CHAPTER 5

LOW-TEMPERATURE GALVANOMAGNETIC
TRANSPORT COEFFICIENTS OF TWO
DIMENSIONAL HOT ELECTRONS IN
GaAs QUANTUM WELLS
5.1 INTRODUCTION

The study of electron transport in semiconductors is immensely important for fundamental as well as applied research. Investigations of these transport phenomena focus light on the basic physics, i.e., the band structure and carrier kinetics of semiconductors, and the performance of the materials in modern solid state devives\[5.1-5.3\]. Recently, with the advent of ultra thin materials the potential of polar semiconductors has been further extended. The motivation of their production went up sharply by the introduction of the idea of low-dimensional systems\[5.4\] formed with two dissimilar semiconducting layers a few nanometers thick. They possess radically different properties from those of bulk semiconductors because they quantum mechanically restrict the degrees of freedom of the conduction electrons to two or one. This change in the effective dimensionality offers fascinating changes in electronic, magnetic, optical, and vibrational properties \[5.5\]. In this era of very large scale integration(VLSI) and superfast computers, the search for high-speed miniature devices has motivated the preparation of such structures. Two-dimensional(2D) electronic transport in polar semiconductor QW has been extensively studied in the absence of a magnetic field \[5.6,5.7\] and in the presence of quantizing magnetic field \[5.8\]. But the same studies for a high electric
field and a crossed low nonquantizing magnetic field are also very important.

In this work, galvanomagnetic transport coefficients of the 2D hot electrons in a square \( QW \) of GaAs are calculated for nonquantizing magnetic fields for lattice temperatures in the range of 4 to 15K using a heated Fermi-Dirac distribution function. Electron energy loss via screened deformation potential acoustic phonon scattering is considered. In electron momentum loss, background ionized impurity scattering is additionally incorporated.

5.2 LITERATURE SCAN:

Hot-electron conditions are usually established if the electric field is sufficiently high so as to cause a pronounced deviation from Ohm's law. The concept of hot electron is associated with a temperature of electron gas which is higher than that of the host lattice. Under hot-electron conditions the character of the electron transport may change radically from that under a low electric field. Different works on hot-electron transport have been reported in various review articles published from time to time[5.9 - 5.17].

Two-dimensional (2D) electronic transport in quantum wells (QWs) has been the subject of many recent investigations[5.18-5.21] because of its importance from physics and device application
points of view[5.22-5.24]. Investigations of the galvanomagnetic transport coefficients give an idea of the different scattering mechanisms controlling the carrier dynamics in the structures and the band details. Electronic transport has been extensively studied in the absence of a magnetic field [5.6,5.7,5.18-5.24] and in the presence of a quantizing magnetic field [5.8]. The magnetic field changes the trajectory of the carriers, thus modifying the transport coefficients. Under the application of a high magnetic field, energy spectrum is split into a set of Landau levels and gives oscillatory phenomena, namely, Shubnikov de Haas oscillations (SdH) and magnetophonon effects [5.25-5.27]. High magnetic field magnetoresistance in GaAs/Ga$_{1-x}$Al$_x$As heterojunction has been calculated by Leadly et al. [5.8] using the momentum balance equation, and they observed that their results are in good agreement with experimental data. Dietzel et al. [5.28] have studied the absorption of a phonon in the frequency range of 100GHz by a 2D electron gas, formed at the heterojunction interface of GaAs/Ga$_{1-x}$Al$_x$As in a quantizing magnetic field. Phonon-drag signal oscillating in phase with the SdH oscillations has been observed during their investigations. The magnetic field dependencies of the shallow donor impurity states in GaAs/GaAlAs have been studied by Kuh et al. [5.29].
Khmelnitskii et al.[5.30] have studied the properties of the diffusional motion in a strong magnetic field. They explained that the conductance fluctuation of the 2D electron gases in the region of well developed SdH oscillation was due to two possible phase memory breaking mechanisms. The oscillatory magnetisation of multiple QW's has been investigated by Oh et al.[5.31]. They explained the phenomena with the hybrid magneto-electric quantization of electron energies owing to the coupling of magnetic confining effects and band structure effects. It appears from the scan of the literature that the study of the transport properties of 2D electron gases are mostly confined to high magnetic fields. But the same studies for a high electric field and a cross low nonquantizing magnetic field are scarce in the literature. This chapter presents the calculations of the 2D hot electron galvanomagnetic transport coefficients, viz., the Hall mobility, the Hall-to-drift mobility ratio, and the magnetoresistance coefficient, for nonquantizing magnetic fields in a GaAs QW for lattice temperatures in the range of 4 to 15K considering a heated Fermi—Dirac distribution function and the prevalent scattering mechanisms. The dependencies of the galvanomagnetic transport coefficients on the magnetic field, the heating electric field, and the system parameters, namely, the
channel width, 2D carrier concentration and lattice temperature are investigated.

