CHAPTER 3

REVIEW OF LITERATURE

3.1 General

Several attempts have been made to understand the behaviour of RC beams and the size effect in shear. In this Chapter, a historical development of RC members in shear and fracture mechanics of concrete has been presented initially. Also, the review pertaining to ductility of RC members and minimum reinforcement in shear is presented. However, the main focus is on the contributions by different researchers on the size effect of RC beams in shear. At the end, the summary of the review on the size effect of RC beams in shear, minimum web reinforcement and shear ductility, hence the motivation for the present study have been described.

3.2 Historical Development

The historical development of shear theories is presented in ACI-ASCE Committee 445 (1998), the concept of diagonal tension (DT) and truss analogy was first presented by Ritter (1899). Mörsch (1902) derived the shear stress distribution in reinforced concrete beam containing flexural cracks. Mörsch showed that the shear stress would reach its maximum value at the neutral axis and then remains constant from the neutral axis down to level of the flexural steel. The magnitude of
this maximum shear stress can be predicted by the following relationship,

\[ \tau = \frac{V}{b \cdot z} \]  

(3.1)

where \( b \) = width of member and \( z \) = flexural lever arm. In 1909 Tablot demonstrated that nominal shear strength depends on the strength of concrete, quantity of longitudinal reinforcement, length of beam and depth or stiffness of beam, which were not formulated in a mathematical form. Again in 1920, Mörsch introduced the concept of shear strength, \( (V_u/ bd) \) as a measure of diagonal tensile (DT) strength and later extended the truss analogy to torsion problems. In 1940, Morretto proposed an empirical equation for predicting the nominal shear strength incorporating the strength of concrete and longitudinal reinforcement. Later, Clark (1951) proposed an expression for predicting the nominal shear strength including compressive strength of concrete, longitudinal reinforcement and shear span-to-depth ratio.

However, it was Jourawski in 1856 (Collins, 2001) suggested that prior to cracking, the maximum shear stress in the web can be calculated by the conventional theory for homogeneous, elastic and uncracked beams, Eq. 3.2.

\[ \tau = \frac{VQ}{Ib} \]  

(3.2)

where \( I \) = moment of inertia of the cross section, \( Q \) = first moment about the centroidal axis of the area located above the point at which
the shear stress is required, and \( b \) = width of the member at that section.

A more realistic approach was reported by Kani (1964), in which the beam segments between the inclined flexural cracks act analogous to tooth in the comb thus behaving like a cantilever. The concrete tooth was a free cantilever fixed in the compression zone of the beams and loaded by the horizontal shear from bonded reinforcement. Although this theory did not cover most of the shear transfer mechanisms, it was probably the start of more rational approaches. Fenwick and Paulay’s (1968) work on Kani’s tooth model, brought out the significance of the forces transferred across cracks in slender beams by crack friction. From the studies on Kani’s model, Taylor (1974) revealed that for normal test beams the contributions of the shear resistance were; compression zone shear (20-40%), crack friction (35-50%) and dowel action (15-25%). Reineck (1991) further refined the tooth model, taking all the shear transfer mechanisms into account, carrying out a nonlinear approach including compatibility, to derive an explicit formula for estimating the ultimate shear force.

For members with web reinforcement, the truss models were used as conceptual tools in the analysis and design of reinforced concrete beams. Ritter (1899) and Mörsch (1902) postulated independently that a cracked reinforced concrete beam, due to diagonal tension, can be thought of as a parallel chord truss with compression diagonals inclined at 45° with the longitudinal axis of the beam (ACI-ASCE
Mörsch (1920, 1922) adopted the truss models to study the torsion. In these models the contribution of concrete in tension is neglected. The diagonal compressive stresses in concrete push the top and bottom faces of the beam apart, while tension in stirrups pulls them together. The effects of these two are the same. According to the 45° truss model, the shear capacity is reached when the stirrups yield; and will correspond to a shear stress of

\[ \tau = \frac{A_v f_y}{b s} = \rho v f_y \]  

(3.3)

where \( A_v \) = area of the transverse reinforcement, \( s \) = spacing of the transverse reinforcement and \( f_y \) = yield strength of the transverse reinforcement, and \( b \) = width of member.

