CHAPTER 7

INVERSE KINEMATICS MODEL

7.1 STAGE IV: GENERATION OF INVERSE KINEMATICS MODEL EQUATIONS

7.1.1 Introduction

The iterative inverse kinematics in the stage V is done in two different approaches. In this stage the inverse kinematics formulae is generated using the inverse matrices and a model is developed using the LabVIEW software. Also, it is validated and verified using RoboCell and AutoCAD. This chapter explains how the inverse kinematics equations are utilised successfully in the LabVIEW model.

7.1.2 IK Model Equations Generation

Using the Inverse kinematics approach, the joint parameters are obtained. The following section elaborates the same. From Chapter 4 (Stage I), the transformation matrix and the inverse of the matrices obtained are as follows:

$$[{}^{o}T_1]^{-1} = \begin{bmatrix} c_1 & s_1 & 0 & -a_1 \\ 0 & 0 & -1 & d_1 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{1}T_2]^{-1} = \begin{bmatrix} c_2 & s_2 & 0 & -a_2 \\ -s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{2}T_3]^{-1} = \begin{bmatrix} c_3 & s_3 & 0 & -a_3 \\ -s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{3}T_4]^{-1} = \begin{bmatrix} -s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ c_4 & s_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{4}T_5]^{-1} = \begin{bmatrix} c_5 & s_5 & 0 & 0 \\ -s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using matrix equality, the inverse kinematics equations are obtained as explained in the following steps.

7.1.2.1 Base Joint Angle

The overall complex homogeneous matrix of transformation is taken from the Stage I in Chapter 4, as said below.

$$T_e = {}^{o}T_5 = {}^{o}T_1\, {}^{1}T_2\, {}^{2}T_3\, {}^{3}T_4\, {}^{4}T_5$$

$${}^{o}T_1\, {}^{1}T_2\, {}^{2}T_3\, {}^{3}T_4\, {}^{4}T_5 = {}^{o}T_5$$

LHS: =

$$\begin{bmatrix} -s_1s_5-c_1s_{234}c_5 & -s_1c_5+c_1s_{234}s_5 & \mathbf{c_1c_{234}} & \mathbf{c_1(a_1+a_2c_2+a_3c_{23}+c_{234}d_5)} \\ c_1s_5-s_1s_{234}c_5 & c_1c_5+s_1s_{234}s_5 & \mathbf{s_1c_{234}} & \mathbf{s_1(a_1+a_2c_2+a_3c_{23}+c_{234}d_5)} \\ -c_{234}c_5 & c_{234}s_5 & -s_{234} & d_1-a_2s_2-a_3s_{23}-s_{234}d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$,$c_{i\,jk}= \cos(\theta_i+\theta_j+\theta_k)$and $s_{ijk} = \sin\theta_i$.

RHS: $= \begin{bmatrix} n_x & o_x & \mathbf{a_x} & \mathbf{p_x} \\ n_y & o_y & \mathbf{a_y} & \mathbf{p_y} \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

θ_1 can be isolated by dividing the second and first elements of fourth column and found out.

$s_1 (a_1 + a_2c_2+ a_3c_{23} +c_{234}d_5) = p_y$

$c_1 (a_1 + a_2c_2+ a_3c_{23} +c_{234}d_5) = p_x$

$s_1 / c_1 = t_1 = p_y / p_x$

θ_1=atan2(p_y,p_x) (7.1)

alternatively

θ_1 can be isolated by dividing the second element and first element of third column and found out.

