IA. COLLECTIVE MODEL
1. Collective Model:

In the shell-model the average nuclear field in which the nucleons move more or less independently is assumed to have spherical symmetry. The equipotentials are spheres described by \( R = R_0 \). Such fields are associated with closed shells. The nucleons outside a closed shell (core) tend to polarise this spherically symmetric field giving rise to a deformed average nuclear field, while the core resists this polarisation and maintain a spherical shape. Thus the nucleus may vibrate about its spherical equilibrium shape giving rise to vibrational spectra, for which there is ample experimental evidence. During the course of vibration the radial coordinates of a point on the surface can be represented by

\[
R(\theta, \phi) = R_0 \left\{ 1 + \sum_{\lambda \mu} \alpha_{\lambda \mu} \varphi_{\lambda \mu}(\theta, \phi) \right\},
\]

where \( \varphi_{\lambda \mu} \) is the normalised spherical harmonic of order \( \lambda, \mu \), and the expansion parameters \( \alpha_{\lambda \mu} \) are functions of time. In fact, these quantities may be looked upon as the collective coordinates that are oscillating. Since \( R \) must describe a real surface, the reality condition

\[
\alpha_{\lambda \mu} = (c^{\dagger})_{\mu} \alpha_{\lambda, -\mu}^{*}
\]

must be satisfied.

The above expression (1) is the most general multipole expansion of a function of \( \theta \) and \( \phi \). The monopole term \( \lambda = \mu = 0 \) corresponds to bodily expansion and contraction of the nucleus. The nuclear matter being very much incompressible, this sort of an oscillation requires a very large energy. Even if...
such a monopole oscillation exists, it will not be found among the low-lying excited states of a nucleus. The next higher multipole $\lambda = 1$ denotes a displacement of the centre-of-mass of the nucleus, and hence does not correspond to any oscillation at all. The lowest multipole that corresponds to a low energy oscillation is thus $\lambda = 2$, i.e., a quadrupole oscillation.

Let us discuss the type of nucleus which exhibits only small instantaneous quadrupole deformation about a spherically symmetric equilibrium nuclear force field, and hence write

$$R(\theta, \phi) = R_0 \left\{ 1 + \sum_{\mu=-2}^{2} \alpha_\mu \mathcal{Y}_\mu^2(\theta, \phi) \right\}.$$  

There are five possible $\alpha_\mu$ corresponding to $\mu = -2, -1, 0, 1, 2$.

Let us now introduce the collective hamiltonian $H$ given as

$$H = V(\alpha_\mu) + T(\dot{\alpha}_\mu, \alpha_\mu).$$  

Here, $V(\alpha_\mu)$ represents the potential energy of the nucleus and is dependent upon the shape of the nucleus through the $\alpha_\mu$. $T(\dot{\alpha}_\mu, \alpha_\mu)$ represents the kinetic energy of the collective motion which takes place as the nucleons follow the relatively slow changes in the quadrupole field. For systems which prefer spherical symmetry, one normally expands $H$ in a power series in the $\alpha_\mu$ and $\dot{\alpha}_\mu$. If the time variation of the dynamical variables $\alpha_\mu$ for the collective motion, is assumed to be adiabatic with respect to the motion of the nucleons, then the lowest-order (zeroth approxi-
The quantities $B$ and $C$ depend on more detailed assumptions regarding the properties of nuclear matter and must be calculated from a more specific model or must be treated as empirical parameters to be determined from experiment. The equilibrium shape of the nucleus being spherical, there is no natural choice of an axis along which the projection $\mu$ has to be taken. Since the axis of projection may be chosen arbitrarily, the frequency of vibration cannot depend on its choice, i.e., on the value $\mu$. This is the reason we have not put the label $\mu$ on $B$ and $C$.

