Chapter 8

Study of Finite Thermal Waves in a Magneto-thermoelastic Rotating Medium

8.1 Introduction

In view of the fact that most of the large bodies like the earth, the moon and other planets have an angular velocity as well as earth itself behaves like a huge magnet, it is more important to study the propagation of thermoelastic waves in a rotating medium under the influence of a magnetic field. So the attempts are being made to study the propagation of finite thermoelastic waves in an infinite elastic medium rotating with an angular velocity. Schenberg and Censor [194] have discussed the propagation of plane harmonic elastic waves in a rotating elastic medium and investigated that the rotation causes the medium to be dispersive. Sinha and Bera [205] have studied the effect of rotation and relaxation time on the propagation of plane waves in generalized thermoelasticity following L-S theory and applying the eigenvalue approach of Das et al. [50]. Baksi et al. [20] have considered a two-dimensional problem of generalized thermoelasticity in the context of L-S theory to study the effect of rotation and relaxation time by...
applying eigenvalue approach of Das et al. [50]. Abd-Alla and Bayones [1] have investigated the influence of rotation and initial stress on the propagation of Rayleigh waves in a homogeneous isotropic, generalized thermoelastic body, subject to the boundary conditions that the outer surface is traction free. Roychoudhuri and Banerjee [191] have studied the propagation of magnetoelastic plane waves in rotating media in thermoelasticity (GNII Model). Kumar and Kansal [115] have showed the effect of rotation on Rayleigh-Lamb waves in an isotropic generalized thermoelastic diffusive plate and Othman and Song [162] have discussed the reflection of magneto-thermoelastic waves in a rotating elastic half-space in generalized thermoelasticity under three theories. Othman [161] has also discussed the propagation of electromagneto thermoelastic disturbances produced by a thermal shock in a perfectly conducting elastic half space when the entire elastic medium is rotating with a uniform angular velocity. Singh and Tomar [204] have studied propagation of plane waves in a rotating thermoelastic solid with voids. Control of rotating beam vibrations by piezoelectric materials has been discussed by Al-Qahtani and Sunar [9]. Nayak and Saha [149] have studied the analysis and design of rotating disks of functional materials of varying thickness.

However, this chapter deals with a problem of wave propagation in an unbounded rotating elastic medium in the context of generalized theory of thermoelasticity for GNII and GNIII model under the influence of magnetic field. Although this is a one-dimensional problem under the influence of a periodically varying heat source distributed over a plane area in a homogeneous, isotropic and unbounded medium, it involves two displacement components due to rotation. The governing equations are expressed in a Laplace-Fourier transform domain. The solutions in Laplace transform domain are obtained by taking the Fourier inversion, which is carried out by using residual calculus, where the poles of the integrand are obtained numerically in the complex domain by using Laguerre's method. The numerical inversion of the Laplace transform is done by using a method based on Fourier series expansion technique (Honig and Hirdes[82]). The
results obtained theoretically have been computed numerically and presented graphically for a copper like material to show the effect of rotation, presence of magnetic field and the damping coefficient on the physical quantities.

8.2 Formulation of the Problem

Let us consider a homogeneous isotropic thermoelastic medium occupying the whole space $-\infty < x < \infty$ whose state depends only on the space variable $x$ and time variable $t$ and which is rotating with an uniform angular velocity $\Omega = \Omega \mathbf{n}$, where $\mathbf{n}$ is the unit vector in the direction of the axis of rotation. A magnetic field of strength $H$ acts in the direction of the axis of rotation. If we take the coordinate axis fixed in the rotating medium the displacement equation of motion in the rotating frame of reference has two additional terms - the centripetal acceleration $\Omega \times (\Omega \times \mathbf{u})$ due to the time varying motion only and the coriolis acceleration $2\Omega \times \dot{\mathbf{u}}$, where $\mathbf{u}$ is the dynamic displacement vector measured from a steady state deformed position and the deformation is assumed to be very small. We shall consider the propagation of plane waves in the context of GNIII model and GNII model in the presence of periodically varying heat sources distributed over a plane area within the medium. Since we are dealing with an isotropic thermoelastic medium, without any loss of generality, we may consider waves propagating in the $x$-direction and all the field variables are supposed to be functions of $x$ and $t$ only, i.e., we may assume that $u = u(x,t)$, $v = v(x,t)$, $w = w(x,t)$ and $T = T(x,t)$ (Roychoudhuri [182]) where $T(x,t)$ denotes the temperature field. In order to examine the effect of rotation on the propagation of plane waves we set $\Omega = (0,0,\Omega)$, where $\Omega$ is a constant. In view of the above assumptions our problem will involve two displacement components $u(x,t)$ and $v(x,t)$ (Sinha and Bera [205]).