$c_1c_{234}= a_x$

$s_1c_{234}= a_y$

$s_1 / c_1 = t_1=a_y / a_x$

θ_1=atan2(a_y,a_x) (7.2)

7.1.2.2 Wrist Roll Joint Angle

The overall complex homogeneous matrix of transformation is modified to find the wrist roll joint θ_5.

$$^oT_1\ ^1T_2\ ^2T_3\ ^3T_4\ ^4T_5 = {}^oT_5$$

$$^1T_2 \times {}^2T_3 \times {}^3T_4 \times {}^4T_5 = {}^1T_5 = [{}^oT_1]^{-1} \times {}^oT_5$$

$$[{}^oT_1]^{-1} \times {}^oT_5 = {}^1T_5$$

LHS:
$$= \begin{bmatrix} c_1n_x + s_1n_y & c_1o_x+s_1o_y & c_1a_x+s_1a_y & c_1p_x+s_1p_y-a_1 \\ -n_z & -o_z & -a_z & -p_z+d_1 \\ -s_1n_x+c_1n_y & -s_1o_x+c_1o_y & -s_1a_x+c_1a_y & -s_1p_x+c_1p_y \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

RHS:
$$= \begin{bmatrix} -s_{234}c_5 & s_{234}s_5 & c_{234} & c_{234}d_5+a_3c_{23}+a_2c_2 \\ c_{234}c_5 & -c_{234}s_5 & s_{234} & s_{234}d_5+a_3s_{23}+a_2s_2 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ_5 can be isolated by dividing the first and second elements of third row and found out.

$s_5 = -s_1n_x + c_1n_y$

$c_5 = -s_1o_x + c_1o_y$

$t_5 = s_5/c_5 = (-s_1n_x + c_1n_y)/(-s_1o_x + c_1o_y)$

θ_5=atan2($-s_1n_x+c_1n_y$,$-s_1o_x+c_1o_y$) (7.3)

7.1.2.3 Sum of Shoulder, Elbow and Wrist Pitch Joint Angles

From the same matrix equality the sum of shoulder, elbow and wrist pitch joint angles can be found in the following steps.

$[{}^{o}T_1]^{-1} x\ {}^{o}T_5 = {}^{1}T_5$

$$\text{LHS:}\quad = \begin{bmatrix} c_1 n_x + s_1 n_y & c_1 o_x + s_1 o_y & \mathbf{c_1 a_x + s_1 a_y} & c_1 p_x + s_1 p_y - a_1 \\ -n_z & -o_z & \mathbf{-a_z} & -p_z + d_1 \\ -s_1 n_x + c_1 n_y & -s_1 o_x + c_1 o_y & -s_1 a_x + c_1 a_y & -s_1 p_x + c_1 p_y \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{RHS:}\quad = \begin{bmatrix} -s_{234} c_5 & s_{234} s_5 & \mathbf{c_{234}} & c_{234} d_5 + a_3 c_{23} + a_2 c_2 \\ c_{234} c_5 & -c_{234} s_5 & \mathbf{s_{234}} & s_{234} d_5 + a_3 s_{23} + a_2 s_2 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$(\theta_2+\theta_3+\theta_4)$ can be isolated by dividing the second and first elements of third column and found out.

$s_{234} = -a_z$

$c_{234} = c_1 a_x + s_1 a_y$

$t_{234} = s_{234} / c_{234} = -a_z \ /(c_1 a_x + s_1 a_y)$

$$(\theta_2+\theta_3+\theta_4) = \text{atan2}(-a_z, c_1 a_x + s_1 a_y) \qquad (7.4)$$

7.1.2.4 Elbow Joint Angle

Elbow joint angle θ_3 can be found as follows.

$[{}^{o}T_1]^{-1} x\ {}^{o}T_5 = {}^{1}T_5$

$$\text{LHS:} \quad = \begin{bmatrix} c_1n_x + s_1n_y & c_1o_x+s_1o_y & c_1a_x+s_1a_y & \mathbf{c_1p_x+s_1p_y-a_1} \\ -n_z & -o_z & -a_z & \mathbf{-p_z+d_1} \\ -s_1n_x+c_1n_y & -s_1o_x+c_1o_y & -s_1a_x+c_1a_y & -s_1p_x+c_1p_y \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{RHS:} \quad = \begin{bmatrix} -s_{234}c_5 & s_{234}s_5 & c_{234} & \mathbf{c_{234}d_5+a_3c_{23}+a_2c_2} \\ c_{234}c_5 & -c_{234}s_5 & s_{234} & \mathbf{s_{234}d_5+a_3s_{23}+a_2s_2} \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ_{23} and θ_2 are separated from the first and second elements of fourth column for convenience.