In order to solve the dynamical problem given by Eq. (4), we introduce the canonical momentum $\Pi_\mu$ conjugate to the coordinate $a_\mu$, defined by

$$\Pi_\mu = \frac{\partial (\tau - \gamma)}{\partial a_\mu}$$

If the number of nucleons outside the deformable core is small, then the zero-point energy of the oscillation is greater than the energy of deformation and the core shape, on the average, does not possess a stable deformation and we must deal with a theory of quantised surface oscillations. If we want to quantise the nuclear surface oscillations (considered as a system of harmonic oscillators) given by Eq. (4), we must require the commutation rule
From the knowledge about the harmonic oscillator in quantum mechanics we know that the energy of the five-dimensional oscillator under consideration, will have values given by

\[ E = \hbar \omega \sum_{\mu} (n_{\mu} + \frac{1}{2}) = \hbar \omega (N + \frac{5}{2}) \]

where \( \omega \) is the frequency of the surface oscillation, given by

\[ \omega^2 = \frac{c}{\beta} \]

and

\[ N = \sum_{\mu} n_{\mu} = 0, 1, 2, \ldots, \infty, \]

is the number of oscillator quanta, each of energy \( \hbar \omega \). \( 5/2 \hbar \omega \) is the zero-point energy for the five-dimensions. Each quantum carries 2 units of angular momentum, and can be created by the 'creation operator'.

\[ \eta^+_{\mu} = \frac{1}{\sqrt{2}} b \left\{ \alpha_{\mu} - i \hbar^{-1} b \pi_{\mu} \right\}, \]

where

\[ b^2 = \frac{\hbar}{\beta \omega} \]

The hermitian conjugate operator, \( \eta_{\mu} \), is the 'annihilation operator'. The meaning of the creation and annihilation operators is as follows: Operating on a state of \( N_{\mu} \) oscillator quanta they give rise to states having \( N_{\mu} + 1 \) and \( N_{\mu} - 1 \) quanta, respectively, by the following rules:

\[ [\alpha_{\mu}, \pi_{\mu}'] = -i \hbar \delta_{\mu \mu'} \]
These operators satisfy the following commutation relations:

\[ [\eta^+_\mu, \eta^+_{\mu'}] = [\eta^+_{\mu}, \eta^+_{\mu'}] = 0 \]
\[ [\eta_\mu, \eta^+_{\mu'}] = \delta_{\mu, \mu'} \]

and the number operator \( \eta_\mu \) of Eq. (6) is defined as

\[ \eta_\mu = \eta^+_{\mu} \eta_{\mu} \]

The ground-state of the nucleus will be characterised by the absence of any oscillator quantum (sometime called 'phonon'), and is denoted by \( |N=0\rangle \). The first excited vibrational state has one phonon, energy \( \hbar \omega \), an angular momentum two and even parity, \( |N=1, I=2^+\rangle \). Now, if we consider vibrational states having two phonons there can be several states corresponding to the coupling of the angular momenta of the two phonons. Coupling two angular momenta each of value 2, we obtain states having total angular momentum \( I = 0, 1, 2, 3, 4 \). Of these states, \( I = 1 \) and 3 will be antisymmetric under exchange of the two phonons and \( I = 0, 2, 4 \) will be symmetric under this exchange. Since the phonons have integral spins, they are bosons, and hence they must be symmetric under exchange. Therefore, there can be three two-phonon states \( I = 0, 2, 4 \), each of positive parity and energy \( 2 \hbar \omega \).
Let us now consider the electric quadrupole transition between vibrational states. By definition, the electric quadrupole transition operator is given by

$$\mathcal{M}_m(E_2) = \int \rho(x) \left( \gamma^2 \gamma^2 \right) \, d^3r$$

where $\rho(x)$ is the charge density at the point $x$ and the integration is extended over the nuclear volume. Under the assumption that the electric charge distribution of the nucleus is that of a uniformly smeared out volume distribution, the $E_2$ operator takes the form

$$\mathcal{M}_m(E_2) = \frac{3Ze}{4\pi R_0} \int_0^R \gamma^2 \gamma^2 \, d^3r$$

$$= \frac{3Ze}{4\pi} R_0 \left( \gamma^2 \gamma^2 \right)$$

$$= \frac{3Ze b R_0^2}{4\pi} \left\{ \gamma \mu + (-1)^m \gamma^+ \gamma^- \right\}.$$