The electromagnetic field is governed by Maxwell's equations (in the absence of the
displacement current and charge density) as

\[ \text{curl } \mathbf{H} = \mathbf{J}, \quad (8.2.1) \]
\[ \text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (8.2.2) \]
\[ \text{div } \mathbf{B} = 0, \quad (8.2.3) \]
\[ \mathbf{B} = \mu_0 \mathbf{H}. \quad (8.2.4) \]

The generalized Ohm's law in deformable continua is

\[ \mathbf{J} = \kappa (\mathbf{E} + \mathbf{u} \times \mathbf{B}), \quad (8.2.5) \]

where the small effect of a temperature gradient on the conduction current \( \mathbf{J} \) is neglected.

Now the displacement equations of motion in presence of body force \( \mathbf{F} \) and heat conduction equation in the context of generalized thermoelasticity developed by Green-Naghdi\[71, 72\] are given as follows:

\[ (\lambda + 2\mu) \frac{\partial^2 \mathbf{u}}{\partial x^2} - \gamma \frac{\partial T}{\partial x} + F_x = \rho [\ddot{u} - \Omega^2 u - 2\Omega \dot{v}] \quad (8.2.6) \]
\[ \mu \frac{\partial^2 \mathbf{v}}{\partial x^2} + F_y = \rho [\ddot{v} - \Omega^2 v + 2\Omega \dot{u}] \quad (8.2.7) \]

where

\[ \mathbf{F} = (\mathbf{J} \times \mathbf{B}), \quad \mathbf{F} = (F_x, F_y, F_z) \]

and

\[ K \frac{\partial^2 T}{\partial x^2 \partial t} + K^* \frac{\partial^2 T}{\partial x^2} + \rho \dot{Q} = \rho c_v \frac{\partial^2 T}{\partial t^2} + \gamma T_0 \frac{\partial^3 u}{\partial t^2 \partial x} \quad (8.2.8) \]

where \( T_0 \) is the reference temperature, \( K \) is thermal conductivity and \( K^* \) is a material constant.
Constitutive equation in the present case becomes

\[ \sigma_{xx} = (\lambda + 2\mu) \frac{\partial u}{\partial x} \gamma T \]  

(8.2.9)

We set \( \mathbf{H} = \mathbf{H}_0 + \mathbf{h} \), where \( \mathbf{H}_0 = (0, 0, H_0) \). The perturbed magnetic field \( \mathbf{h} \) is so small that the product of \( \mathbf{h} \) and \( \mathbf{u} \) and their derivatives can be neglected for linearization of the field equations.

Eq. (8.2.1) gives

\[ J_x = 0, \quad J_y = -\frac{\partial H_z}{\partial x}, \quad J_z = \frac{\partial H_y}{\partial x}, \]  

(8.2.10)

where \( \mathbf{J} = (J_x, J_y, J_z), \mathbf{H} = (H_x, H_y, H_z) \).

Eqs. (8.2.2) and (8.2.4) yield

\[ \frac{\partial H_z}{\partial t} = 0, \quad \frac{\partial E_z}{\partial x} = \mu_e \frac{\partial H_y}{\partial t}, \quad \frac{\partial E_y}{\partial x} = -\mu_e \frac{\partial H_z}{\partial t}, \quad \mathbf{E} = (E_x, E_y, E_z). \]  

(8.2.11)

Eq. (8.2.3) gives \( \frac{\partial h_x}{\partial x} = 0 \) which implies that \( h_x = 0 \), since initially no perturbed field is applied along the \( x \)-axis.

The modified Ohm’s law gives

\[ J_x = \kappa \left[ E_x + \left\{ \mu_e H_z \left( \frac{\partial u}{\partial t} + \Omega u \right) \right\} \right], \]  

\[ J_y = \kappa \left[ E_y + \left\{ \mu_e H_z \left( \frac{\partial u}{\partial t} + \Omega v \right) \right\} \right], \]  

(8.2.12)

\[ J_z = \kappa \left[ E_z + \left\{ -\mu_e H_x \left( \frac{\partial u}{\partial t} + \Omega u \right) + \mu_e H_y \left( \frac{\partial u}{\partial t} - \Omega v \right) \right\} \right]. \]

By eliminating \( J_x, J_y, J_z \) and using Eqs. (8.2.10), (8.2.11) and (8.2.12), we get

\[ \frac{\partial H_z}{\partial t} = \nu_H \frac{\partial^2 H_z}{\partial x^2} - \frac{\partial}{\partial x} \left[ H_z \left( \frac{\partial u}{\partial t} - \Omega v \right) \right], \]  

(8.2.13)
Study of Finite Thermal Waves in a Magneto-thermoelastic Rotating Medium

\[
\frac{\partial H_y}{\partial t} = \nu_H \frac{\partial^2 H_y}{\partial x^2} + \frac{\partial}{\partial x} \left[ H_x \left( \frac{\partial v}{\partial t} + \Omega u \right) \right] - \frac{\partial}{\partial x} \left[ H_y \left( \frac{\partial u}{\partial t} - \Omega v \right) \right] 
\]

(8.2.14)

where \( \nu_H = (\kappa \mu_e)^{-1} \) is called the magnetic viscosity.

