Chapter - 7

Pattern Recognition of Yearly Frequency of Tropical Cyclones Using Harmonic Analysis

7.1 Preamble

Purpose of the present chapter is to introduce the method of Harmonic Analysis (HA) in order to reveal the pattern inherent within the yearly frequency of occurrence of tropical cyclones and severe tropical cyclones over North Indian Ocean (Over and Gupta, 1994). The results reveal the persistence of a cyclic pattern within the dataset pertaining to the occurrence of the said weather hazards. A cycle of 10 years and 25 years span is revealed for severe tropical cyclones.

7.2 Methodology and Implementation Procedure

The data analyzed in the present study pertain to the yearly frequencies of the occurrence of cyclonic storms and severe cyclonic storms within the period from 1891 to 1989 that are confined within the region $5^\circ N$ to $35^\circ N$ latitude and $50^\circ E$ to $100^\circ E$ longitude. Several cycles are framed using sine and cosine functions. The framing procedure basically transforms the original dataset into trigonometrical wave structures (Shapiro, 1982). The wave patterns extracted from the yearly frequencies are compared to identify the best representative cycle.

The data are collected from WMO Technical Document (Mandal, 1991).

The methodology adopted in the present chapter comprises the following:

i. Familiarization with Harmonic Analysis

ii. Implementation procedure of the technique

7.2.1 Harmonic Analysis – A Preview

Analysis of a time-series consists of two approaches;

i. Time-domain approach and

ii. Frequency Domain approach.

The time domain approach adopts Markovian technique if the associated random variable is discrete and autoregression, if the random variable is continuous.

Analysis in the frequency domain involves representing data series at different time scales. A time series of hourly temperature data from a mid-latitude location usually exhibits strong variations both at daily time scale and at the annual time
scale (Buishand, 1978). In the time series approach, there are two types of representation of the datasets. One is Fourier analysis or spectral analysis and another is harmonic analysis. Both the methods represent the harmonics of datasets. In case of Fourier analysis amplitudes are plotted against frequencies (Wilks, 1995). In case of harmonic analysis the amplitudes are represented against time periods. In this study the harmonic analysis is chosen because it has been tried to find the yearly variations of occurrences of different cyclonic storms.

Harmonic analysis consists of representing the fluctuations or variations in a time series having generated for adding together a series of sine and cosine functions. These trigonometric functions are “harmonic” in the sense that they are chosen to have frequencies exhibiting integer multiples of the “fundamental” frequency determined by the sample size of the data series (Harris, et.al., 1997).

A given data series consisting of exactly n points can be represented, meaning that a harmonic function can be obtained that passes through each of the points, by adding together a series of $\frac{n}{2}$ harmonic functions,

$$y_i = y + \sum_{k=1}^{\frac{n}{2}} C_k \cos\left(\frac{2\pi kt}{n}\right) - \phi_k$$  \hspace{1cm} (7.1a)

$$= y + \sum_{k=1}^{\frac{n}{2}} \{A_k \cos\left(\frac{2\pi kt}{n}\right) + B_k \sin\left(\frac{2\pi kt}{n}\right)\}$$  \hspace{1cm} (7.1b)
The cosine wave, constituting $k = 1$ term in equation (7.1a) is the fundamental and represents the first harmonic. The other $\left[ \frac{n}{2} - 1 \right]$ terms in the summation of equation (7.1a) and (7.1b) are higher harmonics, or cosine waves with frequencies 

$$\omega_k = \frac{2\pi k}{n}$$ (7.2)

These are integer multiples of the fundamental frequency $\omega_1$.