5.3: ANALYTICAL DETAILS:

A square QW of GaAs with infinite barrier height is considered.

The carriers are assumed here to occupy the lowest subband only. For the ranges of the system parameters, the lattice temperatures, and the heating electric fields used here, the difference between the lowest subband and next higher one is more than three times the average electron energy in the QW, justifying the foregoing assumptions.

In the rectangular cartesian coordinate system, we take the z-axis perpendicular to the interfacial planes so that the carriers are free to move parallel to the xy plane. The classical magnetic field $B$ and the heating electric field $F$ are assumed to act along the z and x axis, respectively. The electron-electron interaction is much stronger in a QW than in the bulk material due to the weakening of the ionized impurity scattering in the former by modulation doping [5.32]. An electron temperature is, therefore, established in a 2D electron gas system, as revealed in photoluminescence experiments [5.33]. Thus the carrier distribution function is written as:
\[ f(k) = f_0(E) + \frac{e^* \hbar F}{m} \left( \frac{\partial f_0}{\partial E} \right) \left[ k_x \xi_x(E) - \omega_B k_y \xi_y(E) \right] \]  \hspace{1cm} (5.1) 

where \( f_0(E) \) is the Fermi-Dirac distribution function at an electron temperature \( T_e \), \( e \) and \( m^* \) are, respectively, the charge and the effective mass of an electron. \( \hbar^* \) is Planck's constant divided by \( 2\pi \), and \( \omega_B \) is the cyclotron angular frequency. \( k_x \) and \( k_y \) denote, respectively, the \( x \) and \( y \) components of the 2D wave vector \( \vec{k} \) for the electron energy \( E \). \( \xi_x(E) \) and \( \xi_y(E) \) are the perturbation functions.

Here two-term Legendre polynomial expansion of \( f \) is used as the streaming of the distribution function is not important and the scattering processes are not mainly in the forward direction. The heating electric fields and the lattice temperatures are such that the electron temperature is always below 30K. Low temperature is preferred because then the electron mobility is enhanced due to the reduction of phonon scattering and the suppression of impurity scattering due to modulation doping [5.32]. Further more, at low temperatures, noise, and energy spread of electrons involved in the transport are reduced substantially [5.34].

Here we consider the scattering via screened deformation potential acoustic phonons and the background ionized impurities, as they are the major scattering mechanisms at low lattice
temperatures[5.35]. The contribution of the longitudinal optic (LO) phonon scattering is insignificant[5.36] over the temperature range of our interest here, and hence it is not included in the calculations. The effect of remote impurity scattering is not incorporated here as it can be substantially reduced by introducing thick undoped spacer layer[5.37]. The carriers lose energy through the deformation potential acoustic scattering for electron temperatures of less than 40K[5.36]. The piezoelectric scattering contributes an order of magnitude lower[5.36], and hence its contribution is not incorporated in the calculations. The electron temperature $T_e$ and the heating electric field are related through the energy balance equation.

$$\int E \left( \frac{\partial f}{\partial t} \right)_{\text{field}} \, d\mathbf{k} + \int E \left( \frac{\partial f}{\partial t} \right)_{\text{coll}} \, d\mathbf{k} = 0 \quad (5.2)$$

For the inelastic screened deformation potential acoustic scattering, $\left( \frac{\partial f}{\partial t} \right)_{\text{coll}}$ is obtained from the square of the relevant matrix element for the 2D systems [5.38, 5.39]. The procedure is similar to that used for the bulk material [5.2]. Thus $f_0 \left( E + \hbar \omega_q \right)$ is expanded in a Taylor series and the terms with powers of $\hbar \omega_q$ higher than the second are neglected, the acoustic phonon energy $\hbar \omega_q$ being much less than the carrier energy $E$. The integrations over the in-plane and the perpendicular components of the 3D phonon wave vector $\mathbf{Q}$ are carried out
numerically retaining only the highest order terms in $m \nu_L/h$, where $\nu_L$ is the longitudinal acoustic velocity.