Eq. (8.2.6) reduces to

\[
(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} - \mu_e \frac{\partial}{\partial x} \left\{ \frac{1}{2} (H_y^2 + H_z^2) \right\} = \rho [\ddot{u} - \Omega^2 v - 2\Omega \dot{w}] 
\]

(8.2.15)

and Eq. (8.2.7) can be written as

\[
\mu \frac{\partial^2 v}{\partial x^2} + \mu_e H_z \frac{\partial H_y}{\partial x} = \rho [\ddot{v} - \Omega^2 v + 2\Omega \dot{w}] 
\]

(8.2.16)

We set \( H_z = H_0 + h_z \), where the perturbed magnetic field \( h_z \) is small compared to the initial magnetic field \( H_0 \).

Then from Eqs. (8.2.13)-(8.2.16) after linearization, we get

\[
\frac{\partial h_z}{\partial t} = \nu_H \frac{\partial^2 h_z}{\partial x^2} - H_0 \frac{\partial^2 u}{\partial x^2} + H_0 \Omega \frac{\partial v}{\partial x}, 
\]

(8.2.17)

\[
\frac{\partial h_y}{\partial t} = \nu_H \frac{\partial^2 h_y}{\partial x^2}, 
\]

(8.2.18)

\[
(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} - \mu_e H_0 \frac{\partial h_z}{\partial x} = \rho \left[ \frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega \frac{\partial v}{\partial t} \right] 
\]

(8.2.19)

and

\[
\mu \frac{\partial^2 v}{\partial x^2} = \rho \left[ \frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega \frac{\partial u}{\partial t} \right]. 
\]

(8.2.20)

Now for a perfect electrical conductor, \( \nu_H \rightarrow 0 \) as \( \kappa \rightarrow \infty \). Eq. (8.2.17) leads to

\[
\frac{\partial h_z}{\partial t} = H_0 \left[ - \frac{\partial^2 u}{\partial x \partial t} + \Omega \frac{\partial v}{\partial x} \right], 
\]

since there is no perturbation at \( \infty \). Then Eqs. (8.2.19) and (8.2.20) reduce to

\[
c_1^2 (1 + R_H) \frac{\partial^2 u}{\partial x^2 \partial t} - \gamma \frac{\partial^2 T}{\partial x \partial t} - c_1^2 \Omega R_H \frac{\partial^2 v}{\partial t^2} = \frac{\partial^2 u}{\partial t^2} - \Omega^2 \frac{\partial u}{\partial t} - 2\Omega \frac{\partial^2 v}{\partial t^2} 
\]

(8.2.21)

\[
c_1^2 \frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega \frac{\partial u}{\partial t} 
\]

(8.2.22)
Study of Finite Thermal Waves in a Magneto-thermoelastic Rotating Medium

where \( R_H = \frac{\mu_0 H_0^2}{\rho c_1^2} = \frac{v_A^2}{c_1^2}, \) \( c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \) \( c_4 = \sqrt{\frac{\mu}{\rho}} \) and \( v_A = \sqrt{\frac{\mu_0 H_0}{\rho}} \) is the Alfven wave velocity of the medium. The coefficient \( R_H \) represents the effect of an external magnetic field in the thermoelastic processes proceeding in the body.

We introduce the following dimensionless quantities:

\[
x' = \frac{x}{l}, \quad u' = \frac{\lambda + 2\mu}{\gamma T_0 l} u, \quad v' = \frac{\lambda + 2\mu}{\gamma T_0 l} v, \quad t' = \frac{c_1 t}{l}, \quad T' = \frac{T}{T_0}, \quad \Omega' = \frac{l}{c_1},
\]

\[
\sigma_{xx}' = \frac{\sigma_{xx}}{T_0}, \quad 1 + R_H = R_M^2
\]

where \( l = \) some standard length and \( c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}} \) is the standard speed. Now omitting primes, Eqs. (8.2.21), (8.2.22), (8.2.8) and (8.2.9) can be re-written in dimensionless form as

\[
\frac{R_M^2}{\gamma} \frac{\partial^3 u}{\partial x^3} - \frac{\partial^3 T}{\partial x^3} - \Omega (R_M^2 - 1) \frac{\partial^3 v}{\partial x^2 t} - \Omega^2 \frac{\partial^3 u}{\partial x^3} - 2\Omega \frac{\partial^3 v}{\partial x \partial t^2},
\]

(8.2.23)

\[
\frac{c_T^2}{\gamma} \frac{\partial^3 v}{\partial x^2 t} = \frac{\partial^3 v}{\partial x^3} - \Omega^2 \frac{\partial^2 u}{\partial x \partial t^2} + 2\Omega \frac{\partial^3 v}{\partial x \partial t^2},
\]

(8.2.24)

\[
\sigma_{xx} = \frac{\partial u}{\partial x} - T,
\]

(8.2.25)

where

\[
\sigma_T = \frac{K^*}{\rho c_1^2}, \quad \epsilon_T = \frac{\gamma T_0}{(\lambda + 2\mu)\rho c_1}, \quad \kappa_0 = \frac{K}{\rho c_1 c_1 l}, \quad Q_0 = \frac{l}{T_0 c_0 c_1} \frac{\partial Q}{\partial t}, \quad c_T^2 = \frac{c_T}{\gamma} = \frac{\mu}{\rho c_1^2}.
\]