We thus find that the electric quadrupole operator can either create or annihilate one phonon. Hence the two-phonon states $I = 0^+, 2^+, 4^+$, can decay only to the one-phonon state $I = 2^+$, and not to the ground-state $0^+$. The latter transition will necessitate the annihilation of two phonons. The one-phonon state $I = 2^+$, similarly, will decay to the ground-state $0^+$, and this transition will be very strong. Because the operator contains the total nuclear charge $Ze$, the transition matrix element will also be proportional to this quantity and hence very large. Since the whole nuclear charge parti-
icipates in the transition, this is clearly a collective process.

According to the angular momentum and parity selection rule the two-phonon $2^+$ state can decay to the ground-state; however the phonon selection rule discussed above strictly forbids it. This two-phonon state, therefore, predominantly decays to the one-phonon $2^+$ state by an $E_2$ transition. Also, in all the nuclei for which data on two-phonon triplet are available, these levels are never degenerate and the ratio of their energies to the energy of the first $2^+$ excited state deviates from the value two predicted by the harmonic hamiltonian.

2. Splitting of the Two-Phonon States:

Without a more fundamental approach, a purely phenomenological model can attempt to account for energy splitting of the two-phonon states by carrying the expansion for the collective kinetic and potential energies of (3) in the $\alpha\mu$, beyond the zeroth approximation. The extended hamiltonian up to the cubic terms in the expansion (there is only one cubic potential term and only one cubic kinetic energy term linear in $\alpha\mu$), thus becomes

$$H = \frac{1}{2} c \sum_{\mu} |\alpha\mu|^2 + \frac{1}{2} B \sum_{\mu} |\alpha\mu|^2 + \frac{h\omega_c^2}{(\alpha')^3} \sum_{\mu_\nu} \left[ \begin{array}{ccc} 2 & 2 & 2 \\ \mu & \nu & \mu+\nu \end{array} \right]$$

$$\times \alpha\mu \alpha\nu \alpha\mu+\nu - \frac{B (\alpha')^2}{\alpha} \sum_{\mu_\nu} \left[ \begin{array}{ccc} 2 & 2 & 2 \\ \mu & \nu & \mu+\nu \end{array} \right] \alpha\mu \alpha\nu \alpha\mu+\nu,$$ (13)

where the notation $\left[ \begin{array}{ccc} 2 & 2 & 2 \\ \mu & \nu & \mu+\nu \end{array} \right]$ stands for a Clebsch-Gordan coefficient and $\alpha' = \frac{h\omega_c}{c}$. The form of the terms in (13) is
dictated by considerations of rotational invariance and time reversal invariance.

