Lens design is a many dimensional, nonlinear and constrained optimization problem. A typical lens design problem may involve as many as 20 to 100 variables. For a hyper-dimensional problem like this, even the strongest optimization method may fail to find a global optimum in the absence of a suitable starting point. From the point of view of mathematical complexity of the problem, it is really surprising how useful practical solutions are routinely obtained by lens designers. In fact, a judicious combination of Gaussian optics and the theories of aberrations of various orders often provides analytical solutions for the simpler problems of lens design, which in turn, act as spring board for tackling the more complicated lens design problems. The large numbers of patents and lens data library dealing with conventional systems often facilitate practical tackling of the problem.
We are working on an alternative strategy (Hazra et al, 1984, 1986, 1998) for multicomponent lens design. This scheme facilitates systematic search for good starting points to be utilized for final design optimization. For 'ab initio' design problems, a thin lens layout specifying the powers of the individual components and intercomponent separations are worked out analytically (Herzberger, 1943; Delano, 1963; Hopkins 1962). Requirements of central aberration targets for the individual components, in order to satisfy the prescribed overall primary aberration targets for the system, are determined by optimization techniques. The next step involves finding out the optimum structure of an individual component satisfying the primary central aberration targets. If the simpler structures fail to provide a useful solution, an increasing degree of complexity in the structure of the desired component is allowed. The search commences from a singlet (Hazra & Samui, 1986) and culminates as and when the optimum solution is obtained during search in the following sequence – singlet, cemented doublet, broken contact doublet, combination of a cemented doublet and a singlet, combination of two cemented doublets, cemented triplets etc.

Contrary to the usual practice of preassigning glass types for the individual lens elements, our approach enables the treatment of glass types as independent variables at the structural design stage, reducing substantially the role of meta-heuristics in this regard. Another important
advantage of our method is its ability to tackle the problem of optimum
design of lenses with non-zero aberration targets. The latter are often
called for in various electro-optic or optoelectronic systems. On the other
hand, this would facilitate the piecewise treatment of modules in modular
optical/optoelectronic design (Stravroudis, 1982).

In pursuance of the design strategy enunciated above, we have
experimented with a number of quasiglobal and global optimization
algorithms in the treatment of optimization in structural design problems.
We have experimented with two wellknown stochastic GO techniques
(simulated annealing and genetic algorithm. Though they are chiefly
perceived as global optimizers, our interest lies in determining their
effectiveness in exploring the entire design space. This is of interest from a
practical point of view.

2.1 DESIGN EXAMPLES

We have investigated structural design problems of cemented and broken
contact doublets with prespecified Gaussian characteristics and primary
aberration targets. Although doublets constitute an important class of lens
systems providing relatively simple solutions in many imaging problems,
e.g. telescope and microscope objectives and deserve enough attention as
such, they play a very significant role in the synthesis of multicomponent
lens systems. In the latter case, a doublet is often the most preferred choice, next only to a singlet, for practical realization of the individual components. Often in such applications, the required doublets need to satisfy nonzero aberration targets. It is obvious that the multitude of readily available doublet designs catering to zero aberration targets cannot be utilized for this purpose. Our approach to the design problem takes proper account of the aberration targets. The problem is formulated as a multivariate, nonlinear and constrained optimization or nonlinear programming problem. The problem is solved by 1) a variant of damped least squares (DLS) method which applies 'dynamic weights' to the merit function definition to achieve 'floating of aberrations' about their target values, 2) a variant of simulated annealing (SA) which includes a strategy for generating random walk strictly confined within the convex search space and lastly 3) an implementation of genetic algorithm (GA).

It is well known that strict adherence to primary aberration targets at the thin lens design stage is not very meaningful in practice (Khan and Macdonald, 1982). Indeed, optimum solutions even in the extended neighborhood of required targets are often adequate. This 'floating' of aberration targets should preferably be carried out in a manner that can reveal a set of promising configurations - from which a suitable choice can be based on other considerations, e.g. preferred glass types, likelihood of presence or absence of higher order aberrations etc. The latter may be
based on considerations like aperture utilization ratio (AUR). AUR of a spherical interface is given by product of the curvature of a refracting interface and its semidiameter (Khan et al, 1982; Hazra et al, 1986). At this level more justified approach would be to select glasses for the two components such that the target for longitudinal chromatic aberration is more or less satisfied, while allowing a certain amount of floating in the other two primary aberrations. Hence considerations based on the optics part of the problem demand that we should explore the solution space for good local optima.