The 45° truss model yields conservative results as the direction of the diagonal compressive stresses with the longitudinal axis is less than 45°. Mörsch stated that it is impossible to determine the inclination of the secondary cracks for which the stirrups are designed. But Wagner (1929) solved an analogous problem for shear design of stressed skin aircraft. Wagner assumed that after the thin metal skin buckled, it will continue to carry shear by a field of diagonal tension, provided that it was stiffened by transverse frames and longitudinal stringers (ACI-ASCE Committee 445, 1998). The approach used for determining the inclination of the diagonal tension, Wagner used the deformation of the system, which is known as “tension field theory”.
Kupfer (1964) and Baumann (1972) presented a method for determining the angle $\theta$ assuming that the cracked concrete and the reinforcement were linearly elastic. Collins and Mitchell (1974) based on Wagner theory developed a method for determining “$\theta$” applicable over the full loading range for members subjected to Torsion. This procedure is called the “compression field theory (CFT”). Mitchell and Collins (1974) developed the diagonal Compression Field Theory for members subjected to pure torsion. Later, Vecchio and Collins (1986) presented the Modified Compression Field Theory (MCFT) extending the first theory to members subjected to shear. The MCFT is a further enhancement of the CFT that accounts for the influence of the tensile stresses in cracked concrete that was ignored in truss models. It considers the overall load-deformation responses of elements in which the reinforcement acts in uniaxial tension and the concrete works in biaxial tension/compression. The principal stresses and strains in concrete are assumed to be coincident. The equilibrium equations, compatibility relationships, reinforcement stress-strain relationships, and stress-strain relationships for cracked concrete in compression and tension enable to determine the average stresses, average strains, and angle “$\theta$” for any load level up to failure.

Failure of the reinforced concrete element may not be governed by the average stresses, but rather by local stresses occur at a crack. This so-called crack check is a critical part of MCFT and the theories derived from it. The crack check involves limiting the average principal
tensile stress in concrete to the allowable limit determined by considering the steel stress at a crack and ability of crack surface to resist the shear.

Schlaich et al. (1987) extended the truss model for beams with uniformly inclined diagonals throughout the structure in the form of strut-and-tie models (STM). This approach is particularly relevant in regions where the distribution of strains is significantly non-linear along the depth. Schlaich et al. (1987) also introduced the concept of D and B regions, where D stands for discontinuity or disturbed, and B stands for beam or Bernoulli. In D regions the distribution of strains is non-linear, whereas in B regions it is linear. Normally D and B regions co-exist in a structural concrete member rather than occurring independently.

Belarbi and Hsu (1994, 1995) developed a procedure called Rotating Angle Softened Truss Model (RA-STM) to account for tensile stresses in diagonally cracked concrete. Like MFCT, this method assumes that the inclination of the principal stress, \( \theta \), coincides with the principal strain direction. For typical elements, \( \theta \) will decrease as the shear is increased, hence the name ‘rotating angle’.

### 3.3 Behaviour of RC Beams in Shear

#### 3.3.1 Beams without Web Reinforcement

Leonhardt and Walther (1961) reported the size effect by testing of two series of similar beams without web reinforcement. In the first series,
the cross-section was varied between 50mm × 80mm and 200mm × 320mm with bar diameter proportional to the external dimensions and number of reinforcing bars constant. In the second series, the cross-section was varied between 100mm × 180mm and 225mm × 670mm. The ratios of depth were different from the ratios of the widths, the bar diameter was constant but the number of bars was varied to maintain the given percentage of steel. It has been observed that the shear stress at failure was decreased by 37% between small and large size beams in the first series and by 21% in the second series. The greater loss of strength in the first series was attributed to the poor bond with increase in the bar diameter. However, the tests by Kani (1967) to study the size effect in shear adopted constant width of 150mm using the same concrete strength, percentage steel reinforcement and a/d ratio. Kani demonstrated that as the depth of the beam increases the shear stress at failure decreases. The reduction in the margin of safety for large size beam ($D = 1200\text{mm}$) was 40% lower than that of the small size beam ($D = 150\text{mm}$) with all other parameters kept constant. Fig. 3.1 shows the variation of the ultimate shear strength with shear span-to-depth ratio for beams tested by Kani. It has been reported that with further increase in the beam depth a decrease in the strength may occur. Taylor (1972) concluded from the experimental study that by scaling the size of aggregates correctly, the loss of strength is less significant and the size effect observed in Kani’s test beams was due to large depth-to-breadth ratios.
Fig. 3.1 Percentage variation in shear strength with shear span-to-depth ratio

Iguro and Shioya (1985) conducted experiments on RC beams without shear reinforcement with effective depth varying from 100mm to 3000mm and further reported that the shear strength of beams decreases with increase in the beam size. Shioya et al. (1989) showed that the shear strength was reduced by another 25% when the beam depth was increased from 800mm to 3000mm. The high shear strength in small size beams as reported by Swami and Qureshi (1971), was attributed to the extreme strain gradients. The description of the size effect and its applications to concrete structures has been discussed by Karihaloo (1995).