The perturbation functions determined from the Boltzmann transport equation (BTE), are

$$\xi_x(E) = \frac{\tau}{1 + \omega_B^2 \tau^2} \tag{5.3}$$

$$\xi_y(E) = \frac{\tau^2}{1 + \omega_B^2 \tau^2} \tag{5.4}$$

Here $\tau$ is the momentum relaxation time and given by

$$\tau^{-1}(E) = \tau^{-1}_{ac}(E) + \tau^{-1}_{imp}(E) \tag{5.5}$$

where $\tau_{ac}(E)$ and $\tau_{imp}(E)$ are the momentum relaxation times for the deformation potential acoustic phonon and the background ionized impurity scattering, respectively. The detailed expressions for $\tau_{ac}$ and $\tau_{imp}$ can be found in chapter 2 (vide Eqs. 2.28 and 2.33, respectively). The Hall mobility ($\mu_H$), the magnetoresistance coefficient ($R_m$), and Hall ratio ($r_H$) are calculated from the following expressions:

$$\mu_H = \frac{\mu_{xx}(0) |\mu_{xy}|}{B(\mu_{xx}^2 + \mu_{xy}^2)} \tag{5.6}$$
\[ R_m = \frac{\mu_H B \mu_{xx}}{\mu_{xy}} - 1 \]  \hspace{1cm} (5.7)

and \[ r_H = \frac{\mu_H}{\mu_{xx}(0)} \]  \hspace{1cm} (5.8)

where \( \mu_{xx} = \frac{e}{\pi n_{2D} \hbar^2} \int_0^{\infty} \left( -\frac{\partial f}{\partial E} \right) \frac{\tau}{1 + \omega_B^2 \tau^2} E \ dE \)  \hspace{1cm} (5.9)

\[ \mu_{xx} = \frac{\omega_B}{\pi n_{2D} \hbar^2} \int_0^{\infty} \left( -\frac{\partial f}{\partial E} \right) \frac{\tau^2}{1 + \omega_B^2 \tau^2} E \ dE \]  \hspace{1cm} (5.10)

and \( \mu_{xx}(0) \) is the value of \( \mu_{xx} \) for \( B = 0 \).

It may be noted that the results for the 2D transport are different from those for the 3D transport in the bulk material due to the different density-of-states in the two cases.

5.4: RESULTS AND DISCUSSIONS

Numerical results are obtained for a GaAs QW at lattice temperatures of 4, 10, and 15K with the material parameters given in Chapter3 (vide Table 3.1). Although commonly accepted value of the acoustic deformation potential constant \( \omega_1 \) is 11.2x10\(^{-19}\) J we take here a higher value of 17.6x10\(^{-19}\) J obtained from the analysis of energy loss rates of 2D electrons[5.36]. A typical value of 6x10\(^{21}\) m\(^{-3}\) for the background ionized impurity concentration\( n_{bi} \) is used here in the calculations. The magnetic field \( B \) is nonquantizing for which \( \omega_B \) is much less than the
average energy of the electron (at least ten times less in the present case).

Fig. 5.1: Variation of Hall mobility ($\mu_H$) and magnetoresistance ($R_m$) with channel width ($L_z$) at lattice temperatures of 4, 10, and 15K for a typical magnetic field of 0.001T, electric field of 500V/m, and 2D carrier concentration ($n_{2D}$) of $6 \times 10^{15} \text{m}^{-2}$.