We assume that the medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have

\[
u(x, 0) = \bar{u}(x, 0) = v(x, 0) = \bar{v}(x, 0) = T(x, 0) = \bar{T}(x, 0) = 0.
\]

(8.2.27)
Varying Heat Source

Let us assume that the heat source is distributed over the plane $x = 0$ in the following form:

$$Q_0 = Q_0^0 \delta(x) \sin \frac{\pi t}{\tau}, \quad 0 \leq t \leq \tau$$

$$= 0. \quad t > \tau$$

(8.2.28)

8.3 Method of Solution

Let us define the Laplace-Fourier double transform of the function $g(x, t)$ by

$$\tilde{g}(x, p) = \int_0^\infty g(x, t) e^{-pt} dt, \quad Re(p) > 0$$

$$\tilde{g}(\alpha, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{g}(x, p) e^{i\alpha x} dx .$$

Applying the Laplace-Fourier double integral transform to Eqs. (8.2.23)-(8.2.26) and using the relation in Eq. (8.2.27) we get

$$(R_M^2 \alpha^2 + p^2 - \Omega^2) \tilde{u}(\alpha, p) - \left[(R_U^2 - 1) \frac{\alpha^2 \Omega}{p} + 2 \Omega p\right] \tilde{v}(\alpha, p) - i\alpha \tilde{T}(\alpha, p) = 0, \quad (8.3.1)$$

$$2\Omega p \tilde{u}(\alpha, p) + (p^2 - \Omega^2 + \alpha^2 \alpha^2) \tilde{v}(\alpha, p) = 0, \quad (8.3.2)$$

$$i\varepsilon T \alpha^2 \tilde{u}(\alpha, p) - (\Omega^2 \alpha^2 + p^2 + \alpha^2 \alpha^2) \tilde{v}(\alpha, p) + \tilde{\dot{Q}}_0 = 0, \quad (8.3.3)$$

$$\tilde{\dot{g}}_{xx}(\alpha, p) = -i\alpha \tilde{u}(\alpha, p) - \tilde{\dot{T}}(\alpha, p) . \quad (8.3.4)$$

Solving Eqs. (8.3.1), (8.3.2) and (8.3.3) for $\tilde{u}(\alpha, p)$, $\tilde{v}(\alpha, p)$ and $\tilde{\dot{T}}(\alpha, p)$, we get

$$\tilde{u}(\alpha, p) = \frac{i\alpha \tilde{Q}_0 (\Omega^2 - p^2 - \alpha^2 c_2^2)}{M(\alpha)}, \quad (8.3.5)$$

$$\tilde{v}(\alpha, p) = \frac{2\alpha p \Omega \tilde{Q}_0}{M(\alpha)}, \quad (8.3.6)$$
Study of Finite Thermal Waves in a Magneto-thermoelastic Rotating Medium

\[ \hat{T}(\alpha, p) = -\frac{\hat{Q}_0[R^2 M_2 \alpha^2(\Omega^2 + p^2 + \alpha^2 c_2^2) + (\Omega^2 + p^2)^2 - \alpha^2\{(\Omega^2 - p^2)c_2^2 + 2\Omega^2\}]}{M(\alpha)} \]  

(8.3.7)

where

\[
M(\alpha) = -c_2^2 R_M^2 (c_2^2 + p\kappa_0)\alpha^6 - [(c_2^2 + p\kappa_0)(R^2_M(\Omega^2 + p^2) - c_2^2(\Omega^2 - p^2) - 2\Omega^2) + c_2^2 p^2 R_M^2(\Omega^2 + p^2)]\alpha^4 + [(R^2_M + \epsilon_T)(\Omega^2 - p^2)p^2 - (c_2^2 + p\kappa_0)(\Omega^2 + p^2)^2 + \Omega^2 p^2(c_2^2 + 2(1 - R^2_M))]\alpha^2 - p^2(\Omega^2 + p^2)^2 \]

= -c_2^2 R_M^2 (c_2^2 + p\kappa_0)(\alpha - \alpha_1)(\alpha - \alpha_2)(\alpha - \alpha_3)(\alpha - \alpha_4)(\alpha - \alpha_5)(\alpha - \alpha_6)

(8.3.8)

Now the expression for the stress in the Laplace-Fourier transform domain can be obtained from Eq. (8.3.4) using Eqs. (8.3.5) and (8.3.7)

\[ \hat{\sigma}_{xx}(x, p) = \frac{-\hat{Q}_0}{M(\alpha)}[(1 - R^2_M)\{\Omega^2 + p^2 + \alpha^2 c_2^2\}\alpha^4 - (\Omega^2 + p^2)^2 + (\Omega^2 - p^2)\alpha^2 c_2^2] \]

(8.3.9)