The earliest empirical equation to determine the diagonal cracking strength and ultimate strength for slender beams without accounting for the size effect was proposed by Zsutty (1968) based on regression analysis by the following expression
\[ v_{cr} = 2.14 \left( f'_{c} \rho_{l} \frac{d}{a} \right)^{1/3} \text{ MPa} \]  

(3.4)

\[ v_{u} = 2.3 \left( f'_{c} \rho_{l} \frac{d}{a} \right)^{1/3} \text{ MPa} \]  

(3.5)

For short and deep beams

\[ v_{u} = (Eqn. 3.4) \left( \frac{2.5}{a/d} \right) \]  

(3.6)

where

- \( v_{cr} \) = diagonal cracking strength
- \( v_{uc} \) = ultimate strength
- \( f'_{c} \) = cylinder compressive strength of concrete,
- \( a/d \) = shear span to depth ratio
- \( \rho_{l} \) = percentage of longitudinal reinforcement, and
- \( d \) = effective depth of beam.

Subsequently, several expressions were developed to predict the diagonal cracking strength of RC beams without web reinforcement. Okamura and Higai (1980) proposed a design equation based on the analysis of the published data, which was modified later by Niwa et al. (1987), taking into the test results of large size beams into account.

The modified equation proposed by Niwa et al. (1987) is as follows

\[ v_{cr} = 1.125 \frac{\rho_{l}^{1/3} f'_{c}^{1/3}}{d^{1/4}} \left( 0.75 + \frac{1.4}{a'/d} \right) \text{ MPa} \]  

(3.7)

where \( d \) is expressed in mm, \( \rho_{l} \) in percentage and \( f'_{c} \) in MPa.
Bazant and Kim (1984) proposed a formula for estimating the mean ultimate nominal shear strength of RC beams, given by Eq. 3.8,

\[
v_{uc} = \frac{10\rho_{l/3}}{d} \left( \frac{0.083\sqrt{f_c'}}{d} + 20.69 \sqrt{\frac{\rho}{\left(\frac{a}{d}\right)^5}} \right) \text{MPa} \quad (3.8)
\]

Bazant and Sun (1987) modified Eq. 3.8, accounting for the size of aggregate \( (d_a) \) as,

\[
v_{uc} = \left( 0.54\rho_{l/3} \right) \frac{1 + \sqrt{\frac{5.08}{d_a}}}{d} \left( \frac{\sqrt{f_c'}}{d} + 249.2 \sqrt{\frac{\rho}{\left(\frac{a}{d}\right)^5}} \right) \text{MPa} \quad (3.9)
\]

In 1991, Bazant and Kazemi showed that there is a strong size effect on the ultimate strength; while on the diagonal cracking strength the size effect is negligible for geometrically similar RC beams in the size range of 1:16. Similar conclusions were drawn by Tan and Lu (1999) for deep beams and also by Walraven and Lehwalter (1994) for beams with shear span-to-depth ratio 1.0. Further, it has been observed that the maximum size of aggregate between 8-32mm showed no significant size effect. The experimental study by Walraven (1993) also concluded that a strong size effect exists on the ultimate shear strength of RC beams without web reinforcement. Chana's (1987) studies based on splitting failure of beams concluded that owing to complex nature of stress in the dowel splitting region, the
Shear design methods for members without web reinforcement are likely to remain empirical in nature.