Using parameters such as $\beta'$ and $C'$ of (13) as additional empirical parameters whose values are to be found by comparing the theory with the experimental data, Kerman and Shakln were able to obtain a good fit for Ni$^{62}$, but they did not have success in fitting vibrational nuclei in general. However, as they point out, inclusion of cubic terms without quartic terms may not be logically consistent. The cubic terms contribute only matrix elements off-diagonal in $N_j$; thus they contribute to the energy in second order of perturbation theory while quartic terms (even if smaller) have matrix elements diagonal in $N$ and may thus give contributions of the same order of magnitude as the cubic ones. Since there is one quartic potential constant and three additional kinetic energy constants in the next term of the expansion, the number of needed empirical constants becomes much too large. What is obviously needed is an approach which should start with fewer and more fundamental parameters like the strengths of the pairing force and quadrupole force in a microscopic theory. The rest of the thesis is devoted to the fulfilment of this basic idea. In part IB of this thesis we have tried to gain a more basic understanding of how collective effects can develop in the framework of the shell-model, and the exact shell-model configuration-mixing calculations for two (Ni$^{58}$) and four (Ni$^{60}$) identical nucleons have been performed, using various two-nucleon effective interactions. The configuration-mixing calculation for nucleon number exceeding four in one of the unfilled major shell
becomes extremely involved. Such calculations are made possible by the development of the quasiparticle theory. The Part II of this thesis describes the complete development of the quasiparticle modified Tamm-Dancoff approximation (MTDA) and its application to single-closed-shell nuclei (even Ni and Sn isotopes). Its validity has been established by the good agreement obtained for Ni isotopes, by the comparison of the exact shell-model and MTDA results.
IB. EXACT SHELL-MODEL CALCULATION FOR $^{58}$Ni AND $^{60}$Ni
1. INTRODUCTION

The success of the shell-model type approach in nuclear structure theory is well-established. In this model, the nucleons are assumed, in the first approximation, to move independently in an overall average field and exhibit some shell structure. The overall potential is usually assumed, rather than derived, in nuclear structure work; and is generally taken as the isotropic harmonic-oscillator-type. The introduction of the spin-orbit interaction of a nucleon explains all the magic numbers, consisting of a close-lying groups of energy levels constituting major shells. The adjacent major shells are separated by larger energies compared to the level spacings inside a major shell. In the nuclear structure work it is assumed that the major shells form an inert core and the nuclear properties are attributed only to the nucleons present in the unfilled major shell. The effect of the residual interactions (this is the part of the nuclear two-body interaction that is not averaged out in the production of the average field) is then included using approximate methods, which are assumed to be sufficiently accurate for calculating the low-energy properties of the system. An exact treatment must take into account the presence of all the nucleons in the unfilled major shell with some residual interaction between them. In the past, this model has been extensively used for the study of properties of nuclei having only few nucleons outside the closed (major) shell. As long as we are considering low-lying states, and the number of nucleons to be treated are not too many, one is required to do a straightforward diagonalisation of the total shell-model hamiltonian using all the
possible configurations of the few nucleons as basis. However, when
many nucleons (nucleon number exceeding four) are present in one of
the large major shells and are distributed among several single-
particle levels, the number of such configurations of the nucleons
as basis, becomes numerous and the standard shell-model calculations
become prohibitively complicated. The labour involved in calculating
the fractional parentage coefficients and evaluating the matrix
elements of the two-body potential is enormous and thus, for most
nuclei, such a straightforward calculation cannot be performed even
on the present-day fast computers. Therefore, approximate methods
have been developed to make such calculations possible. The most —
important and recent development in this direction is the quasi-
particle theory of the nucleus which neatly takes into account the
strong "pairing interaction" between nucleons by the Bogoliubov-
Valatin transformation\(^3\) resulting in independent quasiparticles.
Since the number of quasiparticle considered is not too many (two
in the case of one-phonon state and four in the case of two-phonon
states of even nuclei and one and three in the case of odd-mass
nuclei) and therefore, the configuration-mixing calculations are
indeed tractable. The details of the quasiparticle calculations on
even Ni and Sn isotopes have been described in Part II of this
thesis, and all the necessary formulae and other details on odd-mass
(Ni isotopes) are contained in the recent publication\(^23\) by the
author. The details of the work on odd-mass Ni isotopes have not
been included in this thesis.
Several authors have calculated the spectra of even Ni isotopes by the application of quasiparticle method. The protons in the Ni isotopes are usually assumed to form an inert core corresponding to the semimagic number 28, while the neutrons in excess of 28 are assumed to be distributed amongst the close-lying $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$ orbitals. Some of the defects of the earlier calculations have been remedied in more recent calculations and have been described in full detail in Part II of this thesis, which use a complete set of nonredundant and orthonormal two- and four-quasiparticle states as basis; the effect of the spurious states from two- and four-quasiparticle basis states has also been eliminated in the work of Refs. 10, 11 and has been given in Part II of this thesis.

