Chapter - 5

Spectral Analysis to Recognize the Pattern of Kinetic Energy and Divergence during Tropical Cyclones

5.1 Preamble

Surface wind structure is a significant component of tropical cyclone (TC) destructive potential. A large or a small storm of equal intensity only cannot generate surge. Storm surge is a very serious threat to coastal regions often causing greater damage than the winds and was dramatically demonstrated by Hurricane Katrina (2005) which caused unprecedented storm surge damage to Louisiana and Mississippi which is rated as in category 3 on the Saffir-Simpson hurricane scale (SSHS) at landfall.

A better physical understanding of the effect that the structure of a cyclone has on its own intensification will eventually lead to improvements in intensity prediction (Epstein, and Barnston, 1988).

The aim of the present chapter is to view the pattern of kinetic energy and divergence during tropical cyclones over Arabian Sea (Fraedrich and Larnder,1993). Statistical method is adopted to attain the objective. Spectral harmonic analysis is used to make out a pattern of kinetic energy and divergence. The study reveals that both the kinetic energy and divergence decrease with decreasing pressure levels up to 750 hPa to 700 hPa and then it start increasing, indicating a strong convergence in the lower levels followed by a upward push and stronger divergence aloft.

Chaudhuri S and De Sarkar A, (2009), “Spectral Analysis to Recognize the Pattern of Kinetic Energy and Divergence during Tropical Cyclones”, (Communicated for publication)
5.2 Methodology and Implementation Procedure

In the present study the kinetic energy and divergence field associated with tropical cyclone occurring over Arabian Sea in the month of June 1979 are computed. The cyclone was reported to originate as a depression over the east Arabian Sea on 16 June and moved towards north-westerly direction. It has intensified into a tropical storm by the morning of 18th June. Later, it moved towards west-north-west till it struck the Saudi Arabian coast on June 20, 1979 (Overland, and Preisendorfer, 1982). We have adopted statistical methods to identify the pattern of kinetic energy and divergence of tropical cyclones (Tracton and Kalnay, 1993).

The divergence pattern is evaluated with help of the expression;

\[ D = \frac{2}{\pi R^2} \int_{0}^{2\pi} V_r ds = \frac{2}{\pi R^2} \int_{0}^{\pi} V_r R d\theta = \frac{2nRV_r}{\pi R^2} = \frac{2V_r}{R} \]  

(5.1)

where \( V_r \) is the radial component of the wind and \( R \) is the radial distance from the storm centre (Simpson, 1974). Similarly kinetic energy is evaluated with the following expression taking pressure as the vertical coordinates;

\[ \frac{\partial k}{\partial t} + V_p \nabla p.k + \frac{w\partial k}{\partial p} = -V_p \nabla \psi + V_p F_p \]  

(5.2)

Where,

\( V \rightarrow \) Horizontal wind vector with components \( u \) and \( v \)
\[ k = \frac{1}{2}(u^2 + v^2) \]

\( \psi = \text{Geo-potential}, \]

\( F = \text{frictional force/unit mass}. \)

By using equation of continuity
\[ \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_{p} + \frac{\partial w}{\partial p} = 0 \]

and integrating over the volume equation (5.2) leads to;
\[ \left[ \frac{\partial K}{\partial t} \right] = -\nabla \cdot \nabla \cdot (wK) + \left[ \nabla_{p} \cdot (wK) \right] + \left[ \nabla_{p} \cdot F_{p} \right] \]

Equation (5.4) is integrated over the mass enclosed within the surface pressure level up to 200 hPa pressure levels. The term in the left hand side of equation (5.4) is known as local rate of change of kinetic energy per unit mass (De Maria, 1996). First term on the right hand side of eqn (5.4) represents how the kinetic energy can be changed and is known as generation of kinetic energy due to cross-isobaric flow. Second term on the right represents horizontal convergence whereas third term represents vertical convergence of the kinetic energy. The last term is the dissipation term and is calculated as residual after balancing the equation (5.4). Sum of the second and fifth term of the eqn (5.4) represents internal energy source.
Data analyzed in the present study is for the different dynamical parameters of Arabian Sea cyclone occurred from 16th June 1979 to 18th June 1979. It has been taken as the case study of an Arabian Sea cyclone. Several cycles are framed using sine and cosine functions. The framing procedure basically transforms the original dataset into trigonometrical wave structures. The wave patterns extracted from the data are transformed into spectral densities. The data are collected from WMO Technical Document (Mandal, 1991).

