frac{1}{2} \tau_q^2 \dddot{\Delta}) = K (\nabla^2 \phi + \tau_q \nabla^2 \dot{\phi}) + K^* (\nabla^2 \ddot{\phi} + \tau_q \nabla^2 \dddot{\phi})
\]

(9.2.4)
where $K^*$ is an additional material constant, $K$ is the thermal conductivity, $c_v$ is the specific heat at constant strain, $c_3 = \left( \frac{K^*}{\rho c_v} \right)^{1/2}$ is the finite thermal wave speed, $\tau_\nu$ is called the phase-lag of thermal displacement gradient, $\tau_T$ is called the phase-lag of the temperature gradient and $\tau_q$ is called the phase-lag of the heat flux. Here $\cdot^\prime$ denotes derivatives with respect to time. For $\tau_T = \tau_q = \tau_\nu = 0$, Eq. (9.2.4) reduces to the 2TGNIII theory, and for $\tau_T = \tau_q = \tau_\nu = 0$ and $K << K^*$, Eq. (9.2.4) reduces to the 2TGNII theory.

The relation between conductive temperature and thermodynamic temperature is,

$$\phi - T = a \phi_{,tt},$$

where $a > 0$, is the temperature discrepancy.

### 9.3 Formulation of the Problem

We now consider a magneto-thermoelastic medium of perfect conductivity in an initial magnetic field $H$. This produces an induced magnetic field $h$ and induced electric field $E$, which satisfy the linearized equations of electromagnetism in a slowly moving continuum as

1. \[ \text{curl} \ h = J, \] (9.3.1)
2. \[ \text{curl} \ E = -\mu_e \frac{\partial h}{\partial t}, \] (9.3.2)
3. \[ E = -\mu_e \left( \frac{\partial u}{\partial t} \times H \right), \] (9.3.3)
4. \[ \text{div} \ h = 0. \] (9.3.4)

Now, we shall consider a homogeneous isotropic thermoelastic perfectly conducting solid occupying the half-space $x \geq 0$, which is initially quiescent and where all the state functions depend only on $x$ and time $t$. 
The displacement vector has components

\[ u_x = u(x,t), \quad u_y = u_z = 0. \]  \hspace{1cm} (9.3.5)

In the context of the linear theory of 2T3P, the equation of motion, heat equation and the constitutive equations can be written as

\[
(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} + F_x = \rho \frac{\partial^2 u}{\partial t^2},
\]  \hspace{1cm} (9.3.6)

where

\[ F = \mu_e (J \times H), \quad F = (F_x, F_y, F_z), \]
\[
K \left( \frac{\partial^2 \phi}{\partial x^2} + \tau_\gamma \frac{\partial^4 \phi}{\partial x^2 \partial t^2} \right) + K^* \left( \frac{\partial^2 \phi}{\partial t^2} + \tau_\gamma \frac{\partial^4 \phi}{\partial x^2 \partial t^2} \right) =
\]
\[
\rho c_v \left( \frac{\partial^2 T}{\partial t^2} + \tau_q^2 \frac{\partial^4 T}{\partial t^2 \partial x^2} + \frac{1}{2} \tau_q \frac{\partial^4 T}{\partial t^2 \partial x^4} \right) + \gamma T_c \left( \frac{\partial^2 u}{\partial t^2 \partial x} + \tau_q \frac{\partial^4 u}{\partial t^2 \partial x^3} + \frac{1}{2} \tau_q \frac{\partial^4 u}{\partial t^2 \partial x^5} \right)
\]
\[
\sigma = \sigma_{xx} = (\lambda + 2\mu) e - \gamma (T - T_0),
\]  \hspace{1cm} (9.3.8)

where

\[ e = e_{xx} = \frac{\partial u}{\partial x}. \]  \hspace{1cm} (9.3.9)

A constant magnetic field with components \((0, H_0, 0)\) is permeating the medium. The current density vector \(J\) has one component in \(z\)-direction and the induced magnetic field \(h\) has one component in the \(y\)-direction, while the induced electric field \(E\) has one component in the \(z\)-direction. i.e.

\[ J = (0, 0, J), \]
\[ h = (0, h, 0), \]
\[ E = (0, 0, E). \]