Figure 5.1 shows the variation of Hall mobility ($\mu_H$) and magnetoresistance ($R_m$) with channel width $L_z$ for a nonquantizing magnetic field $B = 0.001 \text{T}$, 2D carrier concentration $n_{2D} = 6 \times 10^{15} \text{m}^{-2}$ at lattice temperatures of 4, 10, and 15K for a typical electrical field of 500V/m. $\mu_H$ decreases and $R_m$ increases with the rise of $L_z$. This variation is linked with the fact that
as \( L_z \) increases, the phonon scattering becomes weaker\[5.40\] and the impurity scattering gets stronger\[5.41\]. \( \mu_H \) decreases by 25\% at 4K, 40\% at 10K, and 45\% at 15K when \( L_z \) is increased from 8 to 12nm. Whereas the enhancement of \( R_m \) in the range of the channel length used here is more than an order of magnitude in all the cases.

The Hall mobility \( (\mu_H) \) increases and magnetoresistance \( (R_m) \) decreases with increasing 2D carrier concentration as shown in the Fig.5.2. The enhanced screening of the scattering rates with
increasing \( n_{2D} \) and upward shift of the Fermi level \( (E_F) \) at higher \( n_{2D} \) explain these results [5.39, 5.40]. \( \mu_H \) is increased by a factor of about 1.6 at all the lattice temperatures of our investigations when \( n_{2D} \) is changed from 2 to \( 6 \times 10^{15} \text{ m}^{-2} \). But \( \mu_m \) is reduced by an order of magnitude in all the cases when \( n_{2D} \) is increased from 2 to \( 6 \times 10^{15} \text{ m}^{-2} \).

The Hall-to-drift mobility ratio \( (r_H) \) remains practically constant with the increase of the \( L_z \). Interestingly, \( \mu_m \) is more sensitive than \( \mu_H \) to the changes in system parameters like \( L_z \), \( n_{2D} \), and lattice temperature \( (T_L) \).

Figure 5.3 shows the plot of Hall mobility \( (\mu_H) \) and magnetoresistance \( (R_m) \) with the electric field at lattice temperatures of 4, 10, and 15K for a magnetic field of 0.001T. \( \mu_H \) decreases and \( R_m \) increases with the increasing electric field due to enhancement of the electron temperature \( (T_e) \). As the electric field is increased from 250V/m to 750V/m, \( R_m \) increases by about two orders of magnitude. Experimental results on \( R_m \) for 2D systems for classical magnetic fields are not yet available for a comparison with our calculations. A maximum change of 25% of \( \mu_H \) occurs at 4K as the electric field is changed from 250 to 750V/m. The Hall mobility is found to be higher at lower lattice temperature due to the reduction of phonon scattering and suppression of ionised impurity scattering. We find that for a
Fig. 5.3: Variation of Hall mobility ($\mu_H$) and magnetoresistance ($R_m$) with the electric field at lattice temperatures of 4, 10, and 15 K for a typical classical magnetic field of 0.001 T and channel width $L_z = 10$ nm. The other parameters are identical to Fig. 5.1.

Particular electric field, $\mu_H$ changes by about 40% when the lattice temperature is increased from 4 to 15 K. In order to substantiate our model, the Hall mobility is calculated in the Ohmic field region with the parameters for the experimental sample of Ref. 5.42 at $T_L = 10$ K. The calculated Hall mobility is found to agree closely with the experimental data [5.42].
The variation of the Hall-to-drift mobility ratio (also called the Hall ratio) with the electric field is shown in Fig. 5.4. The Hall ratio increases with the electric field owing to the combined scattering mechanisms, but it is not larger than 1.07. Thus the replacement of the Hall mobility by the drift mobility will not introduce great errors. The Hall ratio is very close to unity at low lattice temperatures due to the strong degeneracy of the carrier distribution function.

Figure 5.5 depicts the variation of $\mu_H$ and $R_m$ with magnetic field, for three different lattice temperatures. $\mu_H$ decreases and $R_m$ increases with magnetic field due to the combined scattering
Fig. 5.5: Variation of Hall mobility ($\mu_H$) and magnetoresistance ($R_m$) with the magnetic field $B$ at lattice temperatures of 4, 10, and 15K for a typical electric field of 500V/m. The other parameters are the same as in Fig. 5.3.

Mechanisms. The enhancement of $R_m$ in the range of the magnetic fields used here is more than two orders of magnitude in all the cases. On the contrary, $\mu_H$ decreases quite slowly with increasing magnetic field. When $B$ changes ten folds, the changes in $\mu_H$ is only 7 to 10%.

Conclusion of this study is given in chapter 8.

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