Reineck (1991) reported a mechanical model for slender members without transverse reinforcement. The model incorporates the size effect and also takes into consideration the transfer of shear force in the tension zone by friction along the cracks and by dowel action of the longitudinal reinforcement, given by

\[ v_u = \frac{0.4 \, b_w \, d \, f_{ct} + V_{du}}{(1 + 0.054 \, \lambda)} \]  

\[ \lambda = \text{dimensionless value for crack width} = \frac{f_c'}{E_i \rho_i \, w_u}, \quad \rho_i = \frac{A_{yw}}{b_w \, d} \]

\[ \omega_u = 0.9 \text{ (limited crack width)}, \]

\[ b_w = \text{width of beam and} \]

\[ E_s = \text{Elastic modulus of steel} \]

Rebeiz (1999) proposed an expression for estimating the diagonal cracking and ultimate strength of RC members without stirrups from 350 data points using dimensional and multiple regression analysis,

\[ v_{cr} = 0.4 + \sqrt{f_c' \, \rho_i \, (d/a) \left[ 2.7 - 0.4A_d \right]} \]

\[ v_u = 0.4 + \sqrt{f_c' \, \rho_i \, (d/a) \left[ 10 - 3A_d \right]} \]  

\[ f_{ct} = \text{axial tensile strength} = 0.246 f_c'^{2/3} \text{ MPa} \]

\[ V_{du} = \text{dowel force} = \left( 1.33 - \frac{\rho_i^{1/3}}{f_c^{1/3}} \right) b_w \, d \, f_c' \]  

\[ (f_c' \text{ in MPa and } d \text{ in m}) \]
where \( A_d = \) Shape adjustment factor = \( a/d \) for \( 1.0 < a/d < 2.5 \)

\[ = 2.5 \text{ for } a/d \geq 2.5 \]

Collins and Kuchma (1999) deduced that HSC members are more sensitive to the size and the reduction in shear stress at failure is directly related to the maximum spacing between the longitudinal reinforcement layers than the overall depth of member. Zararis and Papadakis (2001) analytical studies on the size effect showed that shear stress at failure is the product of the ratio of neutral axis depth-to-the effective depth of beam times the splitting tensile strength of concrete. Studies carried out by Yang et al. (2003) on HSC deep beams showed that beams with \( a/h = 0.5 \) are less sensitive to the beam size than those with \( a/h = 1.0 \).

### 3.3.2 Beams with Web Reinforcement

The researchers working in reinforced concrete (RC) before 1900 had two distinct thoughts about the mechanism of shear failures (ACI-ASCE Committee 326, 1962). One section considered the horizontal shear as the basic cause for shear failures considering that concrete alone could resist low horizontal shearing stresses and that vertical stirrups act as shear keys for high shear stresses. However, the second section considered the diagonal tension as the basic cause for shear failures. A clear explanation of diagonal tension was presented by Ritter (1899), stirrups resist tension, not the horizontal shear and an expression for design of vertical stirrups was given by
\[ V = \frac{A_n f_y j d}{s} \quad (3.12) \]

However, Ritter’s view points were not accepted. Mörsch (1902) showed that if a state of pure shear stress exists, then a tensile stress of equal magnitude must exist on a 45° plane. The equation for nominal shear stress, widely used in the design, (Eq. 3.13) was developed

\[ v = \frac{V}{bd} \quad (3.13) \]

At any point in a homogeneous and isotropic beam the diagonal tension stress can be related to the shearing stress, \( v \) and the flexural tension stress, \( f_i \) through the equation,

\[ f_i = \frac{f_t}{2} + \sqrt{\frac{f_t^2}{4} + v^2} \quad (3.14) \]

In RC beams where the shear stress is high, the flexural stress is relatively low and consequently the diagonal tension stress, \( f_i \) is equal to the shearing stress, \( v \). The nominal shear stress proposed by Mörsch is a measure of the diagonal tensile stress.

The 45° truss model conceived by Ritter and Mörsch was generalised by Lampert and Thurlimann (1969). For members subjected to torsion or to the combined torsion and bending, the inclination of the diagonal concrete struts was variable, depending on the volume ratio of the longitudinal steel-to-the transverse steel. This
model is known as the variable-angle truss model. This model was applied to shear and bending by Grob and Thurlimann in 1976 (Belarbi and Hsu, 1990). Since this theory also assumes to have infinite plasticity, it was also called the plasticity truss model.