Now Eqs. (9.3.1)-(9.3.3) yield

\[ J = \frac{\partial h}{\partial x}, \]  \hspace{1cm} (9.3.10)
\[ h = -H_0 e, \]  \hspace{1cm} (9.3.11)
\[
E = -\mu_e H_0 \frac{\partial u}{\partial t}.
\]

(9.3.12)

From Eqs. (9.3.10) and (9.3.11)

\[
(J \times H)_x = H_0^2 \frac{\partial^2 u}{\partial x^2}.
\]

(9.3.13)

Now Eqs. (9.3.6) reduces to

\[
\frac{\partial}{\partial t} \left[ (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} \right] + \mu_e H_0^2 \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}.
\]

(9.3.14)

Eq. (9.3.14) can also be written as

\[
c_1^2(1 + R_H) \frac{\partial^2 u}{\partial x^2} - \frac{\rho}{\rho} \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2},
\]

(9.3.15)

where \( R_H = \frac{\mu_e H_0^2}{\rho c_1^2} = \frac{v_A^2}{c_1^2} \), \( c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}} \) and \( v_A = \sqrt{\frac{\mu_e H_0}{\rho}} \) is the Alfven wave velocity of the medium. The coefficient \( R_H \) represents the effect of external magnetic field in the thermoelastic process proceeding in the body.

Let us introduce the following dimensionless quantities:

\[
x' = \frac{x}{l}, \quad u' = \frac{\lambda + 2\mu}{\gamma T_0^2} u, \quad t' = \frac{c_1 t}{l}, \quad T' = \frac{T - T_0}{T_0}, \quad \phi' = \frac{\phi - \phi_0}{\phi_0},
\]

\[
\sigma' = \frac{\sigma}{\gamma T_0^2}, \quad e' = e, \quad R_M^2 = 1 + R_H, \quad K' = \frac{h}{H_0}, \quad E' = \frac{E}{\mu_e H_0 c_1},
\]

\[
\tau'_T = \frac{c_1 \tau_T}{l}, \quad \tau'_\nu = \frac{c_1 \tau}{l}, \quad \tau'_T = \frac{c_1 \tau_T}{l}.
\]

where \( l = \) some standard length. Now omitting primes, Eqs. (9.3.15), (9.3.7), (9.3.8), (9.3.9), (9.3.11), (9.3.12) and (9.2.5) can be re-written in dimensionless form as

\[
R_M^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2},
\]

(9.3.16)
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\[
\left( 1 + \tau_{\nu} \frac{\partial}{\partial t} \right) \left[ \frac{\partial^2 \phi}{\partial x^2} \right] + \left( 1 + \tau_{T} \frac{\partial}{\partial t} \right) \left[ \kappa_{0} \frac{\partial^2 \phi}{\partial x^2} \frac{\partial \phi}{\partial t} \right] = \\
\left( 1 + \tau_{q} \frac{\partial}{\partial t} + \frac{1}{2} \tau_{q}^2 \frac{\partial^2}{\partial x^2} \right) \left[ \frac{\partial^2 T}{\partial x^2} + \epsilon_{T} \frac{\partial^2}{\partial t^2} \left( \frac{\partial u}{\partial x} \right) \right],
\]

(9.3.17)

\[
\sigma = \frac{\partial u}{\partial x} - T, \quad (9.3.18)
\]

\[
e = \beta_{1} \frac{\partial u}{\partial x}, \quad (9.3.19)
\]

\[
h = -e, \quad (9.3.20)
\]

\[
E = -\beta_{1} \frac{\partial u}{\partial t}, \quad (9.3.21)
\]

\[
\phi - T = \beta_{0} \frac{\partial^2 \phi}{\partial x^2}, \quad (9.3.22)
\]

where \( \epsilon_{T} = \frac{\gamma^2 T_{0}}{(\lambda + 2\mu) \rho c_{v}} \), \( c_{p}^2 = \kappa_{0} \frac{c_{v}^2}{c_{l}^2} \), \( \kappa_{0} = \frac{K}{\rho c_{v} c_{l}}, \beta_{1} = \frac{\gamma T_{0}}{\rho c_{v}^{2}}, \beta_{0} = \frac{a}{l^2} \)

and it is to be noted that 2TGNIII and 2TGNII can be recovered from Eq. (9.3.17) by taking \( \tau_{T} = \tau_{q} = \tau_{v} = 0 \) and \( \tau_{T} = \tau_{q} = \tau_{v} = 0, \kappa_{0} \ll C_{v}^2 \) respectively.