At the subproblem level, our goal is to explore the specified design space as effectively as possible in search of alternative sets of useful solutions. Obviously, these solutions would be local in nature, and are obtained by allowing a tolerable amount of floating in the aberration targets instead of strict adherence to aberration targets. Methods based on DLS techniques being inherently local in scope, can find useful alternative solutions in reasonable time frame if provided with suitable starting points. Choice of a suitable starting point itself requires thorough knowledge of the design space. We have made use of the global optimization (GO) techniques namely, SA and GA, firstly to alleviate this difficulty of choosing suitable starting points in an unknown multidimensional search space, and secondly to explore the same space as completely as possible, in a reasonable time interval. To achieve the last specified goal, we have incorporated a few tricks in our implementation of the said GO techniques.
We have discussed domain partitioning along with our proposed SA technique to obtain different viable local optima. In this context, the GA techniques along with some elegant biological ideas as outlined in Chapter 7, show promise.

The structural design of Cooke triplet is attempted as an example of multicomponent lens design. The optimum configuration for each of the singlets, catering to the required Gaussian specification and primary aberration targets for the Cooke triplet, are determined by the application of our versions of SA and GA.

For both the design examples we have considered, the glasses are treated as variables. In order to thoroughly explore glass characteristics in search of promising solutions, refractive indices and dispersion values for glasses are treated as continuous variables, and are allowed to take values from a specified glass triangle. However, required glasses, as determined thereby, do not always provide practically realizable solutions, for only a finite number of glasses having distinct values of refractive index and dispersion are available. Often a nearest neighbor approach is useful in providing workable solution. However, to circumvent this problem, we have experimented with modified GO techniques that restrict the search for glass from a prespecified list of actual glasses.
2.2 LENS DESIGN: AN ANALYTICAL FORMULATION

Computer-aided design of lens systems implies the solution of a general nonlinear programming problem in many dimensions (Walk & Nicklaus, 1988). The problem belongs to the set of NP-hard problems (Sturlesi & O'Shea, 1990, 1991). It can be formulated as a multiobjective optimization problem (Fonseca & Fleming, 1998). The idea is that of minimizing simultaneously $k$ components of a possibly nonlinear multiobjective function $F$ of a generic decision variable $x$ having $m$ components, in a universe $u$, subject to a positive number $(n-k)$ of conditions involving $x$, where

$$F(x) = (f_1(x), \ldots, f_n(x)) \quad (2.1)$$

and

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \quad (2.2)$$

Mathematically,

Minimize $(f_1(x), \ldots, f_k(x))$

subject to constraints expressed as a functional vector inequality of the type
where the inequality applies componentwise. It is implicitly assumed that there is at least one point in \( u \), which satisfies all constraints, although in practice that cannot always be guaranteed. The problem usually has no unique perfect solution, but a set of nondominated, noninferior alternative solutions. When all constraints cannot be simultaneously satisfied, the number of constraints violated and the extent to which each constraint is violated, are taken into account for looking into the possibility of relaxing some of the constraints.

2.2.1 Defining a merit function

Constraints are seen as high-priority (or hard) objectives, which must be exactly satisfied before any optimization of the remaining, soft objectives. In order to select a suitable compromise solution from all noninferior alternatives, a decision process is necessary. The decision-maker expresses preferences in terms of an aggregating function, which combines individual soft objectives into a single utility value, and ultimately makes the problem single-objective, prior to optimization. Hence optimization of soft objectives
calls for defining a single merit or objective function. Such a merit function
is problem specific, but the form given below is used widely:

$$\psi(x) = \sum_{i=1}^{k} w_i \left[ f_i(x) - y_i^0 \right]^2$$

(2.4)

where $\psi(x)$ is the merit function, $w_i$ - weighting coefficients, a set of real
positive numbers that take into account relative importance of the
objectives and control their involvement in the overall utility measure, $y_i^0$ -
goal/target values indicating desired levels of performance in each
objective dimension.

For the lens design problems to be considered in the following chapters, we
have defined a merit function or performance index $\psi$ of the above form
where, the functions $f_i$ represent primary aberrations (alternatively, Seidel
coefficients or their normalized forms), $x$ gives the parametrical description
of the lens system in terms of shape, power and optical material of the
lens, $y_i^0$ are the user specified targets for the primary aberrations, and the
weights $w_i$ are decided by the relative values of the aberrations.

The merit function characterizing a lens design problem is typically
multimodal. It is defined over a multidimensional parameter space. A
point in this space represents, therefore, a specific configuration of the optical system: tagged to it is a unique value of the merit function. In general, the merit function is nonlinear in the design parameters.

2.2.2 Constraint handling

Cooper and Steinberg (1970) observed that all optimization problems of the real world are, in fact, constrained problems. This view seems to originate from appreciation of the fact that the regions containing the solutions of practical value are in general very small compared to the total volume of space. Hence any practical lens design routine should incorporate some method of constraining the variables within their allowed region.