Russo and Puleri (1997) were of the opinion that inclusion of stirrups in a beam leads to complex interaction between beam, arch and truss resisting mechanisms at failure. Hence, the shear capacity of the beams cannot be estimated by superposing the shear capacity drawn from 45° truss model to the beam without stirrups, as approved by the national codes. Further, a mechanical model was proposed to obtain the mean ultimate shear stress of RC beams with stirrups by considering the stirrup effectiveness ratio, defined as the ratio of effective mean shear stress increase due to stirrup inclusion and the conventional one drawn from 45° truss model. Lee and Watanabe (2000) proposed a shear design method for RC beams with shear reinforcement considering different failure modes. The proposed equation for predicting the shear strength of RC beams was based on two shear failure modes, i) shear failure mode after yielding of shear reinforcement (STF) and ii) concrete crushing failure before yielding of shear reinforcement (SCF), which can be determined as a function of the material properties. In calculating the shear strength of the beam failing in STF mode, the strain hardening effect of steel is also included. Pendyala and Mendis (2000), based on the comparison of test results of beams made from HSC and the existing formula from
five different codes, assessed the margins of safety. The variables include stirrup spacing, strength of concrete and shear span-to-depth ratio \((a/d)\). It has been revealed that the shear strength of concrete does not increase in the range of 50 MPa to 70 MPa. The shear strength of concrete appears to level off above the concrete strength of 90 MPa. Smith and Vantsiotis (1982) have shown that the presence of vertical reinforcement increases the ultimate shear strength but the effectiveness decreases for beams with \(a/d < 1\) and the presence of horizontal reinforcement has no impact on the ultimate strength at \(a/d > 1.0\).

### 3.3.3 Minimum Web Reinforcement

The minimum web reinforcement should be provided as per the codes of practice, since the shear failures of beams without web reinforcement are brittle and sudden. The minimum web reinforcement is intended to prevent such sudden failures immediately after formation of the first diagonal tension cracking and also to control the diagonal cracks at the service loads. Recently, several efforts have been made to understand the behaviour of RC beams with minimum shear reinforcement and a brief review is provided.

Johnson and Ramirez (1989) investigated the adequacy of the minimum web reinforcement in HSC beams. RC beams with concrete compressive strength in the range of 34.5 MPa to 72.5 MPa provided
with shear reinforcement indices SRI, $r_{fy}$ (where $r = A_{sv}/bs_v$) of 0.0 to 0.7 MPa under four-point loading designed to fail in shear. It was concluded that the overall reserve shear strength after diagonal tension cracking diminished with increase in compressive strength of concrete in beams with minimum shear reinforcement as per ACI code. This situation would be more critical for beams with large a/d ratios and small amount of longitudinal reinforcement. Further, it was concluded that i) the ratio of the observed to the predicted ultimate capacity ($V_f/V_n$) increases with increasing the web reinforcement from 0.88 to 1.16 for beams with web reinforcement indices ($r_{fy}$) of 0 to 0.6895 MPa, where ($r = A_{sv}/bs_v$) ii) the reserve strength increases significantly as the web reinforcement index increases from 0.35 to 0.69 MPa. Roller and Russell (1990), based on the experimental study, concluded that the minimum shear reinforcement specified in the codes must increase with the compressive strength. Krauthammer (1992) evaluated the minimum shear reinforcement in RC beams based on the interface shear transfer across a crack. It has been reported that the minimum shear reinforcement by ACI code could be low. Modified the minimum shear reinforcement to produce a shear resistance equal to that provided by effective aggregate interlock along the shear cracks.

An experimental study on ductility of critically reinforced concrete beams in shear with and without web reinforcement has been reported by Xie et al. (1994). The variables were; concrete strength
(40–109 MPa), shear span-to-depth ratio (1.0–4.0) and shear reinforcement index (0.0–0.784). The post-peak deformation characteristics were quantified through shear ductility and HSC beams with $a/d$ ratio equal to 3.0 exhibit plastic post-peak response, when the shear reinforcement is twice that of the ACI code. Yoon et al. (1996) concluded that for HSC members the crack spacing is a function of spacing of longitudinal and transverse reinforcement. The tests conducted by Yoon et al. (1996) on twelve full size beams with different amounts of minimum shear reinforcement revealed that with HSC, the minimum reinforcement specified may not provide adequate reserve strength after shear cracking unless the shear reinforcement can develop significant strain hardening.

The equation proposed by Yoon et al. (1996) was re-evaluated by Ozcebe et al. (1999) and concluded that the reserve strength increases with increase in shear reinforcement index and also proposed an equation for evaluating the minimum shear reinforcement.