Now the problem is to solve the Eqs. (9.3.16)-(9.3.22) subject to the boundary conditions:

(i) Thermal loading:

A thermal shock is applied to the boundary plane \( x = 0 \) in the form

\[
\phi(0, t) = \phi_{0} U(t), \quad (9.3.23)
\]

where \( \phi_{0} \) is a constant and \( U(t) \) is the Heaviside unit step function.

(ii) Mechanical loading:

The bounding plane \( x = 0 \) is taken to be traction-free, i.e.

\[
\sigma(0, t) + T_{11}(0, t) - T_{11}^0(0, t) = 0, \quad (9.3.24)
\]

where \( T_{11} \) and \( T_{11}^0 \) are the Maxwell stress tensor in a elastic medium and in a vacuum.
respectively.

Since the transverse components of the vectors $\mathbf{E}$ and $\mathbf{h}$ are continuous across the bounding plane, i.e. $E(0,t) = E^0(0,t)$ and $h(0,t) = h^0(0,t)$, $t > 0$, where $E^0$ and $h^0$ are respectively the components of the induced electric and magnetic field in free space and the relative permeability is very nearly unity, it follows that $T_{11}(0,t) = T_{11}^0(0,t)$ and consequently Eq. (9.3.24) reduces to:

$$\sigma(0,t) = 0.$$  \hfill (9.3.25)

The initial and regularity conditions can be written as

$$u = T = \phi = 0 \text{ at } t = 0$$
$$\frac{\partial u}{\partial t} = \frac{\partial T}{\partial t} = \frac{\partial \phi}{\partial t} = 0 \text{ at } t = 0$$ \hfill (9.3.26)

and

$$u \to 0, \ T \to 0, \ \phi \to 0 \text{ as } x \to \infty \hfill (9.3.27)$$

### 9.4 Method of Solution

Taking the Laplace transform defined by the relation

$$\tilde{f}(s) = \int_0^\infty e^{-st} f(t) \, dt = L\{f(t)\},$$

of both sides of Eqs. (9.3.16)-(9.3.22), we obtain:

$$R_2^2 \frac{d^2 \tilde{u}}{dx^2} - \frac{\tilde{d}}{dt} = s^2 \tilde{u}, \hfill (9.4.1)$$

$$\frac{\partial^2 \tilde{\phi}}{\partial x^2} + s\kappa_0 (1 + \tau_T s) \frac{d^2 \tilde{\phi}}{dx^2} = s^2 (1 + s\tau_q + \frac{1}{2} \tau_q^2 s^2) \tilde{T} + \epsilon_T s^2 (1 + s\tau_q + \frac{1}{2} \tau_q^2 s^2) \frac{d\tilde{u}}{dx}, \hfill (9.4.2)$$
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where all the initial state functions are equal to zero.

Eliminating \( \ddot{u} \) and \( \ddot{T} \) from Eqs. (9.4.1)-(9.4.7), we obtain

\[
\frac{d^2\phi}{dx^2} = L_1\phi + L_2\sigma, \quad (9.4.8)
\]

where

\[
L_1 = \frac{s^2(1 + \epsilon_T)(1 + s\tau_q + \frac{1}{2}\tau_q^2s^2)}{\sigma_T^2(1 + \tau_q s) + \kappa_0 s(1 + \tau_T s) + (1 + s\tau_q + \frac{1}{2}\tau_q^2s^2)\beta_0 s^2(1 + \epsilon_T)},
\]

\[
L_2 = \frac{\epsilon_T s^2(1 + s\tau_q + \frac{1}{2}\tau_q^2s^2)}{\sigma_T^2(1 + \tau_q s) + \kappa_0 s(1 + \tau_T s) + (1 + s\tau_q + \frac{1}{2}\tau_q^2s^2)\beta_0 s^2(1 + \epsilon_T)}
\]

and

\[
\frac{d^2\sigma}{dx^2} = M_1\phi + M_2\sigma, \quad (9.4.9)
\]

where

\[
M_1 = \frac{s^2 - L_1(R_M^2 - 1))(1 - \beta_0 L_1)}{\beta_0 L_2 + R_M^2(1 - \beta_0 L_2)}, \quad M_2 = \frac{s^2(1 - \beta_0 L_2) - (R_M^2 - 1)L_2(1 - \beta_0 L_1)}{\beta_0 L_2 + R_M^2(1 - \beta_0 L_2)}.
\]