It is, however, interesting to note that as long as Ni is assumed to be a single-closed-shell nucleus, and the number of near-lying orbitals available to the neutron is only three, a direct configuration-mixing calculation without taking recourse to the approximate quasiparticle method, is quite feasible in at least the two lightest isotopes, Ni$^{58}$ and Ni$^{60}$, which have two and four neutrons, respectively, in the unfilled orbitals.

In this part of the thesis we calculate the energy spectra and other properties of Ni$^{58}$ and Ni$^{60}$ by the direct diagonalization of various effective potentials using all the available jj-coupling shell-model neutron configurations. The jj-coupling states included in our study are all those which can arise by distributing two and four neutrons in the orbitals $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$ in all possible
ways. Apart from the general interest in the results as such, this work offers valuable assistance in the quasiparticle calculation for the heavier Ni isotopes, presented in Part II of this thesis, where a direct configuration-mixing calculation becomes extremely laborious. In the first place, a comparison of the results of this work with those obtained by quasiparticle method for Ni$^{58}$ and Ni$^{60}$ tests the accuracy of the MTDA formalism, developed in Part II of this thesis, which is itself approximate. Secondly, the effective potentials that are found to be satisfactory in the present work may then be used in the quasiparticle calculations for the heavier Ni isotopes.

The different types of two-body potentials as the residual interaction used here in the shell-model calculations of Ni$^{58}$ and Ni$^{60}$ can be classified into three main groups.

1) The first group is the smooth local conventional potentials, widely used in the shell-model calculations, involving a number of adjustable parameters, i.e., the range, depth, and the exchange mixtures. These parameters are determined by requiring a fit to certain experimental data, namely, the energy of the low-lying states. Once the parameters are determined, we generate wave functions and use them to calculate other physical properties of the nuclei. The values of the parameters determined in this way are sensitive to the number of single-particle levels included in the shell-model calculation and to the way in which core contributions are treated. The potential so determined can be called an
effective potential and it depends on the extent of configuration-mixing considered in the calculation.

(2) The second group consists of the realistic potentials which fit the two-nucleon scattering data, eliminating the problem of undetermined parameters. The interaction between free nucleons is highly singular and leads to strong short-range correlations between them, therefore, this potential as such should not be used as effective interaction in the shell-model calculations, because the scattering behaviour of free nucleons is quite different from that of two nucleons in a nucleus. Under certain conditions the free two-nucleon interaction is replaced by a reaction matrix (the so-called t-matrix of Brueckner theory). These t-matrix are used as two-body effective matrix elements in the shell-model calculation.

(3) The third is the phenomenological interaction pioneered by Talmi. In this method, instead of introducing an explicit potential, one treats the matrix elements themselves as parameters to be determined directly by requiring a fit to the observed level energies. However, in order that there may not be too many parameters, very little configuration-mixing can be included and thus the matrix elements certainly represent those of an effective potential. The set of numbers obtained in this way for the matrix elements of the effective interaction, are used to predict the spectra of neighbouring nuclei.
Here we present the results of the calculation on Ni$^{68}$ and Ni$^{60}$ with the following potentials: (1) the surface-delta interaction$^{13}$, (2) the Yukawa potential with Serber and Rosenfeld type exchange dependence, (3) the t-matrix elements of the Hamada-Johnston potential$^{15}$, and (4) the nonlocal separable Tabakin potential$^{14}$. These results are compared with the results of Lawson et al.$^{21}$, who do an exact shell-model calculation for all the Ni isotopes using the t-matrix elements of Kuo$^{20}$, the phenomenologically determined two-body matrix elements of Cohen et al.$^{18}$ (EIC), and those (EIA) derived by Auerbach$^{19}$. 