Thus, the solution for the displacements, temperature and stress in the Laplace transform domain can be obtained in terms of the following four integrals:

\[ \hat{u}(x, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} i\alpha \hat{Q}_0(\Omega^2 - p^2 - \alpha^2 c_2^2) e^{-i\alpha x} \, d\alpha, \]  

(8.3.10)

\[ \hat{\theta}(x, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 2i\alpha p\Omega \hat{Q}_0 e^{-i\alpha x} \, d\alpha, \]  

(8.3.11)

\[ \hat{\sigma}_{xx}(x, p) = \frac{-\hat{Q}_0[R^2 M_2 \alpha^2(\Omega^2 + p^2 + \alpha^2 c_2^2) + (\Omega^2 + p^2)^2]}{-\alpha^2\{(\Omega^2 - p^2)c_2^2 + 2\Omega^2\}} e^{-i\alpha x} \, d\alpha, \]

(8.3.12)
\[ 
\sigma_{zz}(x, p) = -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{-(\Omega^2 + p^2)^2 + (\Omega^2 - p^2)^2 c_2^2}{M(\alpha)} \, e^{-i\alpha x} \, d\alpha. \quad (8.3.13) 
\]

where

\[ 
\dot{\hat{Q}}_0 = \frac{Q_0^* \pi (1 + e^{-\nu r})}{\sqrt{2\pi}(\pi^2 + p^2 r^2)}. \quad (8.3.14) 
\]

Thus, the expressions for the displacements, temperature and stress in the Laplace transform domain take the following form:

\[ 
\bar{u}(x, p) = \int_{-\infty}^{\infty} \frac{i\alpha Q_0^* \tau (1 + e^{-\nu r}) (\Omega^2 - p^2 - \alpha^2 c_2^2)}{2(\pi^2 + p^2 r^2) M(\alpha)} \, e^{-i\alpha x} \, d\alpha, \quad (8.3.15) 
\]

\[ 
\bar{v}(x, p) = \int_{-\infty}^{\infty} \frac{i\alpha Q_0^* \tau (1 + e^{-\nu r})}{(\pi^2 + p^2 r^2) M(\alpha)} \, e^{-i\alpha x} \, d\alpha, \quad (8.3.16) 
\]

\[ 
\bar{T}(x, p) = -\int_{-\infty}^{\infty} \frac{Q_0^* \tau (1 + e^{-\nu r}) (\Omega^2 + p^2 + \alpha^2 c_2^2)}{2(\pi^2 + p^2 r^2) M(\alpha)} \, e^{-i\alpha x} \, d\alpha, \quad (8.3.17) 
\]

\[ 
\bar{\sigma}_{zz}(x, p) = -\int_{-\infty}^{\infty} \frac{-(\Omega^2 + p^2)^2 + (\Omega^2 - p^2)^2 c_2^2}{2(\pi^2 + p^2 r^2) M(\alpha)} \, e^{-i\alpha x} \, d\alpha. \quad (8.3.18) 
\]
Applying contour integration to the Eqs. (8.3.15)-(8.3.18) we obtain

\[
\tilde{u}(x,p) = \frac{iQ_0^*\pi \tau (1 + e^{-\nu \tau})}{R_M c_2^2(c_T^2 + p\kappa_0)(\pi^2 + p^2r^2)} \sum_{j=1}^{6} A_j\alpha_j(\Omega^2 - p^2 - \alpha_j^2 c_2^2)e^{-i\alpha_j x} \quad \text{for } x > 0
\]

\[
= -\frac{iQ_0^*\pi \tau (1 + e^{-\nu \tau})}{R_M c_2^2(c_T^2 + p\kappa_0)(\pi^2 + p^2r^2)} \sum_{j=1}^{6} A_j\alpha_j(\Omega^2 - p^2 - \alpha_j^2 c_2^2)e^{-i\alpha_j x} \quad \text{for } x < 0
\]

\[
\tilde{v}(x,p) = \frac{2i\pi \tau Q_0^* (1 + e^{-\nu \tau}) p\Omega}{R_M c_2^2(c_T^2 + p\kappa_0)(\pi^2 + p^2r^2)} \sum_{j=1}^{6} A_j\alpha_j e^{-i\alpha_j x} \quad \text{for } x > 0
\]

\[
= -\frac{2i\pi \tau Q_0^* (1 + e^{-\nu \tau}) p\Omega}{R_M c_2^2(c_T^2 + p\kappa_0)(\pi^2 + p^2r^2)} \sum_{j=1}^{6} A_j\alpha_j e^{-i\alpha_j x} \quad \text{for } x < 0
\]

\[
\tilde{T}(x,p) = -\frac{iQ_0^*\pi \tau (1 + e^{-\nu \tau})}{R_M c_2^2(c_T^2 + p\kappa_0)(\pi^2 + p^2r^2)} \sum_{j=1}^{6} A_j [R_M^2 \alpha_j^2 (\Omega^2 + p^2 + \alpha_j^2 c_2^2) + (\Omega^2 + p^2)^2 - \alpha_j^2 (\Omega^2 - p^2) c_2^2 + 2\Omega^2]e^{-i\alpha_j x} \quad \text{for } x > 0
\]