Choosing as state variables the conductive temperature \( \phi \) and the stress component \( \sigma \) in the \( x \)-direction, Eqs. (9.4.8) and (9.4.9) can be written in the matrix form as:

\[
\frac{d^2\bar{u}(x, s)}{dx^2} = A(s)\bar{u}(x, s), \quad (9.4.10)
\]
where

\[ \overline{v}(x, s) = \begin{bmatrix} \overline{\phi}(x, s) \\ \overline{\sigma}(x, s) \end{bmatrix} \quad \text{and} \quad A(s) = \begin{bmatrix} L_1 & L_2 \\ M_1 & M_2 \end{bmatrix}. \]

The system of Eqs. (9.4.10) may have the solution in the form

\[ \overline{v}(x, s) = \exp[-\sqrt{A(s)}x]\overline{v}(0, s), \quad (9.4.11) \]

where

\[ \overline{v}(0, s) = \begin{bmatrix} \overline{\phi}(0, s) \\ \overline{\sigma}(0, s) \end{bmatrix} = \begin{bmatrix} \phi_0 \\ \sigma_0 \end{bmatrix}, \]

where for bounded solution with large \( x \), we have cancelled the part of exponential that has a positive power.

We shall use the spectral decomposition of the matrix \( A(s) \) and the well-known Cayley-Hamilton theorem to find the matrix form of the expression \( \exp[-\sqrt{A(s)}x] \).

The characteristic equation of the matrix \( A(s) \) can be written as follows:

\[ k^2 - k(L_1 + M_2) + (L_1M_2 - L_2M_1) = 0. \quad (9.4.12) \]

The roots of this equation, namely, \( k_1 \) and \( k_2 \), satisfy the following relations:

\[ k_1 + k_2 = L_1 + M_2, \quad (9.4.13a) \]
\[ k_1k_2 = L_1M_2 - L_2M_1. \quad (9.4.13b) \]

The Taylor series expansion of the matrix exponential in Eq. (9.4.11) has the form

\[ \exp[-\sqrt{A(s)}x] = \sum_{n=0}^{\infty} \frac{[-\sqrt{A(s)}x]^n}{n!}. \quad (9.4.14) \]

Using the Cayley-Hamilton theorem, we can express \( A^2 \) and higher powers of the matrix \( A \) in terms of \( I \) and \( A \), where \( I \) is the unit matrix of second order.
Thus, the infinite series in Eq. (9.4.14) can be reduced to

$$\exp[-\sqrt{A(s)x}] = a_0(x, s)I + a_1(x, s)\sqrt{A(s)} ,$$  \hspace{1cm} (9.4.15)

where $a_0$ and $a_1$ are coefficients depending on $x$ and $s$.

By the Cayley-Hamilton theorem, the characteristic roots $k_1$ and $k_2$ of the matrix $A$ must satisfy Eq. (9.4.15), thus

$$\exp[-\sqrt{k_1x}] = a_0 + a_1\sqrt{k_1} ,$$  \hspace{1cm} (9.4.16)

and

$$\exp[-\sqrt{k_2x}] = a_0 + a_1\sqrt{k_2} .$$  \hspace{1cm} (9.4.17)

The solution of the above system is given by

$$a_0 = \frac{\sqrt{k_1}e^{-\sqrt{k_2}x} - \sqrt{k_2}e^{-\sqrt{k_1}x}}{\sqrt{k_1} - \sqrt{k_2}} ,$$

and

$$a_1 = \frac{e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x}}{\sqrt{k_1} - \sqrt{k_2}} .$$