A proper time series analysis of the dataset is carried out to attain the objective of the present study. Among the time series method, the frequency-domain analysis is chosen because it represents data series in terms of contributions occurring at different time scales or characteristic frequencies (Neumann, 1993). The overall time series is regarded as having arisen from the combined effects of a collection of sine and cosine waves oscillating at different rates. The sum of these waves reproduces the original data, but it is often the relative strengths of the individual component waves are of primary interest. Spectral analysis of frequency domain is a different way to look at a time series as a collection of Fourier coefficients that are a function of frequency. In this study the amplitude of the data series is represented with the functions of frequency. This is a unique feature of the Fourier series or spectral analysis. The advantage of this perspective is that it allows one to see separately the contributions of a time series that are made by processes varying at different speeds; that is, by processes operating at a spectrum.
of different frequencies. In harmonic analysis phase shift of the data series is plotted with the time period. In the present study the kinetic energy and divergence of tropical cyclone is computed with respect to frequency.

The methodology adopted in the present study comprises the following:

i. Familiarization with Spectral Analysis

ii. Implementation procedure

5.2.1 Spectral Analysis – A Preview

The frequency domain analysis involves in representing data series at different time scales. The tropical cyclone of 16\textsuperscript{th} June 1979 to 18\textsuperscript{th} June 1979 over Arabian Sea is considered in this study (Mandal, 1991). The behaviors of kinetic energy during different time intervals of a particular tropical cyclone are viewed by spectral analysis. In the spectral method, different ways to look at a time series as a collection of Fourier coefficients is suggested that represent a function of frequency, rather than a collection of data points measured as a function of time (Chu and Katz, 1989). The advantage of this perspective is that it allows to see separately the contributions of a time series that are made by processes varying at different speeds; that is, by processes operating at a spectrum of different frequencies (Wilks, 1995).

A given data series consisting of exactly n points can be represented, meaning that a harmonic function can be found that passes through each of the points, by adding together a series of n/2 harmonic functions,
\[ y_i = y + \sum_{k=1}^{\frac{n}{2}} C_k \cos\left(\frac{2\pi kt}{n} - \phi_k\right) \] (5.5.a)

\[ = y + \sum_{k=1}^{\frac{n}{2}} \left( A_k \cos\left[\frac{2\pi kt}{n}\right] + B_k \sin\left[\frac{2\pi kt}{n}\right]\right) \] (5.5.b)

The cosine wave, constituting of \( k = 1 \) term in equation (5.5.a) is the fundamental, or represent the first harmonic. The other \( \left(\frac{n}{2} - 1\right) \) terms in the summation of equation (5.5.a) and equation (5.5.b) are higher harmonics, or cosine waves with frequencies

\[ \omega_k = \frac{2\pi k}{n} \] (5.6)

These are integer multiples of the fundamental frequency \( \omega_1 \).

For example, the second harmonic is the cosine function that completes exactly two full cycles over the \( n \) points of the data series. It has its own amplitude \( C_2 \) and phase angle \( \phi_2 \). The factor \( k \) inside the cosine function in equation (5.5.a) is of critical importance. When \( k = 1 \), the angle \( \alpha = \frac{2\pi kt}{n} \) varies through a single full cycle of \( 0 \) to \( 2\pi \) radians as the time index increased from \( t = 0 \) to \( t = n \). In the case of second harmonic where \( k = 2 \), \( \alpha = \frac{2\pi kt}{n} \) executes one full cycle as it increases to \( \frac{n}{2} \), and then executes a second full cycle between \( t = \frac{n}{2} \) and \( t = n \). Similarly, the
third harmonic is defined by the amplitude $C_3$ and the phase angle $\phi_3$, and varies through three cycles as $t$ increases from 0 to $n$.