$c_{234}d_5+\mathbf{a_3}c_{23}+\mathbf{a_2}c_2= c_1p_x + s_1p_y-a_1$

$s_{234}d_5 +\mathbf{a_3}s_{23}+\mathbf{a_2}s_2=-p_z+d_1$

We know that in this case $a_2= a_3$

$c_{234}d_5 +a_3(c_{23}+c_2) = c_1p_x + s_1p_y-a_1$

$s_{234}d_5 +a_3(s_{23}+s_2)= -p_z+d_1$

Rearranging the equations in the following steps,

$a_3(c_{23}+c_2)= c_1p_x + s_1p_y-a_1-c_{234}d_5$

$a_3(s_{23}+s_2)=-p_z+d_1-s_{234}d_5$

$c_{23}+ c_2 = (c_1p_x + s_1p_y-a_1-c_{234}d_5)/ a_3$ (7.5)

$s_{23} + s_2 = (-p_z+d_1-s_{234}d_5)/ a_3$ (7.6)

To eliminate c_3, squaring and adding equation (7.5) and equation (7.6) we get,

$(s_{23} +s_2)^2 + (c_{23} +c_2)^2 = ((c_1p_x + s_1p_y-a_1-c_{234}d_5)/ a_3)^2+ ((-p_z+d_1-s_{234}d_5)/ a_3)^2$

$s_{23}^2+s_2^2+ 2s_{23}s_2 + c_{23}^2+ c_2^2+ 2c_{23} c_2 =((c_1p_x + s_1p_y-a_1-c_{234}d_5)/ a_3)^2+ ((-p_z+d_1-s_{234}d_5)/ a_3)^2$

$s_{23}^2+ c_{23}^2+s_2^2+ c_2^2+2(s_{23} s_2+ c_{23} c_2) =((c_1p_x + s_1p_y-a_1-c_{234}d_5)/ a_3)^2+ ((-p_z+d_1-s_{234}d_5)/ a_3)^2$

$2+2(s_{23} s_2+ c_{23} c_2) =((c_1p_x + s_1p_y-a_1-c_{234}d_5)/ a_3)^2+ ((-p_z+d_1-s_{234}d_5)/ a_3)^2$

$s_{23} s_2+ c_{23} c_2 = \{((c_1p_x + s_1p_y-a_1-c_{234}d_5)/ a_3)^2+ ((-p_z+d_1-s_{234}d_5)/ a_3)^2-2\}/2$

We know that,

cos (A-B) = cos A cos B + sin A sin B

Let us consider $A = \theta_2+ \theta_3$, $B= \theta_2$

$\cos (\theta_2+ \theta_3- \theta_2) = \cos (\theta_2+ \theta_3) \cos \theta_2 + \sin (\theta_2+ \theta_3) \sin \theta_2$

$c_3 = \{((c_1p_x + s_1p_y-a_1-c_{234}d_5)/ a_3)^2+ ((-p_z+d_1-s_{234}d_5)/ a_3)^2-2\}/2$

$\theta_3=\text{acos}(\{((c_1p_x+s_1p_y-a_1-c_{234}d_5)/a_3)^2+((-p_z+d_1-s_{234}d_5)/a_3)^2-2\}/2)$ (7.7)