Hence, we have

$$\exp[\sqrt{A(s)x}] = L_{ij}(x, s), \quad i, j = 1, 2$$

where

$$L_{11} = \frac{e^{-\sqrt{k_2}x}(k_1 - L_1) - e^{-\sqrt{k_1}x}(k_2 - L_1)}{k_1 - k_2}, \quad L_{12} = \frac{L_2(e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x})}{k_1 - k_2} ,$$

$$L_{22} = \frac{e^{-\sqrt{k_2}x}(k_1 - M_2) - e^{-\sqrt{k_1}x}(k_2 - M_2)}{k_1 - k_2}, \quad L_{21} = \frac{M_1(e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x})}{k_1 - k_2} .$$  \hspace{1cm} (9.4.18)

The solution of Eq. (9.4.10) can be written in the form

$$\bar{v}(x, s) = L_{ij}\bar{v}(0, s) .$$  \hspace{1cm} (9.4.19)
Hence, we obtain

$$\ddot{\phi}(x, s) = \frac{(k_1 \ddot{\phi}_0 - L_1 \ddot{\phi}_0 - L_2 \ddot{\varphi}_0)e^{-\sqrt{k_2}z} - (k_2 \ddot{\phi}_0 - L_1 \ddot{\phi}_0 - L_2 \ddot{\varphi}_0)e^{-\sqrt{k_1}z}}{k_1 - k_2},$$  \hspace{1cm} (9.4.20)

$$\ddot{\varphi}(x, s) = \frac{(k_1 \ddot{\varphi}_0 - M_1 \ddot{\varphi}_0 - M_2 \ddot{\varphi}_0)e^{-\sqrt{k_2}z} - (k_2 \ddot{\varphi}_0 - M_1 \ddot{\varphi}_0 - M_2 \ddot{\varphi}_0)e^{-\sqrt{k_1}z}}{k_1 - k_2}. $$ \hspace{1cm} (9.4.21)

By using Eqs. (9.4.20), (9.4.21) and (9.4.7) we get

$$\dot{T}(x, s) = \frac{-(k_2 \ddot{\varphi}_0 - L_1 \ddot{\varphi}_0 - L_2 \ddot{\varphi}_0)(1 - \beta_0 k_2)e^{-\sqrt{k_2}z}}{k_1 - k_2}. $$ \hspace{1cm} (9.4.22)

By using Eqs. (9.4.3), (9.4.4), (9.4.21) and (9.4.22) we get

$$\beta_1\{(k_1 \ddot{\varphi}_0 - M_1 \ddot{\varphi}_0 - M_2 \ddot{\varphi}_0) + (1 - \beta_0 k_2)(k_1 \ddot{\varphi}_0 - L_1 \ddot{\varphi}_0 - L_2 \ddot{\varphi}_0)\}e^{-\sqrt{k_2}z}$$

$$\ddot{\varepsilon} = \frac{-\{(k_2 \ddot{\varphi}_0 - M_1 \ddot{\varphi}_0 - M_2 \ddot{\varphi}_0) + (1 - \beta_0 k_1)(k_2 \ddot{\varphi}_0 - L_1 \ddot{\varphi}_0 - L_2 \ddot{\varphi}_0)\}e^{-\sqrt{k_1}z}}{k_1 - k_2}. $$ \hspace{1cm} (9.4.23)

Using Laplace transformation to the Eqs. (9.3.23) and (9.3.25) we obtain

$$\ddot{\phi}_0 = \frac{\phi_0}{s},$$ \hspace{1cm} (9.4.24)

$$\ddot{\varphi}(0, t) = \ddot{\varphi}_0 = 0. $$ \hspace{1cm} (9.4.25)

Hence, we can use the conditions (9.4.24) and (9.4.25) into Eqs. (9.4.20), (9.4.21), (9.4.22) and (9.4.23) to get the exact solution in the Laplace transform domain as:

$$\ddot{\phi}(x, s) = \frac{\phi_0[(k_1 - L_1)e^{-\sqrt{k_2}z} - (k_2 - L_1)e^{-\sqrt{k_1}z}]}{s(k_1 - k_2)}, $$ \hspace{1cm} (9.4.26)

$$\ddot{\varphi}(x, s) = \frac{\phi_0 M_1(e^{-\sqrt{k_1}z} - e^{-\sqrt{k_2}z})}{s(k_1 - k_2)}, $$ \hspace{1cm} (9.4.27)