\[
= \frac{iQ_0^*\pi \tau (1 + e^{-\nu \tau})}{R_M c_2^2(c_T^2 + p\kappa_0)(\pi^2 + p^2r^2)} \sum_{j=1}^{6} A_j [R_M^2 \alpha_j^2 (\Omega^2 + p^2 + \alpha_j^2 c_2^2) + (\Omega^2 + p^2)^2 - \alpha_j^2 (\Omega^2 - p^2) c_2^2 + 2\Omega^2]e^{-i\alpha_j x} \quad \text{for } x < 0
\]
\[
\varphi_{xx}(x,p) = -\frac{iQ_0^2\pi r(1 + e^{-pr})}{R_M^2\epsilon_{2}(c_T^2 + \rho\kappa_0)(\pi^2 + p^2\pi^2)} \sum_{j=1}^{6} A_j[(1 - R_M^2)(\Omega^2 + p^2 + \alpha_j^2\epsilon_{2})\alpha_j^2]
\]

\[
- (\Omega^2 + p^2)^2 + (\Omega^2 - p^2)^2 \alpha_j^2\epsilon_{2}]e^{-i\alpha_j x}
\]

for \(x > 0\)

\[
= \frac{iQ_0^2\pi r(1 + e^{-pr})}{R_M^2\epsilon_{2}(c_T^2 + \rho\kappa_0)(\pi^2 + p^2\pi^2)} \sum_{j=1}^{6} A_j[(1 - R_M^2)(\Omega^2 + p^2 + \alpha_j^2\epsilon_{2})\alpha_j^2]
\]

\[
- (\Omega^2 + p^2)^2 + (\Omega^2 - p^2)^2 \alpha_j^2\epsilon_{2}]e^{-i\alpha_j x}
\]

for \(x < 0\)

\[ (8.3.22) \]

where \(A_j\)'s are given by

\[
A_j = \prod_{n \neq j}^{6} \frac{1}{(\alpha_j - \alpha_n)} \quad (8.3.23)
\]

### 8.4 Numerical Results and Discussions

To get the solutions for the two thermal displacements, temperature and stress in the space-time domain, we have to apply the Laplace inversion formula to the Eqs. (8.3.19)-(8.3.22), respectively. This has been done numerically using a method based on the Fourier series expansion technique (see Appendix). To get the roots of the polynomial \(M(\alpha)\) in the complex domain, we have used Laguerre's method. The numerical code has been prepared using Fortran 77 programming language. For computational purpose, a copper-like material with a material constant (Roychoudhuri and Dutta[187]) has been taken into consideration.

\[
e_T = 0.0168, \quad \lambda = 1.387 \times 10^{11} \text{ Nm}^{-2}, \quad \mu = 0.448 \times 10^{11} \text{ Nm}^{-2},
\]

\[
\alpha_t = 16.7 \times 10^{-6} \text{ K}^{-1}
\]
Also, we have taken $Q = 1$, $\tau = 1$, $c_2 = 0.4$, $c_T = 2$ and to show the effect of rotation, we have considered $\Omega = 0$ and 8. As it is expected when $\Omega = 0$, the displacement $v$ will be equal to zero and hence the graph corresponding to $v$ has not been available for comparison.

Figure 1 depicts the variation of the thermal displacement $u$ versus distance $x$ to show the effect of rotation for two different theories (GNII and GNIII) in absence of magnetic field (WOMF) by taking $R_M = 1.0$ as well as in presence of magnetic field (WMF) by taking $R_M = 2.0$. It is observed that in absence of magnetic field the displacement increases in the range $0.0 \leq x \leq 0.2$ for non-rotating medium ($\Omega = 0.0$) and in the range $0.0 \leq x \leq 0.3$ for rotating medium ($\Omega = 8.0$) in case of GNII model ($\kappa_0 = 0.0$) and in the range $0.0 \leq x \leq 0.2$ both for $\Omega = 0.0$ and $\Omega = 8.0$ in case of GNIII model ($\kappa_0 = 1.2$). After these ranges it approaches to zero values which is quite plausible since the periodic heat source is given on the boundary $x = 0$. Here the important thing is that
the magnitude of \( u \) is large for non-rotating medium in comparison to rotating medium for both the models GNII and GNIII taking \( R_M = 1.0 \). Moreover for \( \Omega = 0.0, R_M = 1.0 \) and \( \kappa_0 = 0.0 \), the result complies with that of Roychoudhuri and Dutta [187] where they have used analytical method. Also for \( R_M = 1.0 \), the rate of decay is slower in case of \( \kappa_0 = 1.2 \) than the case when \( \kappa_0 = 0.0 \) taking \( \Omega = 0.0, 8.0 \). This result agrees with that of Banik et al. [22] for non-rotating medium (\( \Omega = 0.0 \)). From this figure it can also be observed that the displacement shows the same qualitative behavior as before in presence of magnetic field for both GNII and GNIII models by taking \( \Omega = 0.0 \) and 8.0. Here again for WMF, the magnitude of the displacement is large for \( \Omega = 0.0 \) in comparison to \( \Omega = 8.0 \) both for GNII and GNIII models just like the case of WOMF.