7.1.2.5 Shoulder Joint Angle

Shoulder joint angle θ_2

We know that,

cos (A+B) = cos A cos B – sin A sin B

sin (A+B) = sin A cos B + cos A sin B

Let us consider A = θ_2, B = θ_3

$c_{23} = c_2c_3 - s_2s_3$

$s_{23} = s_2c_3 + c_2s_3$

From equations (7.5) and (7.6) we get,

$c_2c_3 - s_2s_3 + c_2 = (c_1p_x + s_1p_y - a_1 - c_{234}d_5)/ a_3$

$s_2c_3 + c_2s_3 + s_2 = (-p_z + d_1 - s_{234}d_5)/ a_3$

$$(c_3+1)c_2 - s_3s_2 = (c_1p_x + s_1p_y - a_1 - c_{234}d_5)/a_3 \quad (7.8)$$

$$s_3c_2 + (c_3+1)s_2 = (-p_z + d_1 - s_{234}d_5)/a_3 \quad (7.9)$$

Reduce equations (7.8) and (7.9) to find s_2,

(7.8) x a_3s_3:

$$a_3s_3(c_3+1)c_2 - a_3s_3^2s_2 = s_3\,(c_1p_x + s_1p_y - a_1 - c_{234}d_5) \quad (7.10)$$

(7.9) x $a_3\,(c_3+1)$:

$$a_3s_3(c_3+1)\,c_2 + a_3\,(c_3+1)^2s_2 = (c_3+1)(-p_z + d_1 - s_{234}d_5) \quad (7.11)$$

To eliminate $s_{2,}$ subtract equation (7.10) from equation (7.11)

$((c_3+1)^2+s_3^2)\ a_3s_2= (c_3+1)\ (-p_z+d_1-s_{234}d_5)-\ s_3\ (c_1p_x + s_1p_y-a_1-c_{234}d_5))$

$s_2=(c_3+1)\ (-p_z+d_1-s_{234}d_5)-\ s_3\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)/\ \ a_3\ ((c_3+1)^2+s_3^2)$

again reduce equations (7.8) and (7.9) to find c_2

(7.8) x $a_3\ (c_3+1)$:

$$a_3\ (c_3+1)^2c_2 - a_3\ (c_3+1)\ s_3s_2 = (c_3+1)\ (c_1p_x + s_1p_y-a_1-c_{234}d_5) \qquad (7.12)$$

(7.9) x $a_3s_{3:}$

$$a_3s_3^2c_2+ a_3\ (c_3+1)\ s_2s_3 \qquad =s_3(-p_z+d_1-s_{234}d_5) \qquad (7.13)$$

To eliminate $c_{2,}$ add equation (7.12) and equation (7.13)

$a_3\ c_2\ (\ (c_3+1)^2+ s_3^2) =(c_3+1)\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)+ s_3\ (-p_z+d_1-s_{234}d_5)$

$c_2=(\ (c_3+1)\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)+ (s_3\ (-p_z+d_1-s_{234}d_5))/\ a_3(\ (c_3+1)^2+ s_3^2)$

$t_2 = s_2 / c_2= (c_3+1)\ (-p_z+d_1-s_{234}d_5)-\ s_3\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)/$

$(\ (c_3+1)\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)+ (s_3\ (-p_z+d_1-s_{234}d_5))$

$\theta_2=\ atan2([(\ (c_3+1)\ (-p_z+d_1-s_{234}d_5)/\ a_3\ )-\ (s_3\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)/\ a_3)/$ $((c_3+1)^2+s_3^2)\ ],[(\ (c_3+1)\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)/\ a_3)+ (s_3\ (-p_z+d_1-s_{234}d_5)/\ a_3)\ /$

$$((c_3+1)^2+s_3^2)]) \qquad (7.14)$$

7.1.2.6 Wrist Pitch Joint Angle

Wrist pitch joint angle is calculated as follows:

$\theta_4 = \theta_{234} - \theta_2 - \theta_3$

7.2 DEVELOPING INVERSE KINEMATICS MODEL

The solution to the inverse kinematics problem consists of finding the value of the joint parameters from the goal position of the object. This solution is a function of the position and orientation vectors. The known position and orientation of object is given as input and using the inverse kinematics equations, the joint parameters are calculated by developing a LabVIEW model (Refer Figure 7.1). It includes two formula nodes. The first one is used to find the assumed virtual goal position from the assumed joint position (Figure 7.2) and the second node to find the joint parameters from the results of the first node (Figure 7.3). Complete view of the IKM in the block diagram is shown in Figure 7.4.