Based on the comparison of two large size beams tested by Johnson and Ramirez (1989) and the tests on reduced size beams, Frosch (2000) concluded that the beam size did not affect the post cracking behaviour or the shear strength of the stirrups. Experiments conducted by Angelakos (2001) also indicated that the minimum amount of stirrups provided by ACI code have inadequate safety margins.
Lin and Lee (2001) reported that an increase in the quantity of tension reinforcement increases the strength but decreases the ductility of beams. Further, an increase in the quantity of compression steel and concrete strength enhances the ductility of the beams effectively; while the beams with high strength shear reinforcement exhibit the same crack control ability as that of the beams with normal strength shear reinforcement. In another study by Lin and Lee (2003) it has been reported that the factors affecting the ductility are $a/d$ ratio, spacing of stirrups and strength of shear reinforcement. Further, increase in the amount and strength of shear reinforcement do not have apparent influence on the diagonal cracking strength. However, increasing the concrete strength and the strength of shear reinforcement; decrease in shear-span-depth ratio and stirrup spacing increase the ultimate strength. For beams with small $a/d$ ratio the effectiveness of shear reinforcement is much low.

The study carried by Ashour and Morley (1996) has also shown that the effect of horizontal and vertical web reinforcement on the load carrying capacity is mainly influenced by the shear span-to-effective depth ratio. Deeper the beam, less effective is the vertical reinforcement, and more effective is the horizontal web reinforcement.

### 3.3.4 Studies Based on Fracture Mechanics

The expressions proposed for diagonal shear failure were either purely empirical or based on the plastic limit analysis owing to the fact that
flexural cracking precedes the inclined cracking which disrupts the elastic stress field. However, Reinhardt (1981) suggested that the design formula for diagonal shear be based on LEFM. Further, it was established that the size effect by LEFM is too strong for concrete and that brittle failures of concrete structures are better described by NLFM, which is based on the large crack front thus leading to a considerable weaker size effect.

Though the linear elastic fracture mechanics (LEFM) was developed for linear elastic brittle materials, later attempted first to apply to plain concrete by Kaplan (1961), which was unsuccessful due to inherent heterogeneity and development of fracture process zone (FPZ) before failure in a cracked concrete causing tension softening.

Based on non-linear elastic fracture mechanics (NLEFM) Gustafsson and Hillerborg (1988) studied the size effect on shear strength of longitudinally reinforced concrete beams. It was proposed that the beam size should be normalised to an intrinsic length measure ($l_{ch}$) of the concrete as a function of its fracture energy, $G_F$. The form of the intrinsic length, $l_{ch}$ in the proposed shear strength formula is as follows,

$$
\frac{f_s}{f_t} = k \left( \frac{d}{l_{ch}} \right)^{-0.25}
$$

(3.15)

where $l_{ch} =$ characteristic length = $(EG_F/f_t^2)$. The constant of proportionality depends on parameters other than $d, f_t, G_F$ and $E,$
such as shear span-to-depth ratio, percentage of longitudinal reinforcement, yield strength of reinforcement, $f_v$, shear strength = $V_u / bd$; $f_t$ = tensile strength of concrete; $E$ = Modulus of elasticity; $d$ = depth of member.

The ratio $d/l_{ch}$ is the measure of brittleness of structures sensitive to the tensile stress induced fracture, a higher ratio corresponds to more brittle fracture. Fujita et al. (2002) showed that the size effect on shear strength is related to the compressive strength of concrete. The formulae given below were proposed according to the fracture mechanics study,

$$\frac{V}{bd} \propto 0.32 \left( \frac{l_{ch}}{d} \right)^{0.25} \text{ for NSC} \quad \text{and} \quad \frac{V}{bd} \propto 0.30 \left( \frac{l_{ch}}{d} \right)^{0.5} \text{ for HSC} \quad (3.16)$$

where $l_{ch} = 30700 \cdot f_c^{-1.1}$

Gastebled and May (2001) developed a fracture mechanics model for the flexure-shear failure of RC beams without stirrups. The ultimate shear strength was assumed to reach when a splitting crack at the level of longitudinal reinforcement propagates.

$$\frac{V_{cr}}{bd} = 1.109 \left( \frac{H}{d} \right)^{\frac{1}{3}} \left( \rho \right)^{\frac{1}{3}} \left( 1 - \sqrt{\rho} \right)^{\frac{2}{3}} f_c^{-0.35} \sqrt{E} \quad (3.17)$$

Other fracture mechanics models have been proposed to account for the fact that a peak tensile stress near a crack tip and a reduced tensile stress (softening) is located in the crack zone. This approach
explains the size effect in shear. The two popular models are; fictitious crack model (Hillerborg et al. 1976), and crack band model (Bazant and Oh, 1983).