Figure 2 depicts the variation of the displacement \( u \) with distance \( x \) taking \( R_M = 1, 2, 3, 4 \) and keeping the damping coefficient \( \kappa_0 = 1.2 \) (GNIII model) for rotating medium (\( \Omega = 8.0 \)). The effect of magnetic field on displacement \( u \) is such that with the increase
of magnetic field, the magnitude and the rate of decay of the displacement decrease. From Figure 1, we can observe the similar qualitative behavior of the displacement for GNII model ($\kappa_0 = 0.0$) in rotating medium ($\Omega = 8.0$) taking $R_M = 1.0$ and $R_M = 2.0$.

Figure 3 is plotted to show the variation of the displacement $u$ with distance $x$ for $R_M = 2.0$ and $\kappa_0 = 0.0, 0.6, 1.2, 1.8$ taking $\Omega = 8.0$. This figure depicts the effect of damping coefficient on the displacement. Now it is observed that as the damping coefficient increases the rate of decay of the displacement becomes slow.

Figure 4 represents the variation of the displacement $u$ against time $t$ for $x = 0.3$. Here we have considered GNIII model in case of WMF ($R_M = 2.0$). It is observed that for non-rotating medium, the displacement increases first and then reaches to a steady state with the increase of time $t$ whereas for rotating medium oscillatory behavior of
the displacement is observed but the magnitude of the peak of oscillation decreases with time.

Figure 5 depicts the variation of the displacement $v$ versus distance $x$ for rotating medium ($\Omega = 8.0$). Here the displacement $v$ is compressive in nature for all the cases. It is observed that the displacement $v$ increases in magnitude in the range $0.0 \leq x \leq 0.15$ in case of both GNII ($\kappa_0 = 0.0$) and GNIll ($\kappa_0 = 1.2$) models and after this, $v$ begins to decrease and ultimately approaches to zero for WOMF. Similarly in case of WMF, $v$ increases in magnitude in the range $0.0 \leq x \leq 0.21$ in case of both GNII ($\kappa_0 = 0.0$) and GNIll ($\kappa_0 = 1.2$) models and after this, it begins to decrease and ultimately approaches to zero. Although the natures of $v$ are same for $\kappa_0 = 0.0$ and $\kappa_0 = 1.2$ but the rate of damping of $v$ increases as damping coefficient $\kappa_0$ increases for both the cases of WOMF and WMF.
Study of Finite Thermal Waves in a Magneto-thermoelastic Rotating Medium

Fig. 5 Variation of displacement $v$ with distance $x$

$\kappa_s=1.2, c_s=2.0, \xi_s=0.0168, t=0.4$

$-R_s=1.0$
$-R_s=2.0$
$-R_s=3.0$
$-R_s=4.0$

Fig. 6 Variation of displacement $v$ with distance $x$ for GN III model considering rotating medium

$\kappa_s=1.2, c_s=2.0, \xi_s=0.0168, t=0.4$

$-R_s=1.0$
$-R_s=2.0, \kappa_s=1.2, \Omega=8.0$
$-R_s=2.0, \kappa_s=0.0, \Omega=8.0$
$-R_s=2.0, \kappa_s=1.2, \Omega=8.0$

Fig. 6 Variation of displacement $v$ with distance $x$ for GN III model considering rotating medium

$-R_s=1.0$
$-R_s=2.0$
$-R_s=3.0$
$-R_s=4.0$
Figure 6 shows the variation of the displacement $v$ with distance $x$ taking $R_M = 1, 2, 3, 4$ and keeping $\kappa_0 = 1.2$ and $\Omega = 8.0$. The figure shows that the magnitude of the displacement $v$ decreases remaining compressive in nature with the increase of magnetic field. The same effect of magnetic field in rotating medium for GNU model can also be observed in Figure 5.

Figure 7 gives the variation of the displacement $v$ against distance $x$ for rotating medium in presence of magnetic field ($R_M = 2.0$) taking $\kappa_0 = 0.0, 0.6, 1.2$ and 1.8. The effect of damping coefficient on displacement $v$ is such that the rate of decay of the displacement $v$ becomes slow as the damping coefficient increases.

Figure 8 depicts the variation of temperature $T$ with distance $x$ for non-rotating ($\Omega = 0.0$) and rotating ($\Omega = 8.0$) medium in case of two different theories (GNU and GNIII) taking $R_M = 1.0$ and 2.0. Here, the temperature decreases with the increase of
x and finally goes to zero for all the cases. This figure shows that there is no such effect of rotation on temperature.

Figure 9 exhibits the space variation of temperature $T$ in rotating medium for various values of magnetic field where the energy is dissipating ($\kappa_0 = 1.2$). This figure depicts that there is no such effect of magnetic field on temperature for $\kappa_0 = 1.2$ and which is also true for GNII model (Fig. 9) since the magnetic field and the temperature field are independent to each other.