Figure7.1 Front Panel of the IKM

Figure 7.2 First Formula Node

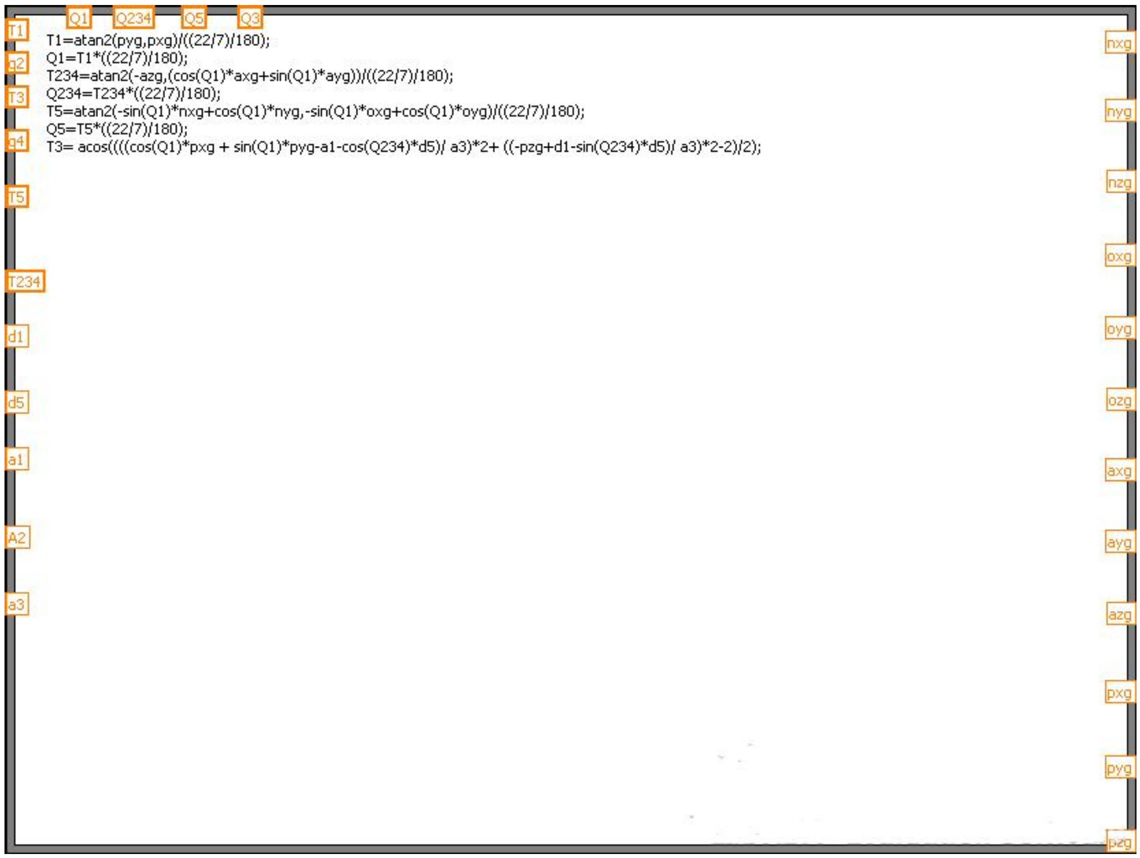

Figure 7.3 Second Formula Node

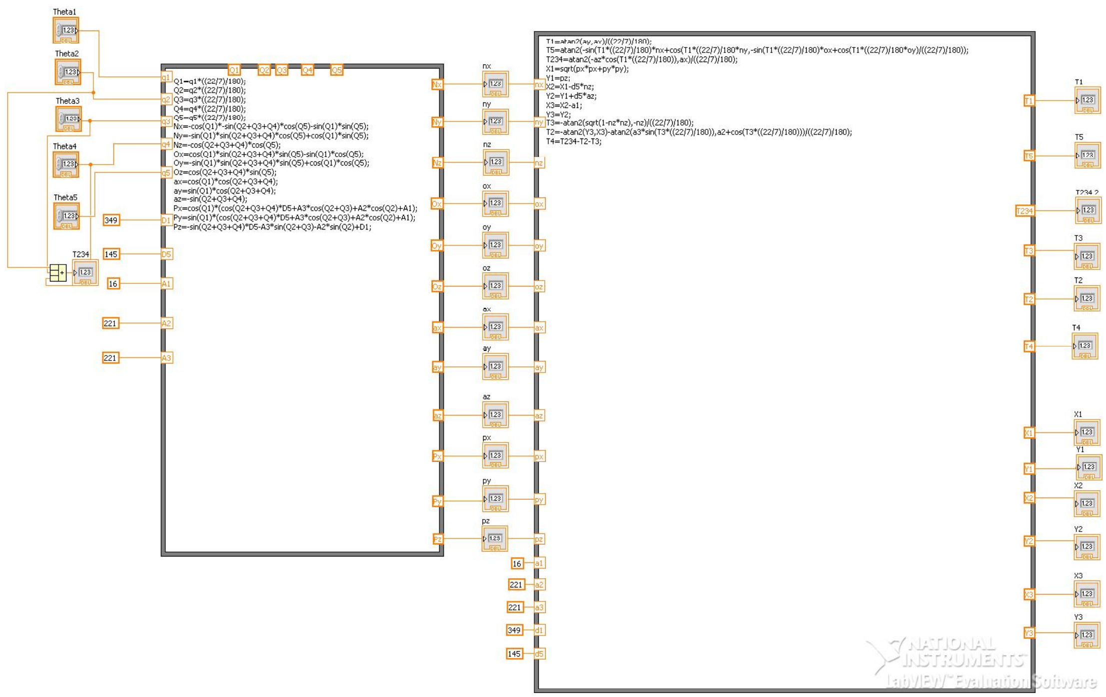

Figure 7.4 Complete View of the IKM in the Block Diagram

7.3 VIRTUAL GOAL POSITION AND ORIENTATION

The 12 parameters are considered as the goal position and orientation for analysis purpose. But in online programming, the position and orientation of the goal position are taken from image processing and similar methods. For analysis purpose, the joint parameters of the SCORBOT ER V Plus Present position is assumed as θ_1=10, θ_2=10, θ_3=10, θ_4=10 and θ_5 =10.

The homogeneous matrix which specifies the location of the fifth coordinate frame with respect to the base coordinate system is $T_e = {}^0T_5 = {}^0T_1\,{}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5$ and is obtained from the FKM as output.

$$
{}^0T_5 = \begin{bmatrix} 0.45491 & -0.0855068 & 0.852754 & 558.241 \\ 0.0855068 & 0.954728 & 0.150425 & 98.4734 \\ -0.852754 & 0.150425 & -0.500183 & 162.466 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Normal vector components:

n_{xg} = 0.45491

n_{yg} = 0.0855068

n_{zg} = -0.852754

Orientation vector components:

o_{xg} = -0.0855068

o_{yg} = 0.954728

o_{zg} = 0.150425

Approach vector components:

a_{xg} = 0.852754

a_{yg} = 0.150425

a_{zg} = -0.500183

Position vector components:

p_{xg} = 558.241

p_{yg} = 98.4734

p_{zg} = 162.466

Results obtained after running the programme.

T1=10

T2=9.86

T3=9.58

T4=10.56

T5=10

T234=30

7.4 RESULTS AND DISCUSSION

The results show that the inverse kinematics equations work successfully for base joint, wrist roll joint and addition of shoulder, elbow and wrist pitch angle. However, other joint values could not be calculated correctly for all the ranges of the position and orientation due to the complexity of the equation and its cosine nature. To address this issue, the iterative method is used in Stage V.