We will now discuss the geometrical significance of inequality constraints and the general techniques to tackle them. Let us consider the \((n-k)\) inequality constraints considered in section 2. In general they take the following form

\[
f_i(x) = f_i(x_1, x_2, \ldots, x_m) \geq 0 \quad i = k+1, \ldots, n \tag{2.5}
\]
A point $z$ is said to lie on the $i$th constraint if $f_i(z) = 0$. Thus the zero contour of the constraint $f_i(x)$ is regarded as a barrier separating the positive values of $f_i(x)$, i.e., feasible points, from the negative values, that is, the infeasible points. Collectively all the $(n-k)$ constraints represent the allowed domain of definition for the merit function in $m$-dimensional real space.

The available constraining techniques depend upon the form of the constraints themselves. In general there are some gradient-based techniques devised for specific applications. These are well documented by Box, Davies and Swann (1969). Current techniques require reformulation of the problem so that the constraints do not appear explicitly. One such technique involves applying penalty to the merit function at nonfeasible points. We will discuss this technique and its limitations in some detail. Another way of implementing constraints is to reject the nonfeasible solutions. This is called the rejection strategy and it is popular with many researchers. The technique is inherently wasteful, since it generates both feasible and infeasible points, then checks them for nonfeasibility, and finally rejects the infeasible points. Some recent techniques incorporate intelligent tools to avoid the generation of infeasible configurations. Since these techniques use problem specific knowledge, they cannot be generalized. They are more
likely to be computationally intensive, and they are more easily designed for convex search domains. But for some problems absolutely rigid boundaries are not desirable, rather an amount of boundary overflow may lead to good solutions. Overall, in lens design problems, the penalty function method turns out to be a prudent choice.

The penalty function method works by allowing boundary violations but penalizing such systems by increasing the value of the merit function according to the magnitude of violation. The typical strategy for implementing this method necessitates incorporation of penalty functions in the expression for merit function. For a lens design application they are called pseudo aberrations. The complete merit function may look like

\[ \psi(x) = \sum_{i=1}^{k} f_i(x) + \sum_{i=1}^{n} w_i [f_i(x)]^2 \]  

The second term in the above equation represents penalty functions and \( w_i \) are real positive numbers, used as weights for various constraints. The problem of optimizing \( \psi(x) \) is thus transformed into an unconstrained problem. As authors like Bazaraa and Sherali (1993), Davis (1987),
Joines and Houck (1994) and Gen and Cheng (1997) had observed, the main limitation of the penalty functions is the degree to which each constraint is penalized. If one imposes a high degree of penalty, more emphasis is put on obtaining feasibility, and the optimization algorithm will move quickly towards a feasible solution. The system will tend to converge to a feasible point even if it is far from optimal. However, if one imposes a low degree of penalty, less emphasis is placed on feasibility, and the system may never converge to a feasible solution.

For the lens design problems considered in this thesis, the constraints are expressed in the form of linear inequalities. These constraints are simple in the sense that they produce only convex hyper-volumes or multidimensional regions. We have experimented with different constraining techniques. Choice of a particular technique depends on the optimization algorithm used and the nature of the variables. Finer details of the constraining techniques are kept aside for the respective chapters dealing with optimization methods. Things will be presented here on a much coarser scale.

But before we venture into our ways of handling constraints a discussion of the nature of design variables we used will be helpful. Design parameters include some constructional parameters, which are in general continuous. This set of variables is allowed to remain
unconstrained when used with DLS method. With SA they are always treated as continuous variables, and their values are constrained within a hyperdimensional rectangular volume. This is expressed with the help of following set of linear inequalities:

$$|x_i| \leq \alpha_i,$$  \hspace{1cm} (2.7)

where $\alpha_i$ is a numerical value, representing the physical boundary for the $i$-th variable. For this problem the generation of infeasible points are avoided intelligently. With GA, this set of variables is treated as discrete variables, and the numerical values of $\alpha_i$s are used in the coding of variables. The encoding of input variables ensures that only the feasible alternatives are searched.

The salient feature of our design method is that we have taken optical materials (glasses) as variables. For some instances we have considered them as continuous variables and discrete for others. In the continuous case, constraints should be imposed to restrain the glasses within the region of available glasses. If the refractive index of glasses at the mean wavelength is plotted against the index difference over the working wavelength range (dispersion), the glasses lie in a region which is roughly triangular in shape (Jamieson, 1971). We have used a
triangular glass boundary in conjunction with DLS technique. The boundary was imposed with the help of penalty function technique. With SA method the same triangular boundary is used but generation of non-feasible glasses is cleverly avoided. One variant each of SA and GA methods use discrete actual glasses by invoking different integer values for them. These techniques naturally come under the class of problems called integer or mixed-integer programming in optimization literature (Schwefel, 1981).