### 3.4 Review of Code Provisions

#### 3.4.1 American Standard – ACI: 318-2008

The design provisions of RC beams for shear by many national codes of practice are conservative. As mentioned in Chapter 2, these provisions are based on the experimental observations from NSC beams of depth less than about 400mm. Further, IS 456 and ACI 318 do not incorporate the size effect in the design provisions. In this context a brief review of different code provisions is discussed below.

The ACI code assumes that flexure and shear can be handled separately for the critical combination of flexure and shear at any section. The interaction between flexure and shear is taken care indirectly by detailing of reinforcement near flexural reinforcement cut-off points. In addition, checks on the level of stresses in concrete in the member are introduced to ensure adequate ductile response and control of diagonal cracking at the service loads. The ACI 318 design approach for shear is based on a parallel truss model with 45° inclined diagonals supplemented by the concrete contribution.

The lower-bound average shear stress at diagonal cracking is given as below
This is well-known ACI formula, which gives a reasonable lower bound value for small slender beams without axial load, provided with at least 1.0% longitudinal reinforcement (ACI-ASCE Committee 445, 1998). Eq. 3.18 is un-conservative for RC beams with very small quantity of reinforcement in HSC beams.

Eq. 3.19 overestimates the contribution of concrete and underestimates the influence of longitudinal reinforcement and
$V_u/M_u d$. The ACI code does not specify any procedure for the design of deep beams, it stipulates a rigorous nonlinear analysis to be carried out or design be based on provisions given in Appendix A of the code (Strut and tie models). The use of strut and tie model based on D and B regions demands for a careful selection of force path based, thus leading to vast variation in the design methodology. Considering these facts and conservative results obtained through the use of strut and tie models the ultimate and diagonal cracking strength of beams in the present study with $a/d < 1.0$ are worked out based on ACI code provisions given below

$$v_c = \left(3.5 - 2.5 \frac{M_u}{V_u d}\right) \left(0.16 \sqrt{f_c} + 17.2 \rho_t \frac{V_u d}{M_u}\right) \leq 0.5 \sqrt{f_c} \quad \text{MPa}$$

where \(3.5 - 2.5 \frac{M_u}{V_u d}\) \leq 2.5

(3.20)

### 3.4.2 British Standard - BS: 8110-1997

BS 8110 (1997) incorporates the size effect in the design expression. For beams with $a/d > 2$, the nominal shear strength is as follows,

$$v_c = 0.79 \left(\frac{100 A_{cu}}{bd}\right)^{1/3} \left(\frac{400}{d}\right)^{1/4} \left(\frac{f_{cu}}{25}\right)^{1/3}$$

(3.21)

$$v_c = (\text{Eq. 3.21}) \left(2 \frac{d}{a}\right) \quad \text{for} \quad \frac{a}{d} < 2.0$$

(3.21a)

where \(\frac{100A_{cu}}{bd} \leq 3.0\) \quad \(\frac{400}{d} \geq 1.0\) \quad $\gamma_m = 1.25$ and $f_{cu} \leq 40.0$ MPa
The condition of $(400/d) \geq 1.0$ restricts the effectiveness of the beam depth up to 400mm only. Further, the concrete strength is limited to 40 MPa and longitudinal reinforcement ratio 3.0% which seems to be conservative.

### 3.4.3 Indian Standard - IS: 456-2000

IS 456 (2000) considers the shear strength of RC beams in the following form,

$$
\nu_{cr} = \frac{0.85 \sqrt{(0.8f_{ck})} \sqrt{1 - 5\beta} - 1}{6\beta} \leq 0.62 \sqrt{f_{ck}} \quad (3.22)
$$

where \( \beta = \frac{0.8f_{ck}}{6.89 \rho_l} \geq 1.0 \)

For \( a/d < 2.0 \) \( \nu_{cr} = (Eq. 3.22) \left[ \frac{2}{a/d} \right] \) \( (3.22 a) \)

in which \( 0.8f_{ck} \) = cylinder strength in terms of cube strength and 0.85 = reduction factor \( (1/\gamma_m) \).

Since the effect of \( a/d \) ratio on the strength of slender beams is not very significant, the term \( a/d \) is not reflected in Eq. 3.22.