The equation (5.5.b) shows that the coefficients $A_k$ and $B_k$ corresponding to particular data series $y$ can be obtained by using multiple regression methods, after the data transformations;

$$x_1 = \cos\left(\frac{2\pi}{n}t\right), x_2 = \sin\left(\frac{2\pi}{n}t\right), x_3 = \cos\left(\frac{2\pi}{n}2t\right), x_4 = \sin\left(\frac{2\pi}{n}2t\right), x_5 = \cos\left(\frac{2\pi}{n}3t\right)$$

and so on.

This is, in fact, the case in general, but if the data series is equally spaced in time and contains no missing values, the coefficients can be written as,

$$A_k = \frac{2}{n} \sum_{i=1}^{n} y_i \cos\left(\frac{2\pi xt}{n}\right)$$

(5.7.a)

and

$$B_k = \frac{2}{n} \sum_{i=1}^{n} y_i \sin\left(\frac{2\pi xt}{n}\right)$$

(5.7.b)

To compute a particular coefficient $A_k$, these equations indicate that an $n$ - term sum is formed, consisting of the products of the data series $y_i$ with values of a cosine function executing $k$ full cycle during $n$ time units. For relatively short data series these equations can be easily programmed and evaluated. Having computed these coefficients, the amplitude - phase form of equation (5.5.a) can be computed as,

$$C_k = \left[ A_k^2 + B_k^2 \right]^{\frac{1}{2}}$$

(5.8.a)
The phase angle is computed as

\[
\phi_k = \begin{cases} 
\tan^{-1} \frac{B_k}{A_k} ; A_k > 0 \\
\tan^{-1} \frac{B_k}{A_k} \pm \pi ; A_k < 0 \\
\frac{\pi}{2} ; A_k = 0
\end{cases}
\]  

(5.8.b)

The spectral density can be obtained as;

\[
d_k = \frac{(n/2)c_k^2}{(n-1)s_y^2}.
\]  

(5.9)

Where

\[
s_y^2 \rightarrow \text{The variance of the sample of size \textit{n}.}
\]

5.2.2 Implementation procedure

The equations (5.5) to (5.8) are used to compute the sine and cosine function for different harmonics of kinetic energy, change in kinetic energy and divergence of tropical cyclone on the selected dates. The spectral densities of different harmonics are then computed using equation (5.9) (Fuller, 1976). The spectral densities of kinetic energy, changes in kinetic energy and divergence with different spectral frequencies are schematically represented in figures 5.1, 5.2 and 5.3. The variations of spectral densities of kinetic energy, rate of change of kinetic energy and divergence with different pressure levels are shown pictorially in the figures 5.4, 5.5 and 5.6.
Flow chart showing the stepwise approach to view the pattern of kinetic energy, change in kinetic energy and divergence during Tropical Cyclones:

1. Collection of Data of Tropical Cyclone days for different time intervals
2. Computation of the Spectral Coefficients $A_k$ and $B_k$ using equations (5.7.a) and (5.7.b) with the collected data.
3. Computation of $C_k$ using equation (5.8.a)
4. Estimation of Spectral Densities of Kinetic Energy at different Pressure Levels
5. Estimation of the Rate of Change of Kinetic Energy with Pressure Levels for the said Tropical Cyclone Days.
6. Recognition of the Pattern of the Variations of Divergence during Tropical Cyclone days with Pressure Levels.
5.3 Results and Discussions

The present chapter deals with a case study of a particular tropical cyclone occurring over Arabian Sea from 16.06.1979 to 18.06.1979. Purpose is to see the pattern of kinetic energy, the rate of change in kinetic energy and the divergence of the tropical cyclone with different pressure levels.