$$\ddot{T}(x, s) = \frac{\phi_0[B e^{-\sqrt{k_2}z} - A e^{-\sqrt{k_1}z}]}{s(k_1 - k_2)}, $$ \hspace{1cm} (9.4.28)

$$\ddot{\varepsilon}(x, s) = \frac{\beta_1 \phi_0[(B - M_1)e^{-\sqrt{k_2}z} - (A - M_1)e^{-\sqrt{k_1}z}]}{s(k_1 - k_2)}. $$ \hspace{1cm} (9.4.29)
where \( A = (k_2 - L_1)(1 - \beta_0 k_1) \), \( B = (k_1 - L_1)(1 - \beta_0 k_2) \).

Using Eqs. (9.4.1) and (9.4.3) the displacement can be written as:

\[
\bar{u} = \frac{1}{s^2} \left[ R_M^2 \frac{d\sigma}{dx} + (R_M^2 - 1) \frac{dT}{dx} \right].
\]

(9.4.30)

Substituting from Eqs. (9.4.27) and (9.4.28) into (9.4.30) we get

\[
\bar{u} = \frac{\phi_0}{s^2(k_1 - k_2)} \left[ \{(R_M^2 - 1)A - R_M^2 M_1\} \sqrt{k_1 e^{-\sqrt{k_1}z}} - \{(R_M^2 - 1)B - R_M^2 M_1\} \sqrt{k_2 e^{-\sqrt{k_2}z}} \right].
\]

(9.4.31)

Using Eqs. (9.4.5) and (9.4.6) the expression for induced magnetic and electric field become

\[
\bar{h}(x, s) = -\frac{\beta_1 \phi_0}{s^2(k_1 - k_2)} \left[ \{(R_M^2 - 1)A - R_M^2 M_1\} \sqrt{k_1 e^{-\sqrt{k_1}z}} - \{(R_M^2 - 1)B - R_M^2 M_1\} \sqrt{k_2 e^{-\sqrt{k_2}z}} \right],
\]

(9.4.32)

\[
\bar{E}(x, s) = -\frac{\beta_1 \phi_0}{s^2(k_1 - k_2)} \left[ \{(R_M^2 - 1)A - R_M^2 M_1\} \sqrt{k_1 e^{-\sqrt{k_1}z}} - \{(R_M^2 - 1)B - R_M^2 M_1\} \sqrt{k_2 e^{-\sqrt{k_2}z}} \right].
\]

(9.4.33)

### 9.5 Numerical Results and Discussions

The copper like material has been chosen for the purpose of numerical evaluations. The constants of the problem are taken as follows: (Ezzat[60])

\[
K = 386 \text{ N/Ks}, \quad \mu = 3.86 \times 10^{10} \text{ N/m}^2, \quad T_0 = 293K, \quad \epsilon_T = 0.0168,
\]

\[
\alpha_{\text{e}} = 1.78 \times 10^{-5} \text{ K}^{-1}, \quad \lambda = 7.76 \times 10^{10} \text{ N/m}^2, \quad \beta_0 = 0.1,
\]

\[
c_v = 383.1 \text{ m}^2/K, \quad \mu_e = 4 \times 10^{-7} \text{ C}^2/\text{Nm}^2,
\]

\[
\beta = 0.1, \quad \beta_1 = 1.0, \quad \phi_0 = 1
\]

To get the solutions for stress, strain, temperatures, induced magnetic field and induced electric field, the Laplace inversion formula to the Eqs. (9.4.26), (9.4.27), (9.4.28),
(9.4.29), (9.4.32) and (9.4.33) has been applied. For numerical purpose, a method based on the Fourier series expansion technique has been used. To get the roots of Eq. (9.4.12) in the complex domain, Laguerre's method has been used. The numerical code has been prepared using Fortran 77 programming language.

Figures 1, 4, 5, 7, 9, 10 are drawn to compare stress, strain, temperatures, induced magnetic field and induced electric field in case of 2TGNIII and 2T3P model, taking $t = 0.1$, Figures 2, 6, 8, 11 are plotted to show the effect of damping coefficient on stress, conductive temperature, thermodynamic temperature and induced magnetic field respectively for both the cases of one type temperature as well as for two type temperature and Figure 3 is plotted to show the variation of stress with respect to time $t$.