Figure 10 is plotted to show the variation of temperature $T$ versus distance $x$ for rotating medium in presence of magnetic field ($R_M = 2.0$) taking $\kappa_0 = 0.0, 0.6, 1.2$ and 1.8. It is observed from the figure that temperature decreases with the increase of distance and finally goes to zero for all the values of the damping coefficient but as the damping coefficient increases, the rate of decay decreases.
Study of Finite Thermal Waves in a Magneto-thermoelastic Rotating Medium

Fig. 9 Variation of temperature $T$ with distance $x$ for GN III model considering rotating medium

$$\kappa_t=1.2, c_p=2.0, t=0.0168, t=0.4$$

- $R_w=1.0$
- $R_w=2.0$
- $R_w=3.0$
- $R_w=4.0$

Fig. 10 Variation of temperature $T$ with distance $x$ considering rotating medium

$$R_w=2.0, c_p=2.0, \kappa_t=0.0168, t=0.4$$

- $\kappa_t=0.0$
- $\kappa_t=0.6$
- $\kappa_t=1.2$
- $\kappa_t=1.8$
Figure 11 exhibits the space variation of stress $\sigma_{xx}$ for non-rotating medium ($\Omega = 0.0$) and rotating medium ($\Omega = 8.0$) taking two different models (GNII and GNIII) and two different values of $R_M$ (1 and 2). It is observed that the stress is compressive first then begins to decrease in magnitude and finally becomes zero for all the cases and the magnitude is maximum near the boundary. Furthermore, magnitude of stress for rotating medium is larger than that of non-rotating medium for two different theories (GNII and GNIII) in case of both WOMF and WMF.

Figures 12 and 13 are plotted to show the variation of stress $\sigma_{xx}$ against distance $x$ for rotating medium taking various values of the magnetic field and the damping coefficient respectively. Figure 12 shows that by increasing magnetic field the magnitude of stress increases near the boundary and Figure 13 shows that by increasing the damping coefficient the magnitude of stress decreases near the boundary, but the rate of decay is reserved in nature. The effect of magnetic field on stress for GNII model and the effect
of damping coefficient on stress in absence of magnetic field for rotating medium are the same as they are in Figure 12 and Figure 13 respectively and these two results can be observed from Figure 11.

Figure 14 depicts the variation of stress $\sigma_{xx}$ with time $t$ for GNIII model in the presence of magnetic field ($R_M = 2.0$) taking $\Omega = 0.0$ and $8.0$. This figure shows that the magnitude of stress increases up to $t = 1.25$ for both $\Omega = 0.0$ and $\Omega = 8.0$ and after $t = 1.25$, it reaches to a constant value for non-rotating medium and shows oscillatory nature for rotating medium but the oscillatory nature vanishes after a considerable duration of time.

In Figures 1, 8 and 11, it can be observed that the results agree with that of Banik et al. [22] when $R_M = 1.0, \Omega = 0.0$ and $\kappa = 1.2$ and when $R_M = 1.0, \Omega = 0.0$ and $\kappa_0 = 0.0$, the result is confirmed by that ofRoychoudhuri and Dutta[187] in which the
Study of Finite Thermal Waves in a Magneto-thermoelastic Rotating Medium

Fig. 13 Variation of stress $\sigma_{xx}$ with distance $x$ considering rotating medium

Fig. 14 Variation of stress $\sigma_{xx}$ with time $t$ for GN III model at $x=0.3$
8.5 Conclusions

This chapter studied the magneto-thermoelastic interaction in a homogeneous isotropic and unbounded rotating medium due to the presence of periodically varying heat source in the context of linear theory of generalized thermoelasticity with energy dissipation and without energy dissipation. The analysis of the results permits some concluding remarks.

1. From the graph it is clear that the magnitude of the displacement $u$ is small for rotating medium than the case of non-rotating medium whereas for non-rotating medium there is no existence of the other displacement $v$. Now for rotating medium, magnitude of stress is larger than that of non-rotating medium.

2. For rotating medium, the displacement $u$ shows oscillatory nature with decreasing amplitude with respect to time and the stress also becomes oscillatory in nature after $t = 1.25$ which again vanishes with time. But for non-rotating medium displacement and stress never be oscillatory if they are plotted against $t$.

3. The damping coefficient has a significant effect on all the physical quantities and the magnetic field has also the effect on all the same except temperature for rotating medium. Now with the increase of damping coefficient the rates of decay of the displacements $u$ and $v$, temperature $T$ and stress $\sigma_{xx}$ decrease and with the increase of magnetic field the magnitudes of the two displacements decreases but the magnitude of stress increases. Magnetic field has no effect on temperature because magnetic field and temperature field are two independent fields.

4. The results obtained in this chapter agree with those of Banik et al. [22] when
both the rotation of the medium and magnetic field are absent but the dissipation of energy is present. Moreover, the solution of Roychoudhuri and Dutta [187] can also be derived from the present solution considering non-rotating medium in absence of both magnetic field and damping coefficient in which closed form solution of the problem has been derived.