The implantation procedure is shown in the above flow chart. The variation of normalized spectral density of kinetic energy with frequency is schematically represented in fig 5.1. It shows that there is a sharp increase in kinetic energy at higher frequency. This higher value of frequency corresponds to the pressure level above 750 hPa. The variation of spectral density of rate of change of kinetic energy with frequency is represented in fig 5.2. It also shows a sharp increasing trend on first two days of cyclone near the higher frequency values, which again corresponds to the pressure level above 750 hPa. Fig 5.3 represents the variations of spectral density of divergence with spectral frequency. It shows a wavy pattern of divergence field along with an increase near 750 hPa level.

The variations of different spectral densities of kinetic energy with pressure levels in three consecutive days are represented fig. 5.4. The trend of the kinetic energy shows that the values of the spectral densities of kinetic energy decreases upward with decrease in pressure. The figure further shows that on the first two days during the preliminary stage of the tropical cyclone the kinetic energy has a little variation near 750 hPa pressure levels. But, in the matured stage of tropical
cyclone on the third day a marked decrease in kinetic energy is observed near 750 hPa pressure levels.

The variation of spectral density of rate of change in kinetic energy with pressure levels for the three days is depicted in fig. 5.5. The figure shows that the overall rate of change of kinetic energy decreases with pressure levels. On the first day, the change in kinetic energy shows a marked fall at 750 hPa pressure levels. The variation is observed to be little pronounced on the other two days.

The variations of spectral density of divergence field of the tropical cyclone with different pressure levels are represented in fig. 5.6. It is observed from the figure that the divergence decreases with pressure levels. On the first day it is observed that the divergence has a sharp drop near 700 hPa pressure levels. On the second day the figure does not show much variation, however on the third day, the divergence value shows a minimum near 750 hPa pressure levels. This figure supports the fact that a tropical cyclone accompanies a stronger divergence aloft.

The study reveals that the tropical cyclone over Arabian Sea during the said period identifies 750 hPa - 700 hPa pressure level a layer of significance. This is the layer where the kinetic energy and the rate of change of kinetic energy attain a minimum value. This is the level where most of the kinetic energy of the tropical cyclone is transformed to potential energy leading to the generation of upward motion. Regarding the divergence field as well, this is the layer where it attains its minimum value converting to more convergence.
5.4 Conclusion

It can be concluded from the present study of the tropical cyclone occurring over the Arabian Sea that the kinetic energy, the rate of change of kinetic energy and the divergence show a decreasing trend with pressure levels up to 750 to 700 hPa pressure levels. The kinetic energy and the divergence attain their maximum aloft.
List of figure captions

Fig 5.1  The Variations of Normalized Spectral Density of Kinetic Energy with Spectral Frequency

Fig 5.2  The Variations of Normalized Spectral Density of Rate of Change of Kinetic Energy with Spectral Frequency

Fig 5.3  The Variations of Normalized Spectral Density of Divergence with Spectral Frequency

Fig 5.4  The Variations of Kinetic Energy with Different Pressure Level on Three Specified Date of A Tropical Cyclone over Arabian Sea

Fig 5.5  The Variations of Change of Kinetic Energy with Time with Pressure for Specified Three Days of a Tropical Cyclone over Arabian Sea

Fig 5.6  The Divergence with Different Pressure Level on Three Specified Days of a Tropical Cyclone over Arabian Sea
Fig 5.1 The Variation of Spectral Density of Kinetic Energy with Spectral Frequencies of Pressure
Fig 5.2 The Variation of Spectral Density of Rate of Change of Kinetic Energy with Frequency

Normalized Spectral Density

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Fig 5.2 The Variation of Spectral Density of Rate of Change of Kinetic Energy with Frequency
Fig 5.3 The Variation of Spectral Density of Divergence with Frequency

Fig 5.3 The Variation of Spectral Density of Divergence with Frequency
Fig. 5.4 The Variations of Kinetic Energy with Different Pressure Levels on Three Specified Dates of a Tropical Cyclone occurring over Arabian Sea.
Fig 5.5 The Variations of Change in K.E. with time and Pressure levels for the Specified Three Days of A Tropical Cyclone over Arabian Sea.
Fig 5.6 The Variations of Divergence with Pressure Levels on
the Three Specified Days of A Tropical Cyclone over Arabian
Sea