Figures 1 exhibits the space variation ($x$) of stress ($\sigma$) in the presence of magnetic field
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0 = 0.1, \( \alpha_0 = 0.0168 \), \( R_m = 2.0 \) by taking \( R_m = 2.0 \) as well as in absence of magnetic field (WOMF) by taking \( R_m = 1.0 \) for the case of two-temperature (\( \beta_0 = 0.1 \)). It is observed from this figure that the magnetic field has a tendency to decrease the stress concentration for both the models (2TGNIII and 2T3P model). This is due to the fact that the magnetic field together with the current density corresponds to the term which behaves like external applied force as suggested by Eq. (9.3.6) and therefore, it tends to accelerate the metal particles. As may be seen from the figure that the stress is continuous when \( \beta_0 = 0.1 \) for both the models (2TGNIII and 2T3P model) in case of WMF as well as WOMF. Now it can be observed that stress is discontinuous for GNU model in the range \( 0.201 < x < 0.270 \) and \( 0.408 < x < 0.550 \) for \( \beta_0 = 0.0 \) in case of WMF as well as WOMF but these results are not depicted here. This figure also shows that the results obtained for 2TGNIII model agree with those corresponding results obtained in sixth chapter.

Figure 2 depicts the variation of stress \( \sigma \) versus distance \( x \) for three-phase-lag model (WMF) by taking \( R_M = 2.0 \) as well as in absence of magnetic field (WOMF) by taking \( R_M = 1.0 \) for the case of two-temperature (\( \beta_0 = 0.1 \)). It is observed from this figure that the magnetic field has a tendency to decrease the stress concentration for both the models (2TGNIII and 2T3P model). This is due to the fact that the magnetic field together with the current density corresponds to the term which behaves like external applied force as suggested by Eq. (9.3.6) and therefore, it tends to accelerate the metal particles. As may be seen from the figure that the stress is continuous when \( \beta_0 = 0.1 \) for both the models (2TGNIII and 2T3P model) in case of WMF as well as WOMF. Now it can be observed that stress is discontinuous for GNU model in the range \( 0.201 < x < 0.270 \) and \( 0.408 < x < 0.550 \) for \( \beta_0 = 0.0 \) in case of WMF as well as WOMF but these results are not depicted here. This figure also shows that the results obtained for 2TGNIII model agree with those corresponding results obtained in sixth chapter.

Figure 2 depicts the variation of stress \( \sigma \) versus distance \( x \) for the damping co-
efficient $\kappa_0 = 1.0, 2.0$ and $3.0$ in the presence of magnetic field ($R_M = 2.0$) for one-type temperature ($\beta_0 = 0.0$) and two-type temperature ($\beta_0 = 0.1$). The figure shows that the stress is reflexive in the range $0.0 \leq x \leq 1.0$ but is compressive in the range $1.0 \leq x \leq 1.4$ for all $\kappa_0$ in case of one type temperature ($\beta_0 = 0.0$). The figure also shows that the stress is compressive for all $\kappa_0$ in case of two type temperature ($\beta_0 = 0.1$). The stress distribution after assuming negative values goes on decreasing, attains maximum value in magnitude and then increases and finally vanishes for both type of temperature.

Figure 3 shows the variation of stress ($\sigma$) with time ($t$) for 2T3P model in case of both WOMF and WMF. It is observed that each $\sigma$ is compressive at the beginning of the thermal shock application to the boundary plane then $\sigma$ becomes positive and finally reaches to the steady state as time $t$ increases in presence of the magnetic field ($R_M = 2.0$) as well as in absence of the magnetic field ($R_M = 1.0$) but with the increase of value $R_M$, the time to reach steady state also increases.
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\[ \beta = 0.1, \ c_l = 0.1, \ c_T = 0.0168 \]

**Fig. 4 Variation of strain \( e \) with distance \( x \) for time \( t = 0.1 \)**

\[ \beta = 0.1, \ c_l = 0.1, \ c_T = 0.0168 \]