### 3.4.4 Eurocode 2 - 2002

The design shear as per Eurocode 2, does not require any shear reinforcement in non pre-stressed members is given by,
$$V_{Rd,c} = \left[ \frac{0.18}{\gamma_c} k(100 \rho_1 f_{ck})^{1/3} + k_1 \sigma_{cp} \right] b_w d \quad (3.23)$$

with a minimum of

$$V_{Rd,c} = \left[ v_{min} + k_1 \sigma_{cp} \right] b_w d \quad (3.23a)$$

Where

$$v_{min} = \left[ 0.035 k^{3/2} f_{ck}^{1/2} \right]$$

$V_{Rd,c}$ = design shear strength of member without shear reinforcement, N

$f_{ck} \leq 100$ MPa

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0, \ d \ is \ in \ mm \quad \rho_1 = \frac{A_{sl}}{b_w d} \leq 0.02$$

$A_{sl}$ = area of anchored tensile reinforcement

$b_w$ = smallest width of cross section in tensile zone

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0.2 f_{cd}$$

$N_{Ed}$ = axial force (N) in the section due to loading or pre-stress, greater than zero for compression

$A_c$ = area of concrete cross section, mm$^2$

$\gamma_c$ = partial safety factor for concrete = 1.5

$k_1 = 0.15$
The design of members with shear reinforcement is based on the truss model. For members with vertical shear reinforcement, the shear resistance, $V_{Rd,s}$ is taken lesser of the two expression below,

$$V_{Rd,s} = \frac{A_{yw}}{s} z f_{ywd} \cot \theta$$

(3.24)

or

$$V_{Rd,max} = \frac{a_c b_w z v f_{cd}}{\cot \theta + \tan \theta}$$

(3.24a)

The limiting values for $\cot \theta$ are given by the expression, $1 \leq \cot \theta \leq 2.5$

where,

$A_{sw} = \text{cross sectional area of the shear reinforcement}$

$s = \text{spacing of stirrups}$

$\theta = \text{angle between the concrete strut and the beam axis perpendicular to the shear force}$

$z = \text{inner lever arm} = 0.9d \text{ for members without axial force}$

$f_{ywd} = \text{yield strength of the shear reinforcement}$

$v = 0.6 \text{ for } f_{ck} \leq 60 \text{ MPa } \& 0.9-f_{ck}/200 (> 0.5) \text{ for } f_{ck} \geq 60 \text{ MPa}$

$a_c = 1.0, \text{ for non pre-stressed structures}$

3.4.5 CEB-FIP Model Code - 1990

According to CEB-FIP model code the shear causing cracking may be estimated as,

$$V_{cr} = \left[ 150 \left( \frac{3d}{a_v} \right)^{1/3} \xi (100 \rho f_{ck})^{1/3} \right] b_w d \text{ kN}$$

(3.25)
where

\[ a_v = \text{distance from major load to support} \]

\[ \zeta = 1 + \sqrt{\frac{200}{d}} \text{ with } d \text{ in mm} \]

\[ \rho = \frac{A_i}{b_w d} \], ratio of flexural tensile reinforcement anchored at the support

\[ \left( \frac{3d}{a_v} \right)^{1/3} \], empirical expression taking the influence of transverse compression from the loads and support reaction.

In CEB-FIP model code, Zsutty’s power laws are adopted, by adding an extra term to account for the size effect.

### 3.5 Summary

The developments made on RC beams in shear without and with web reinforcement have been discussed. The important contributions include; Kani’s tooth model, Compression Field Theory for members subjected to torsion and then the Modified Compression Field Theory for members in shear, Strut-and-Tie models, Reineck’s mechanical model, Rotating Angle Softened Truss Model to account for tensile stresses in diagonally cracked concrete. There are models based on regression analysis and expressions proposed by Bazant and co-
workers. Gastebled and May developed fracture mechanics based model for flexural-shear failure of RC beams without stirrups. Then the different provisions for design of beams in shear by various codes of practice viz., ACI, BS and IS have also been presented.

The problem of RC beams in shear has not yet been resolved completely or there is no single unified solution to predict the shear strength of all types of beam (deep, short and slender). In Chapter 4 an attempt has been made to develop the expressions for predicting the diagonal and ultimate shear strength of RC beams without web reinforcement from nonlinear regression analysis (NLRA) of the experimental data.