Equation (7.1b) shows that the coefficients $A_k$ and $B_k$ corresponding to the data series, $y_i$ can be obtained by using multiple regression methods, after the data transformations;

$$x_1 = \cos\left(\frac{2\pi t}{n}\right), x_2 = \sin\left(\frac{2\pi t}{n}\right), x_3 = \cos\left(\frac{2\pi 2t}{n}\right), x_4 = \sin\left(\frac{2\pi 2t}{n}\right), x_5 = \cos\left(\frac{2\pi 3t}{n}\right)$$

and so on. (7.3)

This is, in fact, the case in general, but if the data series is equally spaced in time and contains no missing values, the coefficients can be expressed as;

$$A_k = \frac{2}{n} \sum_{i=1}^{n} y_i \cos\left(\frac{2\pi kt}{n}\right)$$ (7.4a)

and $$B_k = \frac{2}{n} \sum_{i=1}^{n} y_i \sin\left(\frac{2\pi kt}{n}\right)$$ (7.4b)

To compute a particular coefficient $A_k$, an $n$-term sum is formed consisting of the products of the data series $y_i$ with values of a cosine function executing $k$ full cycle during the $n$ time units. For relatively short data series these equations can
be easily programmed and evaluated (Menabde, et.al., 1997). Having computed these coefficients, the amplitude-phase form of equation (7.1a) can be expressed as:

\[ C_k = \left[ A_k^2 + B_k^2 \right]^{\frac{1}{2}}. \]  

(7.5)

The phase angle is computed as

\[ \phi_k = \begin{cases} 
\tan^{-1} \frac{B_k}{A_k} ; & A_k > 0 \\
\tan^{-1} \left( \frac{B_k}{A_k} \pm \pi \right) ; & A_k < 0 \\
\frac{\pi}{2} ; & A_k = 0 
\end{cases} \]  

(7.6)

The spectral density can be obtained as;

\[ d_k = \frac{(n/2)C_k^2}{(n-1)s_y^2}. \]  

(7.7)

Where

\[ s_y^2 \rightarrow \text{The variance of the sample of size "}n\text{"} \]

**7.2.2 Implementation technique**

The equations (7.1) – (7.7) are used to find out the harmonics of orders 30 years, 25 years, 20 years, 15 years and 10 years respectively. The sine and cosine functions are computed for different harmonics for tropical cyclones and severe tropical cyclones respectively. The cyclic patterns thus obtained are presented in figures 7.1 to 7.10.
7.3 Results and Discussions

The yearly frequencies of occurrences of tropical cyclones over Indian Ocean show the harmonic frequencies of occurrences of tropical cyclones in 30 years, 25 years, 20 years, 15 years and 10 years of cycle respectively (figs 7.1 to 7.5). It clearly depicts that the 10 years cycles possess almost similar pattern for the occurrences of tropical cyclones in the neighborhood of Indian Ocean (fig 7.5). Further, the 25 years cycles of occurrence of tropical cyclones also show similar pattern (fig 7.2). In other words, it can be inferred that the yearly frequencies of tropical cyclones over the zones repeat their patterns after completing both 10 years' and 25 years' cycles. The other cyclical patterns do not portray any similarity (figs 7.1, 7.3 & 7.4).

The yearly frequencies of occurrences of severe tropical cyclones over Indian oceans are computed (figs 7.6 to 7.10). It is observed from the figures that the occurrences of severe tropical cyclones show a similar pattern in 25 years of cycles (fig 7.7). The results thus, reveal that the yearly frequencies of severe tropical cyclones repeat their patterns after completing the 25 years' cycles over Indian Ocean.

7.4 Conclusions

The study recognizes the patterns of data pertaining to the yearly frequencies of tropical cyclones and severe tropical cyclones over North Indian Oceans. The study leads to conclude that both of the events exhibit cyclic pattern to their
yearly frequency. The predictive model for tropical cyclones require 10 years' & 25 years' cycles and severe tropical cyclones requires some methodology that can handle the change in patterns in every 25 years.
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1st 25 yrs
2nd 25 yrs
3rd 25 yrs

Years

Harmonics

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![Graph showing harmonics of 20 years cycles for tropical cyclones](image)

- 1st 20 yrs
- 2nd 20 yrs
- 3rd 20 yrs
- 4th 20 yrs
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Harmonics

Years
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![Graph showing harmonics of 20 years cycles for Severe Tropical Cyclone]

- 1st 20yrs
- 2nd 20yrs
- 3rd 20yrs
- 4th 20yrs
- 5th 20yrs

Harmonics

Years

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