**Fig. 5 Variation of conductive temperature \( \phi \) with distance \( x \) for time \( t = 0.1 \)**
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Figure 4 gives the variation of strain ($\varepsilon$) versus distance ($x$) for $\beta_0 = 0.1$ in the presence of magnetic field ($R_M = 2.0$) and in the absence of magnetic field ($R_M = 1.0$). The strain takes positive values first, then decreases and ultimately disappears with the increase of $x$ for 2TGNIII model both in the presence of magnetic field ($R_M = 2.0$) and in the absence of magnetic field ($R_M = 1.0$). But for 2T3P model, it is observed that strain shows oscillatory nature first then decreases and ultimately vanishes with the increase of $x$ in the presence of magnetic field ($R_M = 2.0$) whereas in absence of magnetic field ($R_M = 1.0$) strain shows the same qualitative behavior as that of in 2TGNIII models.

Figure 5 depicts the variation of conductive temperature ($\psi$) with distance ($x$) for WMF ($R_M = 2.0$) with two-type temperature ($\beta_0 = 0.1$). Here it is observed that conductive temperature decreases with the increase of distance and finally vanishes for both the models (2TGNIII and 2T3P model). In case of 2T3P model, the rate of decay...
is slower than that of 2TGNIII model for WMF. Numerical work shows no variations of values of $\phi$ corresponding to $R_M = 1.0$ (not shown in the figure).

Figure 6 shows the variation of conductive temperature ($\phi$) with distance ($x$) for various values of the damping coefficient keeping $R_M = 2.0$ for both type of temperature ($\beta_0 = 0.0, \beta_0 = 0.1$) in case of 3P model. The figure depicts that conductive temperature increases with the increase of distance and finally disappears for all the values of damping coefficient. But as the damping coefficient increases, the rate of decay decreases in case of both types of temperature. The figure also depicts that for $\beta_0 = 0.1$, the rate of decay of conductive temperature is slower than that for $\beta_0 = 0.0$.

Figure 7 gives the variation of thermodynamic temperature ($T$) with distance ($x$) for 2TGNIII and 2T3P model in the presence of magnetic field ($R_M = 2.0$) and in the absence of magnetic field ($R_M = 1.0$) for two-type temperature ($\beta_0 = 0.1$). Here similar
behavior of thermodynamic temperature is observed as it is for conductive temperature.

Figure 8 exhibits the space variation of thermodynamic temperature ($T$) in the context of one-temperature and two-temperature generalized thermoelasticity in the presence of magnetic field ($R_m = 2.0$) for the different values of damping coefficient $\kappa_0$ taking 3P model and it is noticed that thermodynamic temperature increases when damping coefficient increases.

Figure 9 depicts the variation of induced magnetic field ($h$) versus $x$ for the value of non-dimensional two-temperature parameter $\beta_0 = 0.1$ in case of 2TGNIII and 2T3P model. As seen from the figure, $h$ is compressive, its magnitude gets decreased with $x$ and finally disappears for $R_M = 1.0$ as well as for $R_M = 2.0$.

Figure 10 depicts the variation of induced electric field ($E$) versus distance ($x$) keep-
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\( \delta = 0.1, c_T = 0.1, \beta_s = 0.0168 \)

Fig. 9 Variation of induced magnetic field \( h \) with distance \( x \) for time \( t = 0.1 \)

\( \beta_r = 0.1, C_r = 0.1, \psi = 0.0168 \)

Figure 10. Variation of induced electric field \( E \) with distance \( x \) for time \( t = 0.1 \)
ing $\kappa_0 = 1.0$ for both the models (2TGNIII and 2T3P model) with respect to two-type temperature ($\beta_0 = 0.1$). It is seen that $E$ changes its nature from progressive to compressive after certain $x$ and it disappears for large $x$.

Figure 11 is plotted to show the variation of induced electric field ($E$) with respect to distance ($x$) for various values of damping coefficient in case of one-type temperature ($\beta_0 = 0.0$) and two-type temperature ($\beta_0 = 0.1$) for three-phase-lag model in case of WMF ($R_M = 2.0$). Here also same behavior of $E$ is seen as in Figure 10.

In Figures 1, 4, 5, 7, 9 and 10, the results obtained for 2TGNIII model agree with those obtained in sixth chapter.